The Decline of Polygyny: An Interpretation

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Abstract

Polygyny is thought to be more prevalent where male wealth inequality is greater; however, the decline of polygyny in most of the world occurred with the emergence of capital-intensive farming with unprecedented wealth disparities. The distinction between rival wealth—divided among ones offspring—and non-rival wealth—transmitted to all children irrespective of their number (like a public good)—may resolve this "polygyny puzzle." Our model's marital matching Nash equilibrium replicates the observed incidence of polygynous marriage among the Kipsigis, an African agropastoralist population. We then simulate a hypothetical transition to monogamy among the Kipsigis following an increase in the importance of rival wealth.

Keywords: fitness, polygyny threshold, monogamy, rival wealth, non-rival wealth

JEL Classification: Z1 (Economic anthropology); J12 (Marriage); N3 (Economic history: demography); P51(Comparative analysis of economic systems)

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1. Introduction

"I do not have a satisfactory explanation of why polygyny has declined over time in those parts of the world where it was once more common," wrote Gary Becker (1974, pp. 240). By the decline of polygyny we—and we trust Becker, too—mean a reduction in the prevalence of recognized simultaneous marriages of a man to more than one wife, where the children of each union have the right to inheritance of the father's wealth. This once-prevalent form of social union became a rather uncommon form of marriage in most parts of the world long prior to monogamy being socially and legally imposed. The candid assessment by Becker more than four decades ago, echoed a decade later by the evolutionary biologist Richard Alexander (1986), still rings true. This conundrum is what we term the polygyny puzzle.

It is commonly hypothesized that substantial levels of inequality in the material wealth held by males is required for polygynous marriage to be prevalent. But as we will show presently, the extent of polygyny is greater in horticultural (low technology, land abundant) economies than in more capital intensive farming and industrial economies, systems typically characterized by greater inequality. Where these latter economic systems became predominant—for example, in Europe prior to and during the first millennium of the Common Era—polygyny virtually disappeared, even among elites (Scheidel, 2009).

We provide a possible resolution to this puzzle. In view of the fact that—with the exception of the Middle East, some remaining small-scale horticultural and pastoral societies, and parts of Africa where polygyny persists to this day—the decline of polygyny generally took place well before the demographic transition, and prior to the effective state regulation of marriage (Scheidel, 2009), we propose a model in which both males and females (or their families) autonomously make voluntary marital choices to maximize their expected reproductive success. Our model links marital choices to the effects of wealth on reproductive success, and the population-level inequality with which that wealth is held.

We draw on the literature that establishes links between inheritance patterns and marriage systems (e.g., Goody, 1973; Hrdy and Judge, 1993; Fortunato and Archetti, 2010). However, our model aims to disentangle the effects of reproductively rival and non-rival forms of wealth, similar to the focus of Ingela Alger (2016) on the extent to which shared use of parental resources leads to depletion. Rival wealth—such as land or livestock—is divided among the offspring of a polygynous father. Non-rival wealth, in contrast, is analogous to a public good, in that the amount inherited by one's offspring is independent of their number. An advantageous family name or lineage, or a male's alleles that are expressed in some fitness enhancing phenotypic trait, are examples of non-rival forms of wealth. The variable relationship between polygyny and different kinds of wealth is well recognized in the ethnographic record (e.g., White et al., 1988).

The distinction between rival and non-rival wealth is important, because the extent to which wealth is rival directly affects the degree to which polygyny reduces the quantity available to each wife and her offspring in polygynous marriages. The distinction between these two kinds of wealth is illustrated in a study of English families in the late 18th and early 19th centuries (Clark and Cummins, 2015); the average (rival) material wealth of sons is significantly less in families with more sons, but this is not true of the sons' level of (non-rival) educational attainment, an advantage typically afforded—in that period—through family connections and status rather than the expenditure of rival wealth.

Our model shows that as the importance of rival wealth for reproductive success increases, the fitness returns to taking on multiple wives decreases. This trade-off affects both male demand for polygynous marriages and the availability of women to men wishing to take on additional wives. We show that for a given level of rival wealth inequality among males, the Nash equilibrium extent of polygynous marriages is less in economies in which rival wealth is more important as a determinant of reproductive success. Thus, an increase in the importance of land and other rival forms of wealth relative to non-rival forms of wealth—such as hunting ability, social network position, or genetically transmitted advantages—may have contributed to the decline of polygyny.

To explore the empirical plausibility of our proposed resolution of the polygyny puzzle, we estimate the effects of rival and non-rival wealth on the reproductive success of males among the Kipsigis, a polygynous population of African agropastoralists (Borgerhoff Mulder, 1987, 1990). We then use these estimates and the first order conditions for fitness maximization of both men and women to simulate the Nash equilibrium distribution of marriages in this population. We find a substantial correspondence between the real and predicted marital status of Kipsigis men. We then show that a hypothetical increase in the importance of rival wealth relative to non-rival wealth, holding constant the level of wealth inequality, would convert this population to one in which the only Nash equilibrium is near complete monogamy.

2. Interpretations of polygyny and its decline

Economists have attributed the decline in polygyny to a variety of dynamics including a decline in the relative value of women's labor (e.g., Boserup, 1970; Becker, 1974, 1991, consistent with Massell, 1963 and Jacoby, 1995), increasing democracy (e.g., Lagerlof, 2005), an increase in the value of child 'quality' (e.g., Gould et al., 2008), shifting distributions of wealth among men and women (e.g., De la Croix and Mariani, 2015), changes in male trade-offs between how much to invest in children and how much to invest in the payments required to secure a new wife (Bergstrom, 1996), and the success of monogomous societies in intergroup competition (Bowles, 2004; following Alexander, 1979).

As in Becker (1974, 1991), polygyny in our model arises from wealth inequality among men, similar to the polygyny threshold model used by behavioral ecologists (Verner and Willson, 1966;

Orians, 1969). However, in our model, individuals maximize their expected reproductive success, rather than their consumption of household commodities as in Becker's model. Additionally, we do not explore the effects of differences among women. In both respects, our model is closer to Bergstrom (1996). But, unlike Bergstrom, we do not assume well functioning markets for marital partners, opting instead to explore bilateral Nash equilibrium assignments of partners based on both male and female choice, given a range of customary bride-prices.

Our interpretation—and the complementary explanation of Boserup and Becker based on declines in the value of female labor—differs from several recent contributions of economists, in that it explains the *de facto* decline of polygyny in the setting in which it most likely occurred, namely a land-abundant horticultural economy in transition to more capital-intensive farming. Other accounts, by contrast, may be more relevant to its much later *de jure* elimination.

Nils-Petter Lagerlof (2005) hypothesizes that a trend towards greater equality in the last two centuries accounts for the demise of polygyny. Laura Betzig (2002, pp. 85), whom he cites, puts this view simply: "What put an end to... polygyny? Democracy." Betzig and Lagerlof may be right about the more recent formal prohibitions of polygyny; however, it seems unlikely that European democracy played a role in the historical demise of polygyny, given that the practice of polygyny had been uncommon in Europe long before the rise of democracy. Assessing the extent of polygyny in the distant past is, of course, challenging, so one cannot offer a very definitive judgement, but there is no evidence that polygyny was common in the Mediterranean prior to the Common Era and the same is true for most other parts of Europe with Celts and Germans as possible exceptions in the very early period (see Scheidel, 2009). Even among the well to do, polygyny was apparently not widely practiced long before it was legally prohibited.

Gould et al. (2008) explain the demise of polygyny by the increased value of quality relative to quantity of children as human capital increased in importance (see also Alger, 2016). In a similar vein, evolutionary social scientists often investigate the idea that parents trade off quality for quantity (e.g., Lawson et al., 2012), although evidence for this trade-off is generally quite limited (Borgerhoff Mulder, 1998; Lawson and Borgerhoff Mulder, 2016). More importantly for the questions considered here, this explanation—like Langerlof's hypothesis—places the demise of polygyny as common form of marriage in the post-demographic transition populations of the last few centuries, long after it appears to have occurred. Moreover, if our model is correct, the rise of the importance of human capital—a relatively non-rival form of wealth as our above example from 18th and 19th century Great Britain suggests—should have had the opposite effect, providing conditions more favorable to polygyny.

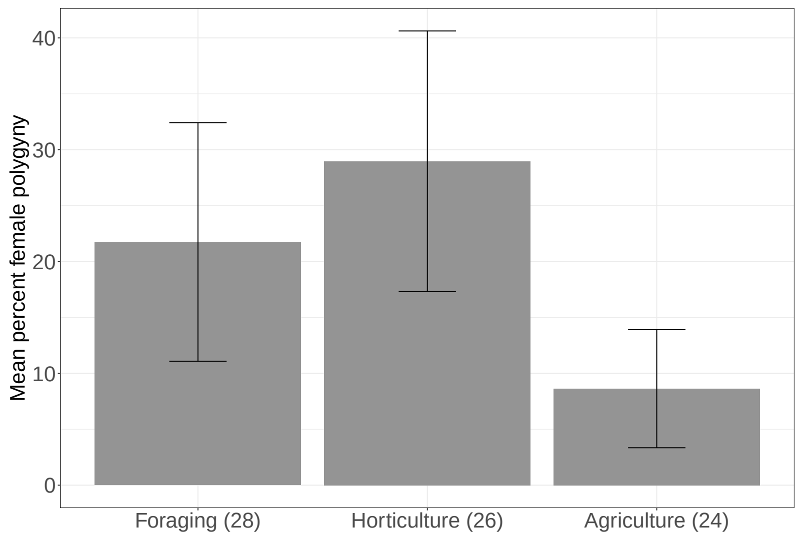
De la Croix and Mariani (2015) model the effects of an increasing fraction of rich men in a population where a democratic electorate legislates legally enforced monogamy, followed by the emergence of more permissive divorce and *de facto* serial monogamy. The empirical timing in this explanation is more plausible, as the social imposition of monogamy long post-dates its widespread acceptance. The approach of De la Croix and Mariani (2015) is similar to ours in that they investigate the effects of different kinds of wealth (with different intergenerational transmission dynamics) on polygyny, consider both male and female interests, and focus on the fraction of the wealth held by a class of rich males. However, insofar as they situate their explanation in a political-economic framework within which men and women vote for favored institutional framework, their approach has its limitations. First, unless one hypothesizes (implausibly in our judgment) that most men and women were politically represented long prior to near-universal suffrage—that is prior to the 20th century, which the authors do—it is difficult to apply this model to empirical contexts concerning the early demise of polygyny, such as the social imposition of monogamy in Rome early in the Common Era (Scheidel, 2009). Second, their approach cannot explain the limited prevalence of polygyny (meaning simultaneous not sequential marriage to multiple women) among many populations, especially foragers, that lack a class of rich men. Finally, their approach posits that women hold wealth (independently) in contexts where this is unlikely.

