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Adaptive Parties in Spatial Elections

We develop a model of two-party spatial elections that departs from the standard model in three respects: parties' information about voters' preferences is limited to polls, parties can be either office-seeking or ideological, and parties are not perfect optimizers, i.e. they are modelled as boundedly-rational adaptive actors. We employ computer search algorithms to model the adaptive behavior of parties and show that three distinct search algorithms lead to similar results. Our findings suggest that convergence in spatial voting models is robust to variations in the intelligence of parties. We also find that an adaptive party in a complex issue space may not be able to defeat a well-positioned incumbent.

Introduction: A Theoretical Problem

Since Anthony Downs' Economic Theory of Democracy (1957), a spatial theory of elections has occupied a prominent theoretical status within political science. Modelers use the intuitive notion of ideological distance to develop explanations for observable electoral trends. The most famous of these trends is the Downsian idea that in a two-party system, given certain assumptions, the parties will converge towards a median position on the continuum of possible voter positions. Although simple spatial models have produced this result, extensions to the models have questioned the robustness of this prediction. Following the voting paradox and the results of Plott (1967) and McKelvey (1976), some scholars have speculated that chaotic results are possible, and in some cases, likely. In two or more dimensions, the top cycle set consists of the entire space of issue positions.[1] Bates (1990, 45) summarizes the more general result: "The principle lesson is that, in general, one cannot expect an equilibrium to exist; and, because any outcome can be defeated, political decisions represent arbitrary outcomes." Whereas some scholars lament the predicted instability in multidimensional voting models (Riker 1982), others see the Downsian model's stability and convergence as more empirically accurate and (perhaps) more normatively desirable.

Some theorists, inclined to believe that electoral chaos is extremely unlikely, have incorporated various complexities to explain stable, often centrist, outcomes. Coughlin (1990a) divides these models into four general categories: a) models that allow for mixed strategies by parties, b) models that track dynamic trajectories of party locations, c) models that search for uncovered or undominated sets, and d) models that include candidate uncertainty over voters' behavior (probabilistic voting models). Coughlin writes: "(I)t is hard to resist the alternative inference that the primary contribution of recent work on the majority rule relation is as a grand 'reductio ad absurdum' that tells us to go back to the basic model that has been used to see how it should be modified in order for theory and empirical observations to match up" (1990a, 164).

Two-party electoral outcomes appear more stable than the chaotic results predict, so the task of seeking alternative assumptions to get more realistic outcomes has a sound scientific basis. Nevertheless, both original spatial models and many contemporary revisions rely on unrealistic assumptions to produce equilibria. They often rely on fully informed and optimizing voters and parties (Davis, Hinich, and Ordeshook 1970). Spatial modelling seems wedded to the defining assumptions of rational choice: that people and organizations are self-interested, have complete information, and can locate optimal strategies regardless of complexity. Chaos results, for example, rely on unrealistic assumptions about the abilities of parties to locate winning platforms. Probabilistic voting models assume parties position themselves optimally given complete knowledge of the probabilities of voters' actions (Coughlin 1990b). We find implausible even the restricted assumptions that parties have sufficient information and analytical abilities to locate optimally. Perhaps different assumptions about the abilities of parties or voters can lead to stable outcomes.

In this paper, we use the idea of boundedly-rational agents to model the behavior of two political parties in a fixed-party system. Instead of modelling parties as full-information global maximizers, we model them as incompletely informed and adaptive. They move incrementally towards better regions of the space through the use of search algorithms. Our model involves a dynamic interaction between parties who make decisions in an evolving environment. Much like Kramer (1977), we are interested in the trajectory of party platforms over a sequence of elections. Using our methodology, we are able to explore a variety of questions: Do imposed informational and computational constraints lead to arbitrary outcomes, with winning party platforms scattered throughout the space? Do boundedly rational parties converge towards central platforms? Do parties always defeat incumbents, as many models suggest? Does altering the preferences of parties from vote maximizing to winning with ideals affect the behavior of parties? Our use of adaptive agents allows us to address these questions without wholly abandoning formal modelling.

In the next section we discuss the advantages of using adaptive artificial agents in the social sciences. The following two sections presents our basic model and a description of the two types of party preferences we consider. We then present the three search procedures used by our parties and describe our results. Finally, we conclude with a discussion of the role of information in elections and possible directions for future research.