The comparative cross-cultural approach favored by anthropologists has also been of only limited value in resolving Becker's puzzle. The fact that practice of polygyny is more common in horticultural populations than among foragers is thought to be readily explained by two facts. First, in both theoretical (Bergstrom, 1996; Fortunato and Archetti, 2010; Grossbard, 1976; Orians, 1969; Verner and Willson, 1966; Ptak and Lachmann, 2003) and empirical studies (Borgerhoff Mulder, 1987; Flinn and Low, 1986; Sellen and Hruschka, 2004) polygyny is typically associated with what George P. Murdock (1949, pp. 206–207) termed "movable property or wealth which can be accumulated in quantity by men." Second, these forms of material wealth are more unequally held in horticultural economies than among foragers, which could explain the greater extent of polygyny in the former.

While this model has been effective in predicting the distribution of polygynous males within populations (e.g. Borgerhoff Mulder, 1990), and between foragers and horticulturalists, it accounts poorly for the contrast in the distribution of polygyny between horticultural and agricultural populations, given the high levels of inequality characterizing agricultural societies. Material wealth is more unequally held in intensive agricultural economies than it is among horticulturalists in both the ethnographic (Shenk et al., 2010; Borgerhoff Mulder et al., 2009) and the archaeological (Fochesato and Bowles, 2017) record, but polygynous marriage is even more rare in intensive agricultural populations than it is among foragers [see Fig. (1)]. Our proposed explanation of the decline of polygyny thus runs differently than Murdock's; we suggest that monogamy might have become the prevalent form of reproductive pairing when the importance of rival wealth to reproduction became large enough to offset the effects of wealth inequality in driving polygyny.

Fortunato and Archetti (2010) develop a theoretical framework linking inclusive fitness and paternity uncertainty to show that increased importance of material wealth can lead to the emergence of monogamy. However, increasing marginal returns to material wealth are needed for monogamy to emerge in their model. Our model, in contrast, shows that monogamy can emerge even with decreasing marginal returns to material wealth—this is important because our empirical estimates below and in Ross et al. (2017) cast doubt on the idea that increasing marginal fitness returns to material wealth occur in human populations.

Figure 1: Average fraction of married women who are married polygynously by production system (± two standard errors)—using the Standard Cross Cultural Sample. Rates of polygyny are measured with variable ♯872 (see notes in the Supplemental Materials). We note strong differences in the frequency of polygynous marriage between the horticultural and agricultural production systems.



A third class of explanations for the rise of monogamy dates to Alexander (1979) and has recently been formalized by Henrich et al. (2012). In their argument, a social norm favoring monogamy conferred widely-shared benefits on members of a population adopting the norm, eventually leading monogamy-practicing populations to be emulated or to prevail in intergroup contests, thereby diffusing the norm through cultural group selection (Richerson et al., 2015). Alexander (1979, pp. 72) proposed that monogamy might be "a basis for social unity in the face of extrinsic threats," in "large cohesive modern nations that wage wars and conduct defense with their pools of young men." Other cultural group selection accounts—in the spirit of Alexander's initial hypothesis—include the idea that monogamous populations had greater success in their political projects (including warfare) when elites renounced privileged sexual and reproductive access to women (Bowles, 2004), that such societies could more successfully avoid endemic sexually-transmitted infections (Bauch and McElreath, 2016), or that monogamous societies would support higher levels of investment in either physical or human capital and therfore experience more rapid growth (Tertilt, 2005; Edlund and Lagerlof, 2012), which if true would allow them to out-compete polygynous societies either militarily, by emulation, or even—in a Malthusian world—demographically.

These cultural group selection models provide plausible accounts of the decline in polygyny, complementary to our interpretation. Transitions to virtually universal monogamy occurring within a few populations by the process that we model would then provide the between group differences in marriage practices essential to a cultural group selection process in which monogamous populations could out-compete the more common extant polygynous populations.

3. The polygyny threshold with rival and non-rival wealth

3.1. The male demand for polygynous marriages

In our model, the fitness maximizing choices of both men and women determine the equilibrium extent of polygyny in a population. We consider a population consisting of N men and N women. We assume men seek to obtain a given number of wives and confer upon them the fitness-relevant resources needed for production of offspring. We consider two types of fitness-relevant resources of the male: non-rival wealth, denoted as g,e.g "genes" and rival wealth, denoted as m, e.g "material wealth".

We represent the total mating investment devoted to acquiring a wife by a cost equal to c units of the rival resource per wife—we call this a bride-price for brevity, although it includes all mating investment in addition to what is ethnographically called a 'bride-price' or 'bridewealth'. The remaining rival wealth available to each wife is thus $\frac{m-cn}{n}$, where n is the number of wives. We assume that rival wealth is allocated equally among wives who are themselves identical, and who in turn do not differentiate investment among their offspring. The non rival wealth available to each wife is g. We assume the bride price, c, is a constant independent of male wealth in this model.

We assume that the number of offspring that each man raises to reproductive age is a function of his rival and non-rival wealth, the productive labor and reproductive value provided by his wives (e.g., in food acquisition and child bearing and rearing respectively), and the number of wives he has obtained. Because wives are assumed to be identical, their labor input, l, will not contribute to differential reproductive success among men in our model. Assuming that fitness is produced according to a Cobb-Douglas function, the male's fitness, denoted by w, can be described as follows:

$$w = \underbrace{n^{\delta}}_{\text{Effective number of wives}} \cdot \underbrace{l^{\lambda} g^{\gamma} \left(\frac{m - nc}{n} \right)^{\mu}}_{\substack{\text{Fitness} \\ \text{per wife}}} \tag{1}$$

The coefficients, λ, γ, μ are the importance of each component.The exponent δ determines the rate at which the marginal fitness returns to the number of wives declines as a husband takes on more wives holding constant rival wealth available to each wife. If $\delta = 1$, then doubling the number of wives will double male fitness. However, when $\delta < 1$,there are diminishing fitness returns to additional wives. For simplicity, we normalize l to be the unit, $l = 1$, which allows the male fitness function to be rewritten as:

$$w = n^{\delta} g^{\gamma} (\frac{m - nc}{n})^{\mu} = n^{\delta - \mu} g^{\gamma} (m - nc)^{\mu} \tag{2}$$

We refer to μ as the importance of rival wealth and similarly with the other exponents and on empirical grounds we assume that $\mu < 1$ so that the marginal fitness returns to the husbands rival weatlh are decreasing.[1] We assume that $\delta > \mu$ to ensure that the elasticity of fitness on wives is positive.

An implication of this male fitness function is that there are two sources of diminishing marginal fitness returns to additional wives. One is the fact that the male's rival wealth is shared among them, reducing the rival wealth per wife. The rival nature of the male's own time and attention can be the another source of diminishing returns unrelated to the need to share a males rival wealth among wives.(Grossbard, 1980)

The fitness of a male with a single wife in this model is: $g^{\gamma}(m - c)^{\mu}$. Adding an additional wife has two effects on the male's fitness, and they are of opposite sign: 1) it increases the number of women producing his offspring, and 2) it reduces the expected reproductive success of each woman, because it reduces her pro-rated share of his rival wealth. Thus, the contribution of an additional wife to his fitness will be the derivative[2] of Eq. (2) with respect to n:

$$\frac{\partial w}{\partial n} = \frac{n^{\delta} g^{\gamma} (\frac{m-nc}{n})^{\mu}}{n} (\delta - \frac{\mu m}{m - nc}) \tag{3}$$

The first term on the right hand side, $\frac{n^{\delta} g^{\gamma} (\frac{m-nc}{n})^{\mu}}{n}$, is the fitness of a wife with $n - 1$ cowives. So the marginal fitness effect of an additional wife is just δ times this quantity minus the negative effect on the fitness of each wife that comes from the partitioning of the husband's rival wealth among a larger number of wives. The size of the negative effect is increasing (in absolute value) with μ, as can be seen from Eq. (3). So, the greater is the importance of rival wealth as a determinant of fitness, the more rapidly do the marginal fitness returns to additional wives fall

[1]For some levels of wealth in some populations, increasing returns to scale: $\lambda + \gamma + \mu > 1$, or even increasing marginal returns to rival wealth: $\mu > 1$, could occur. For example, for very low levels of material wealth, a small increment might yield very substantial fitness benefits. However, on empirical grounds, increasing marginal returns are unlikely to hold for most individuals in most populations. In Ross et al. (2017), we find that in 29 populations estimates of μ average 0.08 (0.05, 0.13).

[2]Here we allow n to be a real number. We could replace this unrealistic assumption by letting the male choose a mixed strategy with support defined by the feasible range of wives (as integers), and then allow n to represent the mean of the distribution of his weights on each number of wives.

with the number of wives.

The demand for wives, denoted by n_d, is the number of wives that maximizes male fitness, and is given by setting the right-hand side of Eq.(3) equal to 0 and solving for n:

$$n_d = \frac{m(\delta - \mu)}{c\delta} \tag{4}$$

We draw two implications from Eq. (4) about the relationship between the importance of rival wealth and the male demand for polygynous marriage. First, by rewriting Eq. (4) as $cn_d = m(1 - \frac{\mu}{\delta})$, we have the optimal level of mating investment for the male—the total amount of bride-price he should be willing to pay—which is just a fraction $(1 - \frac{\mu}{\delta})$ of his total rival wealth; this implies that mating investment is less when the importance of rival wealth is greater. Second, as $\mu \to 0$ it becomes optimal for the male to devote all of his wealth to mating investment, so that $n_d = \frac{m}{c}$. The fitness maximizing number of wives is then simply the maximum number that the male's rival wealth will "purchase" given the bride-price.

3.2. Rival wealth inequality and the female supply condition

We now model the conditions under which women would be willing to engage in polygynous marriage. Specifically, we determine the fraction of women willing to be married polygynously, conditional on the bride-price and the distribution of male rival and non-rival wealth. Each male in the population, we have just seen, will seek to marry n_d wives. However, it remains to be determined if there will be women available to meet each male's demand.

To address this, we need to consider the prospective brides' choice between being the first wife of a given man, or the n^{th} wife of some other man. Being the n^{th} wife of a man will only be attractive to a woman if there are men sufficiently rich that she can achieve higher fitness in a polygynous marriage to a wealthy man than in a monogamous marriage to a poor man. As a result, where male rival wealth inequality is limited, even rich men may not be able to find willing polygynous partners, even though their fitness would be greater in a polygynous marriage.

We do not consider the effects of an age-structured population or differential age at marriage. We further simplify the model by assuming that there are only two types of males, poor and rich, the latter constituting a fraction θ of the population of males. We denote the rival and non-rival wealth of the poor and the rich using subscripts p and r respectively: (m_p, g_p), (m_r, g_r). There are two forms of marriage, monogamy and n-polygyny.

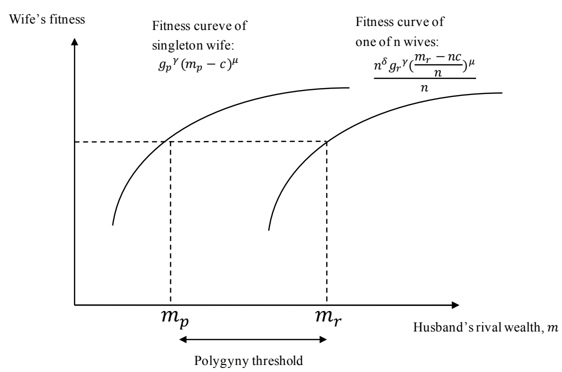
A woman is willing to be the n^{th} wife of a rich man, rather than a singleton wife of a poor man as long as the following condition holds:

$$\underbrace{g_p^{\gamma}(m_p - c)^{\mu}}_{\text{Fitness as a singleton wife}} \quad \leq \quad \underbrace{\frac{n^{\delta} g_r^{\gamma} (\frac{m_r - nc}{n})^{\mu}}{n}}_{\text{Fitness as one of n wives}} \tag{5}$$

We consider the rival wealth of the rich and the poor such that $m_r > nc$ and $m_p > c$. The inequality in Eq. (5) is the female supply condition for n-polygyny because it indicates whether a woman is willing to be the n^{th} wife of a man. From this condition, we can determine the level of rival wealth inequality that—holding constant non-rival wealth inequality—will permit n-wife polygynous marriage.

We can use the female supply condition to derive what we call (adapting a term from anthropology) the n-polygyny threshold. The n-polygyny threshold is the minimum rival wealth difference between the rich and the poor that will permit n-wife polygynous mating. Fig.2 shows an n-polygyny threshold when the equality condition in Eq.(5) holds. The figure can be read as follows. Suppose the income of the poor man is m_p as indicated on the X axis; then we ask how rich must the rich man be in order that his n wives will have the same fitness as the singleton wife of the poor man. The difference between that number (m_r) and the poor man's wealth (m_p) is the polygyny threshold.

Figure 2: Women's fitness, men's rival wealth, and the n-polygyny threshold. The horizontal dotted line denotes a case where the fitness of a singleton wife of the poor is the same as the fitness of each of the n multiple wives of the rich—i.e. where $g_p^\gamma (m_p - c)^\mu = n^{\delta-1} g_r^\gamma ((m_r - nc)/n)^\mu$



To define the supply of women to polygynous marriages, females are assigned to either a rich or a poor husband in such a way as to maximize their fitness. In our simplified example, the first θN females would obviously choose to be assigned to one of the rich men. Then, female $(\theta N + 1)$ must choose to either be a second wife of one of the rich men, or to marry one of the poor men. If the fitness of being the second wife of a rich man is higher than the fitness offered by monogamous marriage to one of the poor men, then all the rich will get the second wife, as long as there are females available. As long as unmarried females remain, a similar process continues, and so long as the inequality in Eq. (5) is satisfied, women will choose to be the n^{th} wife of the rich.

Let n_s be the maximum value of n satisfying Eq. (5), then n_s is the maximum number of women willing to be married to each rich man. According to the female supply condition for n_s-polygyny, $\theta N n_s$ women will seek polygynous marriage to θN rich men and (if male demand is sufficient) the rest of the women will be monogamously married to the poor men or remain unmarried. This condition will hold exactly, so long as the percentage of polygynously married women, $\frac{\theta N n_s}{N} = \theta n_s$, is strictly less than 1, implying that at least some women marry monogamously.

The assignment is a Nash equilibrium in the unilateral sense that no woman would prefer to change her assignment given the existing assignment of the $N-1$ other women. Female availability alone, however, does not determine the equilibrium assignment, because the Nash assignment by the women may be inconsistent with the male demand for polygynous marriage. To determine the bilateral Nash equilibrium assignment, we consider both the male demand and the female supply conditions.

3.3. Mating equilibrium: determination of n-polygyny

A man whose demand for wives is $n_d \geq 1$ can acquire his desired number of wives only if there are women willing to be one of his n_d partners. Thus, the marriage equilibrium will be determined jointly by the male demand and the female supply conditions, given the levels of wealth inequality and the importance of each wealth component.

To find the n-polygyny level of the society, we solve the equilibrium mating in terms of the rich man's fitness maximization problem given the constraint implied by the female supply condition. Let $w(n) = n^\delta \ g_r^\gamma \ (\frac{m_r - nc}{n})^\mu$ be the function that defines fitness for a rich male, and let $\phi(n) = g_p^\gamma(m_p - c)^\mu - n^{\delta-1} g_r^\gamma(\frac{m_r - nc}{n})^\mu$ be the fitness advantage of being the singleton wife of a poor man relative to being among the n wives of the rich man.

We then have the following maximization problem:

$$\max_n \ w(n) = n^\delta \ g_r^\gamma \ (\frac{m_r - nc}{n})^\mu \tag{6}$$
$$s.t. \quad \phi(n) = g_p^\gamma(m_p - c)^\mu - n^{\delta-1} g_r^\gamma(\frac{m_r - nc}{n})^\mu \leq 0$$

where female supply is constrained. Since we have a nonlinear optimization problem with an inequality constraint, we employ the Kuhn-Tucker conditions to solve the problem. We provide the solution in Proposition 1.

Proposition 1. *By solving the optimization problem in Eq. (6), we find the equilibrium n-polygyny level, n^*, to be as follows:*

$$(i) \ \textit{male demand is binding: } n^* = n^d = \frac{m_r(\delta - \mu)}{c\delta} \tag{7}$$
$$(ii) \ \textit{female supply constraint is binding: } n^* = n^s \textit{ such that } \phi(n^s) = 0 \tag{8}$$
$$(iii) \ \textit{market clears: } n^* = n^{**} \textit{ such that } n^{**} = n^d = n^s \tag{9}$$

Proof. We write the Lagrangian as:

$$\mathcal{L}(n, \lambda) = n^\delta \ g_r^\gamma \ (\frac{m_r - nc}{n})^\mu - \lambda[g_p^\gamma(m_p - c)^\mu - n^{\delta-1} g_r^\gamma(\frac{m_r - nc}{n})^\mu] \tag{10}$$

By applying the Kuhn-Tucker first order conditions, we have the following set of conditions:

$$(a)\frac{\partial \mathcal{L}}{\partial n} = \frac{\partial w}{\partial n} - \lambda\frac{\partial \phi}{\partial n} = 0$$
$$(b)\lambda\phi = 0,$$
$$(c)\lambda \geq 0;$$
$$(d)\phi \leq 0$$

We consider the situation where the female supply constraint is either binding ($\lambda > 0$) or not ($\lambda = 0$). We can find n^* satisfying the following conditions for each the three cases below: (i) male demand constraint binding; (ii) female supply constraint binding, and (iii) both binding, namely market clearing.

$$\begin{aligned} &(i) && \frac{\partial w}{\partial n} = 0;\ \lambda = 0 \\ &(ii) && \phi = 0;\ \lambda = \frac{\partial w}{\partial n} / \frac{\partial \phi}{\partial n} \\ &(iii) && \frac{\partial w}{\partial n} = 0;\ \phi = 0 \end{aligned}$$

For case (i), we find n^* from $\frac{\partial w}{\partial n} = 0$, which is simply the male demand that maximizes his reproductive success. For case (ii), we find n^* from $\phi = 0$. For case (iii), n^* satisfies both male demand and female supply constraints. ■

Since female supply constraint is binding in case (ii), the rich gets fewer wives than would maximize their reproductive success. By contrast, in case (i), there are sufficient women willing to be the wives of the rich. It might be that c in a well functioning market would be negotiated so that the market clears, but we will assume herein that bride-price is determined by custom, or in some other exogenous manner. This is consistent with ethnographic evidence; in many cases bridewealth is fixed (Goody and Tambiah, 1973; Anderson, 2007), but even where bargaining occurs, payments show quite limited variability within culturally prescribed norms (Borgerhoff Mulder, 1995), or are clearly related to exogenous economic factors (Goldschmidt, 1974).

It is clear from Proposition 1, that given a constant level of non-rival wealth inequality and a fixed bride-price, higher rival wealth inequality leads to elevated levels of polygyny (larger n^*). To see why, assume we change the rival wealth vector to new values, $(m'_p,\ m'_r)$, to yield a higher level of wealth inequality, such that $m'_p \leq m_p$, and $m'_r > m_r$. The equilibrium n^* will be higher at $(m'_p,\ m'_r)$ for all three cases. For the case (i): $\mathrm{n}^{\delta-1} g_r^{\gamma}(\frac{m'_r - nc}{n})^{\mu} > n^{\delta-1} g_r^{\gamma}(\frac{m_r - nc}{n})^{\mu}$ and $g_p^{\gamma}(m'_p - c)^{\mu} < g_p^{\gamma}(m_p - c)^{\mu}$. It is obvious that since the fitness of a singleton wife is lower while the fitness of the one of n wives is higher, women prefer to marry to a rich male. For the case (ii) : $\frac{m_r'(\delta-\mu)}{c\delta} > \frac{m_r(\delta-\mu)}{c\delta}$, i.e. the rich male's demand for wives increases as his rival wealth increases.