The Use of Adaptive Artificial Agents

Holland and Miller (1991) advocate a class of models using adaptive artificial agents (AAA), a technique in which "the unfolding behavior of the models can be observed step-by-step." Based on computer languages, these models are flexible enough to capture a variety of situations, yet maintain precision and logical consistency.[2] Computational models allow the exploration of systems of well-defined agents in a perfectly controlled environment that can be easily and rapidly replicated under a variety of conditions. Moreover, any conceivable analysis is feasible since the state of the system is fully recoverable. This allows the researcher to generate, develop, and test theoretical hypotheses quickly. Although optimization is the key benchmark in much social science theory, learning, adaptation, and bounded rationality are recognized as important processes. Through the use of AAA models, questions about the relationship between optimization and adaptation can be explicitly explored. We adopt the AAA technique because of these advantages and because of our belief that political parties are better characterized as partially informed, finite-memory information processors who rely on rules of thumb than as completely informed rational agents.

Underlying our methodology is the notion that there exist important classes of behavior that can be captured in models too complex for traditional mathematical analysis. Absence of equilibria in a model (or equilibria that human agents could locate only by chance) does not necessarily imply a lack of predictability. Using AAA allows us to search previously inaccessible models for patterns of generic behavior. Our choice of technique has costs. Restriction to any one type of algorithm can yield ad hoc results, and predictions are often less precise than those of rational actor models. To control for these possibilities, we compare three distinct types of AAA, and we arrive at strikingly similar results for all three. The use of multiple algorithms to discover equivalence classes of adaptive behavior distinguishes our model and strengthens our results.

There are many precedents for the use of AAA and similar techniques in the social and behavioral sciences. For example, economists have applied AAA to evolve efficient bilateral trading rules (Marimon, McGrattan, and Sargent 1989) and to explore learning in varying economic environments (Miller 1986, Arifovic 1989). In political science, Cohen

(1984) has used computer models to explore competing theories of organizational decision making, and Whicker and Strickland (1990) used computer simulations to test the importance of opinion distribution and decision rules on the constitutional amendment process. Axelrod (1986) simulated the evolution of norms, and has also used a genetic algorithm (see below) to develop new strategies that were superior to Tit-for-Tat in his 1984 computer tournaments (Axelrod 1987). Miller (1987) analyzed the co-evolution of strategies and the emergence of cooperation in noisy repeated prisoners' dilemma games using adaptive algorithms.

Thus, we agree with Coleman (1989) that simulation models, with humans, computers, or both, can be useful in the construction of social theory. In this paper, we use computer models to trace the emergent behavior of boundedly-rational parties in the context of spatial voting models. Since our parties are limited in both the information they possess and in how they process the information, there is no guarantee that in finite time they will find optimal locations in the policy space. We can, however, compare the general movements of the parties over time to the predictions of mathematical models of party competition. The model we put forth does not stretch the boundaries of our technique. Our present purposes, however, addressing a central problem in positive political theory and showing the strength of our approach, are best served by a simple model.

The Basic Model

Our model incorporates many of the assumptions of spatial voting models, including voters who are perfectly informed about candidate platforms. We also follow more recent models and relax the assumption of identical voter preference intensities. We call a voter's preference intensity a strength, which is interpreted as the importance the voter attaches to an issue. Strengths vary, so voters will have noncircular indifference curves. Our model departs from standard spatial models in three respects. First, we consider both purely office-seeking parties and ideological parties. Second, our parties are not perfectly informed, and do not know individual voter utility functions. Rather, they obtain information through "test" elections (like opinion polls) that measure how well their current platform would do relative to their opponent's platform. During a campaign, parties test their platforms on voters, receive feedback in the form of vote totals, and alter their platforms to improve vote totals. Finally, as discussed above, our parties are not full-information global optimizers. Instead, using only vote totals from test elections they adaptively alter their platforms, trying to defeat their opponent. They do not attempt to select an optimal platform given their limited information.

Formally, there are two parties competing for V votes in an n-dimensional issue space. Each voter's preferences are represented by two vectors of n integers, which give the voter's ideal positions and strengths on the n issues. We assume that there are k possible positions on each issue $\{0,1,..,k-1\}$ and s possible strengths $\{0,1,...s-1\}$. The utility to a voter from a party's platform, $y \in R^n$, is given by the negative of the squared weighted Euclidean distance, where the weight on the ith issue is the strength associated with that issue by the voter. If s_{ji} denotes the jth voter's strength on the ith issue, and x_{ji} is the jth voter's ideal point, then a voter's utility is given by:

$$u_j(y) = -\Sigma_i \, s_{ji} \cdot (x_{ji}-y_i)^2 \qquad (1)$$

In this model we assume that both strengths and ideal points are independently and uniformly distributed. The election results we present consider 251 voters, 15 issues, 7 positions per issue, and 3 possible strengths (see below for an explanation and discussion of parameter values). Therefore, on average each voter will have five issues of major importance ($s_{ji}=2$), five of minor importance ($s_{ji}=1$), and five of no importance ($s_{ji}=0$). The assumption that voter ideal points are uniformly distributed does not necessarily imply regularity. A relatively small number of voters are generated in a large space, so a spray of points is a more appropriate way to think of the distribution.[3] Each voter casts a ballot for the party giving him or her the higher utility. To evaluate the trajectory of democratic outcomes, we introduce centrality, a measure of the goodness of each outcome. Without such a measure we cannot compare our model analytically to any other model. We calculate centrality by computing both the sum of the utilities of the individual voters if the winning party were located at the median on all issues and the sum of utilities resulting from the winning party in the election; we then divide the first number by the second.