4. Rival wealth and the limits to polygyny

The importance of rival wealth (μ) has an impact on the n-polygyny threshold. By rearranging Eq. (5), and taking the natural logarithms of both sides, we have:

$$\log m^* \geq \frac{1-\delta+\mu}{\mu} \log n - \frac{\gamma}{\mu} \log g^* \tag{11}$$

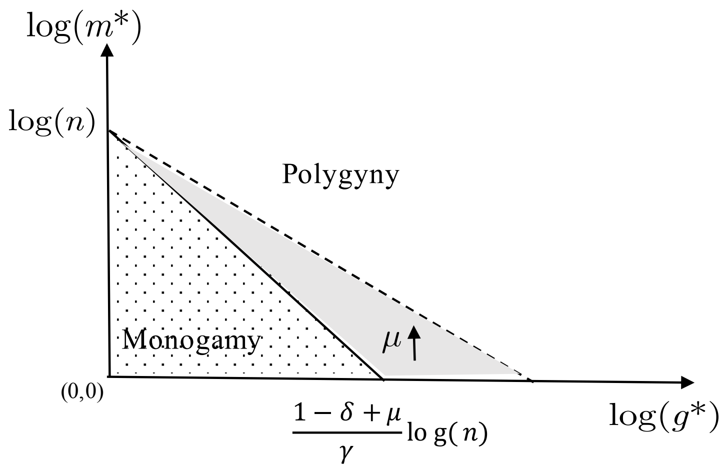
where $m^* = \frac{m_r - nc}{m_p - c}$, and $g^* = \frac{g_r}{g_p}$. Here we define m^* as the rival wealth threshold of n-polygyny under the female supply constraint. Given g^*, the rival wealth threshold increases as μ increases.

Figure 3 illustrates the polygyny threshold and the effect of an increase in the importance of rival wealth, μ. Notice first, that in contrast to the usual representation of the threshold as simply a difference in material wealth, here it is two dimensional, giving the combinations of inequality in both rival and non rival wealth sufficient to allow polygyny to be attractive to females. Thus, as would be expected, a decrease in the inequality level of one wealth type requires an increase in the inequality level of the other wealth type if polygynous marriage is to remain fitness-enhancing for women.

We can see that if there are no differences in non-rival wealth across the male types ($g_p = g_r$), then inequality in rival wealth alone will determine the marriage outcomes in the population, with the threshold being located at $\log(n)$, as is indicated by the intersection of the threshold lines and the y-axis on Figure 3. If as appears to be the case empirically, the two forms of wealth are positively correlated so that $g_p < g_r$, then the n-polygyny threshold will be less than $\log(n)$, as is evident from the downward sloping nature of the threshold line.

The important result for our interpretation of the decline of polygyny is that as the importance of rival wealth increases, the rival wealth threshold of n-polygyny (m^*) also increases. This means that as rival wealth becomes more important to reproduction, higher levels of wealth inequality are required to make polygyny a fitness enhancing strategy. Additionally, when the importance of rival wealth is elevated, the threshold declines less as the rich male's non-rival wealth increases. This occurs because increases in μ diminish the relative importance of the non-rival forms of the husband's wealth for which each wife's share is undiminished by the number of wives. As a result, as the importance of rival wealth for fitness increases, the fitness cost to a woman of joining a polygynous relationship—and having access to only a proportional share of the husband's rival wealth—also grows. The effect of the increase in the importance of rival wealth, μ, on marriage forms is shown in Figure 3. As μ increases holding constant the importance of non-rival wealth γ, the parameter space favoring monogamous marriage expands.

Figure 3: The n-polygyny threshold. The origin represents equality in both wealth dimensions. The solid line indicates the polygyny threshold at some given values of μ and γ. Along the threshold, being a singleton wife of the poor is indifferent from being the n-th wife of the rich. The y-intercept at $\log(n)$ is derived from Eq. (11). When the importance of rival wealth, μ, increases, holding constant the importance of non-rival wealth, γ, polygyny threshold tilts upwards, as indicated by the dotted line. The parameter space favoring monogamous marriage expands..



It is clear from Eq.(4) that as the importance of rival wealth to reproduction increases, the fitness maximizing number of wives for a given man decreases. With respect to the female supply condition, we see that an increase in μ raises the n-polygyny threshold, meaning that it takes greater rival wealth inequality to guarantee female supply for n-polygyny. Thus, both male demand and female availability for n-polygyny diminish as μ grows. We establish these claims formally in Proposition 2.

Proposition 2. *An increase in the importance of rival wealth, μ, will decrease the extent of polygyny.*

Proof. We differentiate n^* in Eq.(8) and Eq.(7) with respect to μ.

(i) when the male demand is binding, we have

$$\frac{dn^*}{d\mu} = -\frac{m_r}{c\delta} < 0$$

(ii) when the female supply constraint is binding, we do not have a closed form solution for n^*. Let $\phi(n^*, \mu) = 0$. By total differentiation we have:

$$\frac{dn^*}{d\mu} = -\frac{\frac{\partial \phi}{\partial \mu}}{\frac{\partial \phi}{\partial n^*}} = \frac{n^{\delta-1} g_r^{\gamma} (\frac{m_r - n^* c}{n^*})^{\mu} \log(\frac{m_r - n^* c}{n^*}) - g_p^{\gamma} (m_p - c)^{\mu} \log(m_p - c)}{g_r^{\gamma} n^{\delta-\mu-2} (m_r - n^* c)^{\mu-1} [\mu n^* c + (m_r - n^* c)(1 - \delta + \mu)]} \tag{12}$$

The sign of the denominator $\frac{\partial \phi}{\partial n^*}$ is positive because we have $m_r - n^* c > 0$ and $1 - \delta + \mu > 0$.

The sign of the numerator will be determined by the size of the two terms, $\log(\frac{m_r - n^* c}{n^*})$ and $\log(m_p - c)$ because we have $n^{\delta-1} g_r^{\gamma} (\frac{m_r - n^* c}{n^*})^{\mu} = g_p^{\gamma} (m_p - c)^{\mu}$ from $\phi(n^*, \mu) = 0$. On empirical grounds we assume that men with rival wealth also have more non rival wealth, $g_r > g_p$. So, in equilibrium, the rival wealth of the poor must exceed the per wife rival wealth of the rich, i.e. $\frac{m_r}{n^*} < m_p$, to provide a singleton wife an offsetting increase in the fitness. If $\frac{m_r}{n^*} < m_p$, then $\log(\frac{m_r - n^* c}{n^*}) < \log(m_p - c)$, so we have $\frac{dn^*}{d\mu} < 0$.

(iii) when the market clears, we also have $\frac{dn^*}{d\mu} < 0$ from the results of (i) and (ii).

Thus, for any given levels of rival and non-rival wealth ($m_r > m_p$ and $g_r > g_p$), an increase in μ will decrease n^*, and so too the fraction of polygynously married women in the population. ■

5. Polygyny in an African agropastoralist population

Now we ask: is our theoretical model capable of predicting the distribution of marriage types in a natural-fertility human population? Specifically, we focus on modeling marriage outcomes in a population of Kenyan agropastoralists, and investigate how these outcomes are predicted to change as a function of hypothetical changes in the importance of rival wealth.

5.1. Wealth and reproductive success among the Kipsigis

The Kipsigis are a farming and cattle herding population living in southwestern Kenya (Kericho, Rift Valley Province), who have a tradition of polygyny (Borgerhoff Mulder, 1987); the extent of polygyny during the time of ethnographic investigation was high, with some men marrying up to 12 wives and others remaining wifeless (Borgerhoff Mulder, 1990). At the time of study (1981-1984), marriages were arranged mostly by parents, whose sons would offer a bride-price to the parents of their marriage interests. In the Kipsigis, there is a cultural norm that requires husbands to split their wealth equally among wives (Peristiany, 1939); this behavior is consistent with the assumptions of our model. Land and livestock are the primary sources of rival wealth for the Kipsigis' subsistence (Manners, 1967; Mwanzi, 1977), and height and number of cattle trading partners are hypothesized to constitute major forms of non-rival wealth.

To estimate the importance of each wealth type to reproductive success in the Kipsigis, we use a Cobb-Douglas function, which implies that the log of predicted reproductive success in individual i, ψ_i, is given by:

$$\log(\psi_i) = \log(w_0) + \gamma \log(G_i) + \mu \log(M_i) + \beta \log(N_i + \eta) + \epsilon \log(E_i) \tag{13}$$

where each male's exposure time to risk of reproductive success (i.e., years lived in the age range between 15 and 60 years) is given by E_i, non-rival wealth is given by G_i, rival wealth is given by

M_i, and number of wives is given by N_i. Note that the parameter $\eta \in (0, 1)$ is used to adjust the zero of the wife vector, allowing men with no wives to produce offspring; as such, η represents the effective exposure to mating chances outside of marriages, and is constrained by fiat to be less than the mating chances inside of a marriage. Since we do not have bride-price data on the majority of these men's marriages, we drop bride-price from the estimation. Eq. (13) is thus an empirically estimable approximation to the male reproductive success function described in Eq. (2).

Rival wealth holdings are approximated using land-holding size and the number of cattle owned by a man. Non-rival wealth holdings are described using height (a heritable indicator of physical stature and the social benefits it confers)[3], as well the number of men in a male's cattle-sharing network (a common insurance mechanism among pastoralists, indicative of the social connections of an individual). To convert wealth proxies into a single kind of rival or non-rival currency, we integrate endogenously estimated shadow prices, ς, into Eq. (13), using the following identities:

$$G_i = (P_i + \varsigma_1 H_i) \tag{14}$$

$$M_i = (C_i + \varsigma_2 L_i) \tag{15}$$

where the variables are cattle trading partners P_i, height H_i, cattle C_i, and land L_i. The likelihood function for reproductive success, R_i, is then defined using a Negative Binomial outcome distribution:

$$R_i \sim \text{Negative Binomial}(\psi_i B, B) \tag{16}$$

where the term $\psi_i B$ defines the shape parameter of a Gamma distribution, and B defines the inverse scale parameter. The Negative Binomial model thus follows a Gamma-Poisson parameterization, as has been proposed specifically for use in modeling fertility outcomes in polygynyous societies (Spencer, 1980). Our model structure, through its use of shadow prices, implies that non-rival wealth is measured in units of cattle partner equivalents, and rival wealth is measured in cattle equivalents. To measure reproductive success, we use the number of children surviving to age 5. In order to use all cohorts of the adult male population, all relevant measures—wives, cows, and land—are age-adjusted in a Bayesian framework to represent their predicted values at the age of 60 years (see Ross et al., 2017).

We use the full posterior estimates of G, age-adjusted M, μ, and γ to ground our simulation models of Kipsigis marriage dynamics. We run our simulation on each Markov Chain Monte Carlo sample, and then summarize the resultant predictive distribution. The posterior mean estimates

[3]The height measure is normalized by subtracting 140 cms so that its zero measure is an estimate of the least height consistent with successful reproduction.

Table 1: Descriptive statistics of age-adjusted Kipsigis data. Measures reflect the hypothetical distribution of wife number and wealth-holdings at age 60 were all individuals to reach that age. Rival wealth is in cattle equivalent units, and non-rival wealth is in cattle trading partner equivalent units. Values are posterior median and 95% confidence estimates of the mean quantity of the indicated column label in each of the subsets given by the row labels.

Marriage Form	Number of Males	Rival Wealth	Non-Rival Wealth
Unmarried	73 (63, 82)	43.4 (23.1, 80.8)	6.1 (2.9, 12.5)
Monogamy	313 (275, 357)	71.0 (34.5, 136.7)	6 (2.8, 12)
Polygyny	463 (413, 503)	137.3 (70.1, 278.3)	6.1 (3.1, 12.4)
n-wives=2	291 (269, 316)	112.8 (56.3, 225.6)	6.1 (3.1, 12.4)
n-wives=3	118 (92, 141)	139.5 (66.2, 279.2)	6.1 (2.8, 12.2)
n-wives=4	39 (28, 52)	217.8 (98.2, 435.5)	6.1 (2.7, 12.1)
n-wives=5	8 (3, 13)	169.3 (77.8, 362.8)	6.0 (2.6, 12.2)
n-wives=6	2 (1, 5)	197.2 (79.8, 429.6)	6.2 (2.2, 12.6)
n-wives=7			
n-wives=8	1 (1, 2)	1238.1 (443.2, 2417.2)	7.6 (4.1, 14.6)
n-wives=9			
n-wives=10	1 (1, 1)	919.2 (441.7, 1728.9)	8.4 (5.5, 15.3)
n-wives=11			
n-wives=12	1 (1, 1)	1278.3 (581.7, 2492.8)	8.4 (4.1, 17.6)

and 95% confidence intervals (in parentheses) of the regression parameters are $\beta = 0.49$ (0.41, 0.56), $\mu = 0.18$ (0.14, 0.22) and $\gamma = 0.36$ (0.10, 0.64). The substantially greater importance of non-rival wealth is robust to a number of alternative estimation strategies (including alternate age-adjustment methods).

Note that the estimate of β is substantially less than one minus the estimate of μ, the value implied by Equation (2) if the only source of diminishing marginal fitness returns were the necessity to share the husband's rival wealth among the wives. This result—which we find more generally in a cross cultural study of fitness, polygyny, and rival wealth in 29 populations—indicates that there are diminishing marginal returns to additional wives accounted for by some kind of rival resource—the husband's own time and attention for example—that we have not accounted for in the model. For more detailed information on the Kipsigis data and statistical methodology see Supplementary Materials and Ross et al. (2017).

In Table (1) we present estimates of the hypothetical extent of polygyny among people at age 60 were all individuals in the population to reach that age. This is not a measure of the extent of polygyny in the real population with its empirical age distribution, and is not comparable with unadjusted empirical data. Rather these measure provide a set of values against which our simulations of the same quantity can be assessed. We report the distribution of men in each marital

Table 2: Polygyny among the Kipsigis. Measures reflect the hypothetical extent of polygyny at age 60 were all individuals to reach that age. Values are posterior medians and 95% confidence estimates.

Measure	Estimate
Fraction of age-adjusted women in polygyny	0.79 (0.75, 0.83)
Fraction of age-adjusted men in polygyny	0.55 (0.49, 0.59)
Fraction of age-adjusted monogamous households	0.37 (0.32, 0.42)
Gini of age-adjusted wives	0.33 (0.32, 0.34)
Average number of age-adjusted wives per man	1.76 (1.64, 1.88)
Variance in number of age-adjusted wives per man	1.36 (1.21, 1.50)

type and the average rival and non-rival wealth for each.[4] There were 463 (413, 503) polygynously-married men, 313 (275, 357) monogamously-married men, and 73 (63, 82) unmarried men in this age-adjusted data-set. Among those men with polygynous marriages, we also report the number of men with n-wives. Most men have 2 or 3 wives, and only few men with extremely large wealth holdings have more than 4 wives. As we can see from Table (1), men having more wives tend to have greater rival and non-rival wealth on average. Our age-adjusted data set includes 848 men who are married to a total of 1492 (1393, 1603) age-adjusted wives. These numbers do not imply a sex imbalance in the population, but rather reflect the implied empirical circumstances where polygyny among Kipsigis elders creates a demographic queue of younger unmarried men (Spencer, 1980)—accordingly, many of the wives generated by our age-adjustment methods can be assumed to come from exogenous, younger cohorts, which in a growing population can be particularly large.

In Table (2) we provide several measures of the extent of Kipsigis polygyny in our hypothetically age-adjusted data-set. We first present the extent of polygyny as measured by the fraction of males and females in polygynous marriages. In the mathematical model presented in Section (3), the fraction of men married polygynously was simply the fraction of rich men in the population, who each had n wives; in this empirical case, however, wife number and wealth are variable across men. The total fraction of men with multiple wives (after age-adjustment) is 0.55 (0.49, 0.59), and the empirical sample statistic for this measure among men age 60 or older is 0.62. The total fraction of age-adjusted wives in polygynous marriages (i.e. wives with cowives) is 0.79 (0.75, 0.83), and the empirical sample statistic for this measure among the wives of men age 60 or older is 0.88.

A second—and quite different—indicator of polygyny is how unequally women are distributed among men. To estimate this quantity, we calculate the Gini coefficient of the distribution of the number of wives attributed to each male, and we call it the wife Gini. In our age-adjusted data set, this is a measure of male inequality in the number of wives, not the extent of polygyny *per se*.

[4]The correlation of these data vectors across all men is small, $\rho = 0.05$ (-0.01, 0.11).

The Gini coefficient, 0.33 (0.32, 0.34), on age-adjusted number of wives for the Kipsigis is quite low, especially when considering that the extent of polygyny is very high. Even though polygyny is widely practiced by the Kipsigis, after age-adjustment to control for demographic effects influencing lifetime acquisition of wives—as recognized ethnographically long ago (Spencer, 1980)—the number of wives achieved by in-sample males is fairly evenly distributed compared to the distribution of rival wealth for which the Gini coefficient is 0.46 (0.44, 0.49). Using the mean number of age-adjusted wives and the corresponding Gini coefficient to compute the average mean difference among all pairs of Kipsigis men, we see that age-adjusted Kipsigis men differ on average by slightly under a half of a wife.

5.2. Simulating a Nash equilibrium Kipsigis marriage distribution

We use a simulation model to determine the predicted Nash equilibrium assignment of marital partners in the Kipsigis, conditional on the posterior estimates of the regression parameters (μ and γ) and wealth vectors (G and M) as calculated from the empirical data. We iterate the estimation procedure over a range of empirically plausible bride-prices.

We use our model to calculate fitness with Eq. (2), and from this we calculate male demand, $n_d = \frac{m(1-\mu)}{c}$. Since our simulation must use integral values for number of wives desired, $\hat{n}_d$, we calculate each man's fitness at both the floor and ceiling of n_d, $\lfloor n_d \rfloor$ and $\lceil n_d \rceil$, and define $\hat{n}_d$ to be the value which leads to higher fitness.

For female supply, we simulate a voluntary assignment of women over men as explained in Section (3.2). In the mathematical model, there were only two male wealth types, but in this empirically-parametrized example, each man has a unique combination of rival and non-rival wealth. Thus, the marital assignment algorithm is defined so that each woman compares her expected fitness under marriage to each and every male in the population who has unsupplied demand, and then selects marriage with the male with whom she will have maximized fitness.

Let n_i be the number of wives of the i^{th} man, and $\omega(i, n_i)$ be the expected fitness of a woman married to male i as his n^{th} wife. Since every wife shares her husband's wealth equally, all of the wives of a given man will have the same fitness. Suppose a yet to be assigned woman is to choose whom to marry given the distribution of wives already assigned to men. She will want to be wife $(n_i + 1)$ of male i—and male i will accept her proposal—if and only if:

$$\omega(i, n_i + 1) = \max[I_1\omega(1, n_1 + 1), I_2\omega(2, n_2 + 1), \ldots, I_n\omega(N, n_N + 1)] \quad (17)$$

and:

$$\sum_{i=1}^{N} I_i > 0 \quad (18)$$

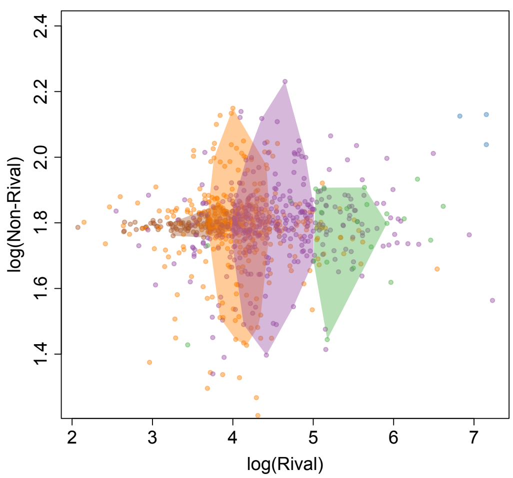
where the terms $(I_1, I_2, \ldots, I_N)$ are binary indicators that take value 0 if a male is already supplied with as many wives as he demands, and value 1 if he still demands at least 1 additional wife; this model works similarly to other marital matching algorithms (e.g. Bergstrom and Real, 2000). Under the assumptions of our model, men care only about wife number—not identity—so the fact that assignment in our algorithm is bilaterally voluntary means that no man would prefer to have fewer wives than in the equilibrium allocation, and no woman would prefer to switch to a new partner. A male may, however, prefer to have additional wives if the female supply constraint is binding, and there may be a surplus of unmarried women if male demand is filled before all women are distributed.

To select a numerical value for the bride-price, we use two methods. In the first, we vary the bride-price for each $c \in \{10, 20, \ldots, 200\}$, calculating the predicted equilibrium number of wives, n_i, for each man, for each bride-price. Using this method, we find that when the bride-price is: $c \approx 60$, several metrics measuring the divergence between predicted and actual wives reach their minimum. Accordingly, we present the simulation results at $c = 60$. Notice from Table (1) that when $c = 60$, the poorest men must exhaust their wealth in order to pay the bride-price. In the second method, we allow bride-price to depend on a male's rival wealth. This model makes empirical sense if higher ranking males pay larger bride-prices to secure 'higher quality' mates, which was quite typical among the Kipsigis until the 1980s (Borgerhoff Mulder, 1995). In this case, we vary the bride-price for each $c_i \in \{m_i^{0.1}, m_i^{0.2}, \ldots, m_i^{0.9}\}$, calculating the predicted equilibrium number of wives, n_i, for each man, for each bride-price. Using this method, we find that when the bride-price is: $c_i \approx m_i^{0.7}$, the divergence between predicted and actual wives reaches its minimum. Accordingly, we present the simulation results at $c_i = m_i^{0.7}$.

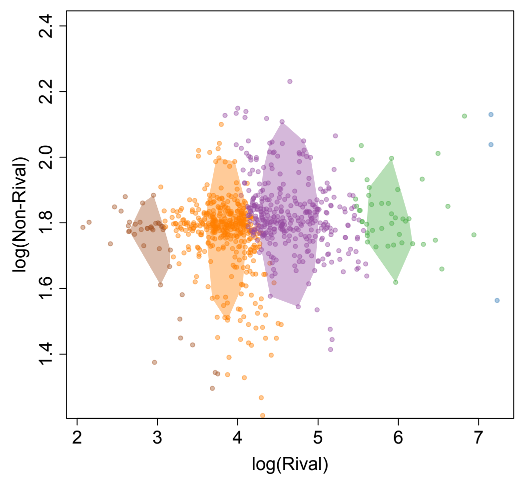
The marital status of both observed (age-adjusted) Kipsigis individuals, and the predicted marital status of these same individuals based on our simulation models are presented in Fig. (4). We show the marital status of all men using the same log(non-rival) and log(rival) wealth plane on which we earlier illustrated the polygyny threshold. We note, in Fig. (4), the existence of approximate n-polygyny thresholds for the empirical age-adjusted Kipsigis data and explicit n-polygyny thresholds for the simulation results, as can be seen by noting the spatial patterning of points on the wealth plane. The empirical data in Fig. (4a) have a more scattered distribution of marriage forms than the simulation results, but nevertheless suggest the presence of thresholds which correspond to those from our model.

In the simulation with a fixed bride-price (Fig. 4c), we over-predict the number of wives of wealthy men, and predict many more unmarried men than we observe empirically. These excess unmarried men are seen in the simulation under both high and low fixed bride-prices, because either: 1) the ultra-wealthy simulated Kipsigis men take many more wives than they do empirically, leaving no supply for the poorer males (low bride-price), or 2) because the poorer men simply cannot pay a bride-price of a level that is sufficient to prevent the ulta-wealthy men from

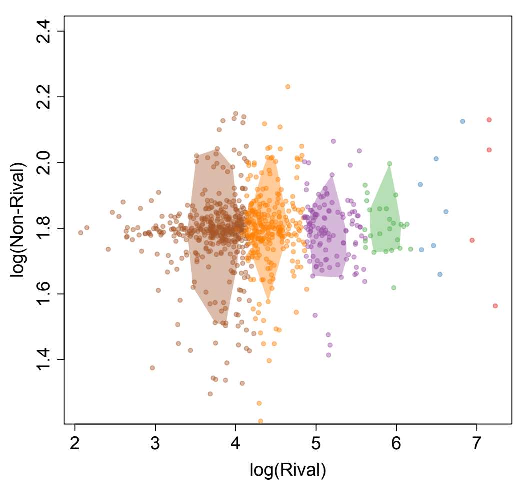
Figure 4: Posterior medians of the real and predicted marriage distributions of Kipsigis men on the log(non-rival) by log(rival) wealth plane. For the simulation in Frame (4b), bride-price was set to $c_i = m_i^{0.7}$, and for the simulation in Frame (4c) it was set to $c = 60$. The full posterior estimates of G, M, μ and γ were used, with $\mu = 0.18$ (0.14, 0.22) and $\gamma = 0.36$ (0.10, 0.64). Each dot denotes a man and each shaded region denotes the minimal convex polygon that encloses 60% of the data points for each marriage-type category. Brown points denote unmarried men, orange points denote monogamously married men, purple points denote polygynously married men (with 2-3 wives), green points denote highly polygynous men (with 4-6 wives), blue points denote ultra-polygynous men (with 7-12 wives), and red points denote men with 13 or more predicted wives. There is a strong correspondence between the data in Frame (4a) and the predictions in Frame (4b), and a weaker correspondence with the predictions in Frame (4c). We note that there is more overlap between marriage-type categories in the age-adjusted data than in the model predictions.



(a) Real (age-adjusted) marriage distribution.



(b) Predicted marriage distribution. Variable bride-price.



(c) Predicted marriage distribution. Fixed bride-price.

dominating the marriage market (high bride-price). Although predictive error is much better than random—average error of 1.02 wives, versus the error of 1.31 (1.26, 1.37) expected under random allocation of the same number of wives across the same number of men—and the correlation between real and predicted wives substantial—$\rho = 0.52$—the model over-predicts the wives of the wealthiest 5% of men by a large margin (4.16 wives on average). In the simulation with a variable bride-price (Fig. 4b), we make much more accurate predictions—an average predictive error of 0.70 wives per man, and a correlation of $\rho = 0.47$ between real and predicted. Under this simulation, we also reduce the over-prediction of wife number among the wives of the wealthiest 5% of men to only 1.58 wives. Empirically, however, although Kipsigis bridewealth payments and overall investments in marriage vary in size (Borgerhoff Mulder, 1995), such investments are constrained by custom to such an extent that the original ethnographer claimed bridewealth to be a fixed payment (Peristiany, 1939). This suggests that some other factor, exogenous to the model, is limiting polygyny among very wealthy males, an implication we return to in the conclusion.

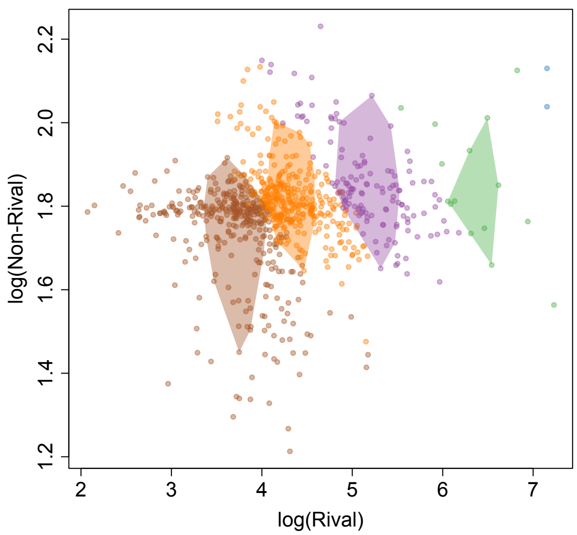
6. A simulated decline in polygyny

Now, we investigate how the importance of rival wealth affects the distribution of marriages in a simulated population. We use the empirical levels of rival and non-rival wealth in the Kipsigis for this hypothetical population, but because we want to study the extent of polygyny in some observable (not hypothetically age-adjusted) population, we assign an equal number of women as men (i.e. 848 men and 848 women).

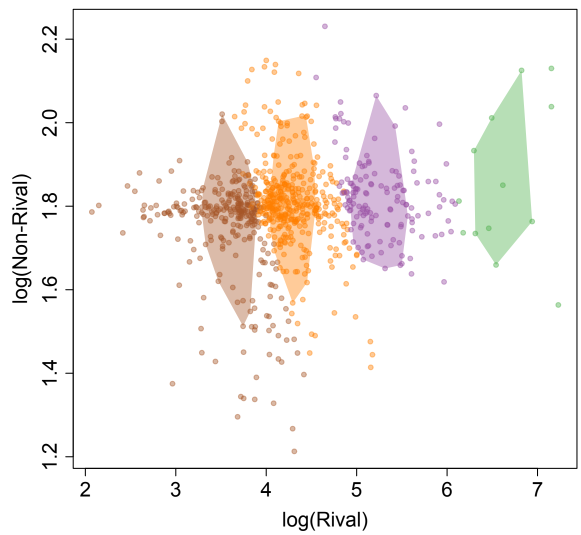
We vary the importance of rival wealth keeping the sum of two exponents constant such that: $\mu + \gamma = 1$ (i.e. we investigate the effect of changing μ while maintaining constant returns to scale). Figs. (5a), (5b), (5c), and (5d) show the predicted marital status of all men when: $\mu = 0.2$, $\mu = 0.4$, $\mu = 0.6$, and $\mu = 0.8$, respectively. We have two important observations. First, we note that as the importance of rival wealth increases, there is more monogamy and less polygyny; the highly-polygynous men disappear as μ increases toward 1. Second, we note that the slope of the boundaries between marriage forms become steeper as μ increases. In our model, the slope of the polygyny threshold is equal to the rate of substitution between non-rival wealth and rival wealth in enabling a man to be polygynously married, and will thus be steeper as μ increases. At higher levels of μ, men require a smaller increase in rival wealth as substitution for a decrease in non-rival wealth in order to keep the same number of wives. When $\mu = 0.8$, the polygyny thresholds are approximately vertical, implying that the number of wives of a given male is almost fully determined by his rival wealth.

Finally, we present various polygyny measures, as discussed in Section (5.1), with respect to μ in Fig. (6). As the model predicts, the fractions of men and women that are polygynously married, as well as the wife Gini, are decreasing in μ. We also show that the fraction of monogamous

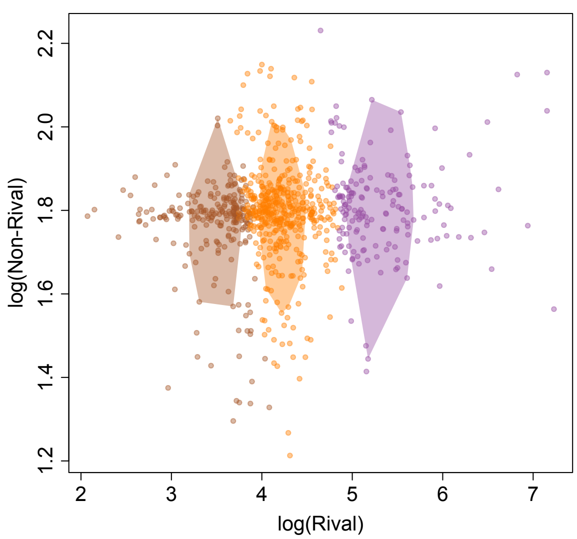
Figure 5: The effect of the $\frac{\mu}{\gamma}$ ratio on the predicted marriage distributions of Kipsigis men. The parameter values for each simulation are described in the frame descriptions. Each dot denotes a man and the shaded regions denote the minimal convex polygons that enclose 60% of the data points for each marriage-type category. Brown points denote unmarried men, orange points denote monogamously married men, purple points denote polygynously married men (with 2-3 wives), green points denote highly polygynous men (with 4-6 wives), blue points denote ultra-polygynous men (with 7-12 wives), and red points denote men with 13 or more predicted wives. As μ increases, the polygyny threshold becomes increasingly determined by rival wealth, and the frequency of polygyny declines.



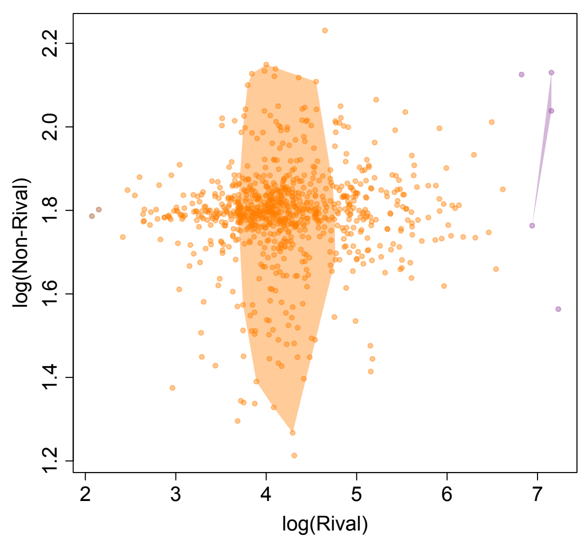
(a) Predicted marriage distribution of Kipsigis men—$\mu = 0.2$, $\gamma = 0.8$, $c_i = m_i^{0.7}$.



(b) Predicted marriage distribution of Kipsigis men—$\mu = 0.4$, $\gamma = 0.6$, $c_i = m_i^{0.7}$.



(c) Predicted marriage distribution of Kipsigis men—$\mu = 0.6$, $\gamma = 0.4$, $c_i = m_i^{0.7}$.



(d) Predicted marriage distribution of Kipsigis men—$\mu = 0.8$, $\gamma = 0.2$, $c_i = m_i^{0.7}$.

households increases in μ. Note from Fig. (6a) that a variable bride-price allows all men to obtain at least a single wife, and as such, increasing μ can drive monogamy to fixation. In Fig. (6b), under a fixed bride-price, polygyny is also eliminated by increasing μ, but the fraction of unmarried men remains high, since a large fraction of the population cannot pay the brideprice.

7. Conclusion

Figs. (5) and (6) illustrate how our proposed model might resolve Becker's polygyny puzzle. We present a model for a pre-demographic-transition marriage system that may be representative of those societies in which a decentralized shift from extensive polygyny to extensive monogamy took place. Our econometric estimates and our ability to simulate an empirically realistic Nash equilibrium assignment of marital partners in a real population suggest that the causal mechanisms postulated by our model are plausible; by increasing μ, our model can produce a hypothetical transition from polygyny to monogamy, suggesting that a changing level of rival wealth importance in early agricultural populations might account for the change in marital norms in those societies.

We observe a tight correspondence between the empirical data and our predictions under a variable brideprice, but in the more ethnographically realistic model with a fixed brideprice, we substantially over-predict the number of wives of the very wealthy. The fact that exceptionally wealthy Kipsigis males take fewer wives than expected, given their wealth, reflects the fact (implied by our estimated fitness function, as already noted) that there are diminishing marginal fitness returns to polygyny due to reasons above and beyond the necessary reduction in each wife's share of the male's rival wealth that is entailed by taking on additional wives.

These unaccounted for diminishing marginal returns to polygyny could be explained by unmeasured and unmodeled variables that are suppressing the polygyny level of the ultra-rich Kipsigis. These variables could include unmeasured sources of rival wealth held by men—including the fact that men's time and attention is rival—or negative externalities resulting from competition among cowives, as well as other costs associated with polygyny—such as higher risks of socially transmitted infections, or social norms penalizing highly polygynous men or a preference among wealthy men for aspects of child quality independent of reproductive success.

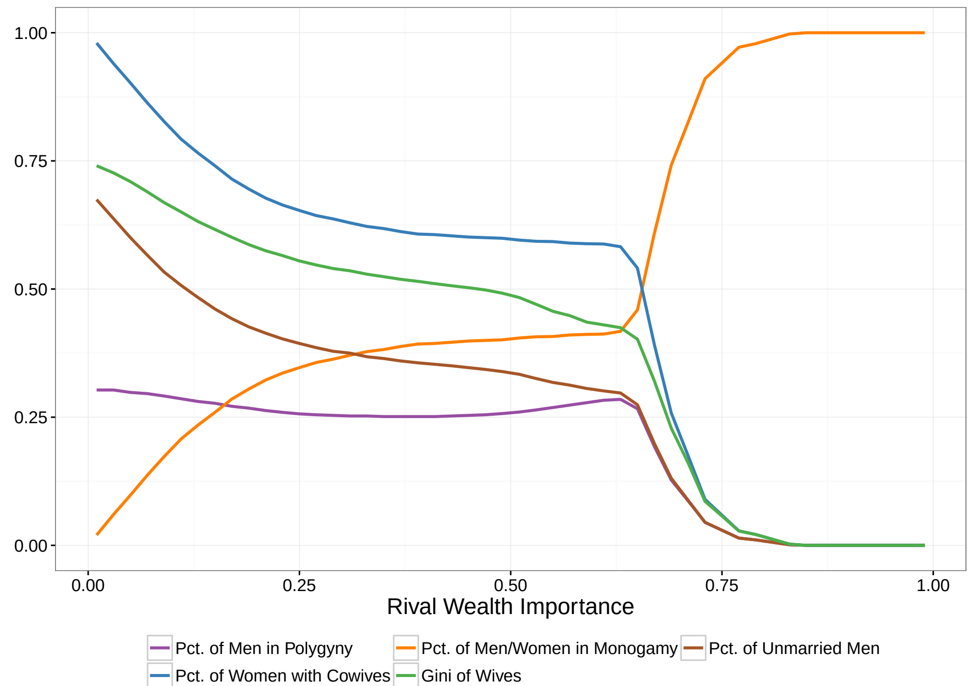
These elements missing from our model could be the basis of a complementary explanation of the decline of polygyny, explored cross-culturally in Ross et al. (2017). This further contribution to resolving the "polygyny puzzle" is that the highly concentrated form of wealth inequality associated with capital intensive farming (by contrast to horticulture) is unfavorable to polygyny because a very small number of exceptionally rich individuals will have fewer wives in total than would be the case if the wealthy class were larger, and individually less wealthy.

Our work could be fruitfully extended in a number of other ways. A more adequate historical test of our interpretation would be based not on a hypothetical population simulated using parameters estimated from an age-adjusted population cross-section, but instead from observations over time on population dynamics. This being said, our static approach is the best that can be done given existing sources of data. Anthropologists conducting longitudinal studies may eventually produce the data needed to test our ideas with dynamic measures (McElreath, 2016), but the fact that few if any of the relevant populations are unaffected by the demographic transition (especially among the wealthy) will present a challenge to this solving this puzzle.

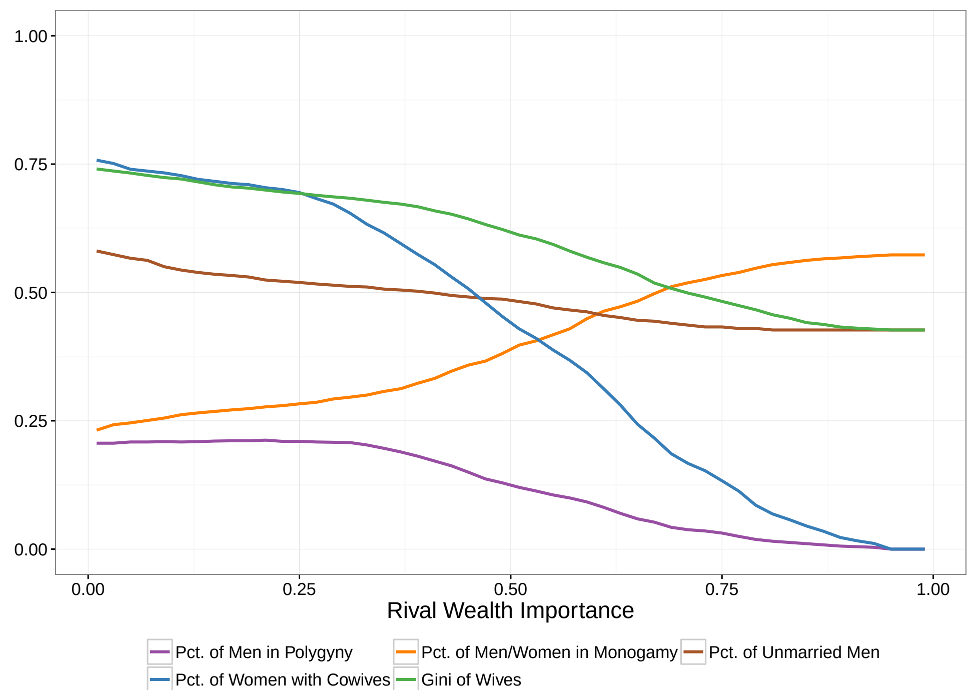
Construction of a more inclusive model of polygyny and its decline would require consideration and estimation of two important relationships that are likely to affect empirical marriage market outcomes. First, one would need to consider how variation in the reproductive importance of rival wealth is associated with corresponding variation in the degree of inequality in this wealth type. Modeling efforts suggest that the importance of rival wealth may strongly covary with its inequality, in part because the degree of investment by parents will be affected by the importance of each wealth type to their offspring (Hartung et al., 1982; Solon, 2000). Data from small scale societies is consistent with this expectation (Borgerhoff Mulder et al., 2009).

Finally, a more general equilibrium approach would take account of the impact of polygyny itself on the long-term levels of material wealth inequality in a given society. It is easily shown using the model of Becker and Tomes (1979) that if wealthier men have more offspring, this will, *ceteris paribus*, enhance regression to mean wealth, and hence reduce the intergenerational transmission elasticity, as pointed out in Lagerlof (2005). The effect of this will be to reduce inequality in the ergodic distribution of material wealth. Thus, any dynamical model of marriage types must treat the prevalence of polygyny and the degree of rival wealth inequality as endogenous, each affected by the level of the other. Modeling this process is entirely feasible, but would take us well beyond the scope of the current paper. Calibrating or testing such a model may be feasible in light of the growing body of individual-level data on relevant measures of wealth and reproductive success.

Figure 6: Posterior mean estimates of various measures of polygyny as the importance of rival wealth, μ, shifts $0.01 \rightarrow 0.99$, under the constraint that: $\mu + \gamma = 1$. As μ increases, we see the frequency of monogamy increases substantially, especially for very high μ.



(a) Variable bride-price, $c_i = m_i^{0.7}$.



(b) Fixed bride-price, $c = 60$.

References

Alexander, R. D. (1979). *Darwinism and Human Affairs*. University of Washington Press: Seattle.

Alexander, R. D. (1986). *The Biology of Moral Systems*. New York: Adine de Gruyter.

Alger, I. (2016). How many wives? on the evolution of preferences over polygyny rates. *Working paper, https://www.tse-fr.eu/sites/default/files/TSE/documents/doc/wp/2015/wp_tse_586.pdf*.

Anderson, S. (2007). The economics of dowry and brideprice. *The Journal of Economic Perspectives 21*(4), 151–174.

Bauch, C. and R. McElreath (2016). Disease dynamics and costly punishment can foster socially imposed monogamy. *Nature Communications 7*, 1–9.

Becker, G. S. (1974). A Theory of Marriage: Part II. *Journal of Political Economy 82(2)*, S11–S26.

Becker, G. S. (1991). *A Treatise on the Family*. Harvard University Press: Cambridge.

Becker, G. S. and N. Tomes (1979). An equilibrium theory of the distribution of income and intergenerational mobility. *Journal of Political Economy 87*, 1153–1189.

Bergstrom, C. T. and L. A. Real (2000). Towards a theory of mutual mate choice: Lessons from two-sided matching. *Evolutionary Ecology Research 2*, 493–508.

Bergstrom, T. C. (1996). Economics in a family way. *Journal of Economic Literature 34*(4), 1903–1934.

Betzig, L. (2002). British polygyny. In M. Smith (Ed.), *Human Biology and History*, pp. 30–97. London: Taylor and Francis.

Borgerhoff Mulder, M. (1987). On cultural and reproductive success: Kipsigis evidence. *American Anthropologist 89*, 617–634.

Borgerhoff Mulder, M. (1990). Kipsigis women's preferences for wealthy men: evidence for female choice in mammals? *Behavioral Ecology and Sociobiology 27*(4), 255–264.

Borgerhoff Mulder, M. (1995). Bridewealth and its correlates: quantifying changes over time. *Current Anthropology 36*(4), 573–603.

Borgerhoff Mulder, M. (1998). The demographic transition: are we any closer to an evolutionary explanation? *Trends in ecology & evolution 13*(7), 266–270.

Borgerhoff Mulder, M., S. Bowles, T. Hertz, A. Bell, J. Beise, G. Clark, I. Fazzio, M. Curven, K. Hill, P. L. Hooper, W. Irons, H. Kaplan, D. Leonetti, B. Low, F. Marlowe, R. McElreath, S. Naidu, D. Nolin, P. Piraino, R. Quinlan, E. Schniter, R. Sear, M. Shenk, E. A. Smith, C. von Rueden, and P. Wiessner (2009). Intergenerational wealth transmission and the dynamics of inequality in small-scale societies. *Science 326*(5953), 682–688.

Boserup, E. (1970). *Woman's role in economic development.* George Allen Unwin, London.

Bowles, S. (2004). *Microeconomics: Behavior, Institutions, and Evolution.* Princeton: Princeton Univ. Press.

Clark, G. and N. Cummins (2015). The child quality-quantity tradeoff, England, 1770-1880: A fundamental component of the economic theory of growth is missing. University of California, Davis.

De la Croix, D. and F. Mariani (2015). From polygyny to serial monogamy: a unified theory of marriage institutions. *Review of Economic Studies 82*, 565–607.

Edlund, L. and N.-P. Lagerlof (2012). *Polygyny and Its Discontents: Paternal Age and Human Capital Accumulation.* Columbia University Academic Commons.

Flinn, M. and B. Low (1986). Resouce distribution, social competiton, and mating patterns in human societies. In D. I. Rubenstein and R. W. Wrangham (Eds.), *Human Nature: A critical reader*, pp. 217–243. Princeton Univ. Press(NJ).

Fochesato, M. and S. Bowles (2017). Technology, institutions and inequality over eleven millennia. *Santa Fe Institute*.

Fortunato, L. and M. Archetti (2010). Evolution of monogamous marriage by maximization of inclusive fitness. *Journal of Evolutionary Biology 23*(1), 149–156.

Goldschmidt, W. (1974). The economics of brideprice among the sebei and in east africa. *Ethnology 13*(4), 311–331.

Goody, J. (1973). Polygyny, economy, and the role of women. In J. Goody (Ed.), *The Character of Kinship*, pp. 175–190. London: Cambridge University Press Cambridge.

Goody, J. and S. J. Tambiah (1973). *Bridewealth and dowry.* Cambridge University Press.

Gould, E. D., O. Moav, and A. Simhon (2008). The mystery of monogamy. *The American Economic Review 98*(1), 333–357.

Grossbard, A. (1976). An economic analysis of polygyny: The case of maiduguri. *Current Anthropology*, 701–707.

Grossbard, A. (1980). The economics of polygamy. *Research in population economics 2*, 321–350.

Hartung, J., M. Dickemann, U. Melotti, L. Pospisil, E. C. Scott, J. M. Smith, and W. D. Wilder (1982). Polygyny and inheritance of wealth [and comments and replies]. *Current anthropology 23*(1), 1–12.

Henrich, J., R. Boyd, and P. J. Richerson (2012). The puzzle of monogamous marriage. *Philosophical Transactions of the Royal Society B 367*(1589), 657–669.

Hrdy, S. B. and D. S. Judge (1993). Darwin and the puzzle of primogeniture. *Human Nature 4*(1), 1–45.

Jacoby, H. G. (1995). The economics of polygyny in sub-saharan africa: Female productivity and the demand for wives in cte d'ivoire. *Journal of Political Economy 81(2)*, S14–S64.

Lagerlof, N.-P. (2005). Sex, equality, and growth. *The Canadian Journal of Economics 38*(3), 807–831.

Lawson, D. W., A. Alvergne, and M. A. Gibson (2012). The life-history trade-off between fertility and child survival. *Proceedings of the Royal Society of London B: Biological Sciences*, rspb20121635.

Lawson, D. W. and M. Borgerhoff Mulder (2016). The offspring quantity–quality trade-off and human fertility variation. *Phil. Trans. R. Soc. B 371*(1692), 20150145.

Manners, R. A. (1967). Contemporary change in traditional societies. pp. 207–359. University of Illinois Press Urbana.

Massell, B. F. (1963). Econometric variations on a theme by schneider. *Economic Development and Cultural Change 12*(1), 34–41.

McElreath, R. (2016). *A long-form research program in human behavior, ecology, and culture.* Max Planck Institute for Evolutionary Anthropology.

Murdock, G. P. (1949). *Social structure.* Macmillan.

Mwanzi, H. A. (1977). *A History of the Kipsigis.* East African Literature Bureau.

Orians, G. H. (1969). On the evolution of mating system in birds and mammals. *American Naturalist*, 589–603.

Peristiany, J. G. (1939). *The Social Institutions of the Kipsigis*. Humanities Press.

Ptak, S. E. and M. Lachmann (2003). On the evolution of polygyny: a theoretical examination of the polygyny threshold model. *Behavioral Ecology 14*(2), 201–211.

Richerson, P., R. Baldini, A. Bell, K. Demps, K. Frost, V. Hillis, S. Mathew, E. Newton, N. Narr, L. Newson, and C. Ross (2015). Cultural group selection plays an essential role in explaining human cooperation: A sketch of the evidence. *Behavioral and Brain Sciences*, 1–71.

Ross, C. T., M. B. Mulder, S.-Y. Oh, S. Bowles, B. Beheim, J. Bunce, M. Caudell, G. Clark, H. Colleran, C. Cortez, P. Draper, R. D. Greaves, M. Gurven, T. Headland, J. Headland, K. Hill, B. Hewlett, H. S. Kaplan, N. Howell, J. Koster, K. Kramer, F. Marlowe, R. McElreath, D. Nolin, M. Quinlan, R. Quinlan, C. Revilla-Minaya, B. Scelza, R. Schacht, M. Shenk, R. Uehara, E. Voland, K. Willfuehr, B. Winterhalder, and J. Ziker (2017). The limits of polygyny. *Santa Fe Institute Working Paper*.

Scheidel, W. (2009). A peculiar institution? Greco-Roman monogamy in global context. *The History of the Family 14*(3), 280–291.

Sellen, D. W. and D. J. Hruschka (2004). Extracted-food resource-defense polygyny in native western North American societies at contact. *Current anthropology 45*(5), 707–714.

Shenk, M. K., M. B. Mulder, J. Beise, G. Clark, W. Irons, D. Leonetti, B. S. Low, S. Bowles, T. Hertz, A. Bell, and P. Piraino (2010). Intergenerational wealth transmission among agriculturalists. *Current Anthropology 51*(1), 65–83.

Solon, G. R. (2000). Intergenerational mobility in the labor market. In A. O and D. Card (Eds.), *Handbook of Labor Economics*. Amsterdam: North-Holland.

Spencer, P. (1980). Polygyny as a measure of social differentiation in Africa. In J. Mitchell (Ed.), *Numerical Techniques in Social Anthropology*. Philadelphia: Institute for the Study of Human Issues.

Tertilt, M. (2005). Polygyny, fertility, and savings. *Journal of Political Economy 113(6)*, 1341–1371.

Verner, J. and M. F. Willson (1966). The influence of habitats on mating systems of North American passerine birds. *Ecology 47*(1), 143–147.

White, D. R., L. Betzig, M. Borgerhoff Mulder, G. Chick, J. Hartung, W. Irons, B. S. Low, K. F. Otterbein, P. C. Rosenblatt, and P. Spencer (1988). Rethinking polygyny: co-wives, codes, and cultural systems [and comments and reply]. *Current Anthropology 29*(4), 529–572.