$$c(y) = [\Sigma_j u_j(median)]/[\Sigma_j u_j(y)] \qquad (2)$$

It follows that $c(median) = 1$. This normalization has the following interpretation: the higher the centrality, the closer the winning candidate is to the weighted center of voter preferences, and therefore the more responsive the democratic outcome. We attach no normative significance to the median itself as an outcome. We merely exploit the fact that it will generally be of high aggregate utility. There may exist platforms with centralities greater than one. Ideally, we would find the platform of minimal average distance and use its utility as the numerator, but the costs in computer time outweigh any advantages. Regardless of the numerator, we have a measure of aggregate utility, or the average weighted distance to a voter.

Parties are initially represented by randomly selecting an "ideal" platform. Since parties are aggregates of individuals, we assume they have uniform strengths across all issues. We consider two types of parties: ***ambitious parties*** and ***ideological parties.*** Ambitious parties care only about winning elections, and their ideal platform serves only as a starting point for the initial campaign. Ideological parties also want to win the elections, but they want to win with a platform that is spatially close to their ideal platform. Formally, if $v(y{:}x)$ is the number of votes the challenger party receives if it takes platform y and the incumbent is at x, and Y is the challenger party's ideal point, then the objective functions for the ambitious and ideological parties can be written as:

$$Obj_{am}(y) = v(y{:}x)$$

$$Obj_{id}(y) = \begin{cases} v(y{:}x) & \text{if } v(y,x) \leq V/2 \\ V/2 + nk^2 - \Sigma_i (y_i - Y_i)^2 & \text{if } v(y{:}x) > V/2 \end{cases}$$

Recall that V is the total number of voters, n the number of issues, and k the number of positions per issue. This implies that $nk^2 \geq \Sigma_i (y_i - Y_i)^2$ for any y and Y. Thus, ambitious parties attempt to maximize votes in the hopes that a larger margin of victory makes them more difficult to defeat in subsequent elections, while ideological parties have lexicographic preferences. Their primary goal is to win the election: $v(y{:}x) > V/2$. Once this is accomplished, they attempt to get as close to their ideal platform as possible.[4]

During each election the incumbent party's platform is fixed and the challenger party attempts to find a platform in the issue space that defeats the incumbent. In the first election the incumbent party (arbitrarily chosen) sits at its ideal platform. Thereafter, the incumbent sits at the platform where it won the previous election. The challenger party attempts to defeat the incumbent by choosing new platform. The challenger party, during a finite campaign, tests new platforms on voters who are assumed to have perfect information about both platforms. These tests are like perfect polls of political popularity.

Both ambitious and ideological parties will be constrained in how they search the issue space for good platforms. First, the campaigns are of finite length, so parties are limited in the number of polls they can take. For example, a party may only be able to take forty polls before the election. Second, during any platform adaptation, our parties are limited by the number of issues they can change and the degree of change on any such issue.[5] Even for ambitious parties the ideal platform functions as an ideological tether in early elections.

Positioning constraints and finite campaign length imply that our parties will fail to fulfill their goals optimally. Ideological challengers should typically win elections because ideological incumbents attempt to stay near their ideal platforms, which generally lie in regions of average centrality. However, ideological challengers adapt platforms further from their ideal platforms than optimal. Ambitious challengers lose; after a few elections, the incumbents are located in regions of high centrality and therefore are difficult to defeat.

How Parties Find Platforms

Once we relax the assumptions that parties have complete information and the ability to locate optimal platforms, we can model our parties in a myriad of ways. There are many ways to not be perfectly rational. We chose three types of parties, each having a different platform search procedure: random adaptive parties (RAP), climbing adaptive parties (CAP), and genetic adaptive parties (GAP). The search procedures were constructed to be crude approximations of actual procedures. More important, they provide reasonable bounds on the ability of parties to locate platforms. The procedures themselves are mechanisms for the party to choose the platform it will present to the voters against the incumbent. All three procedures will be discussed below primarily within the context of ambitious parties. The extension to ideological parties is easy.

RAP are the least adaptive of our parties. Letting L represent the length of the campaign, RAP randomly generate L platforms in the neighborhood of their previous platform and choose the platform that receives the most votes against the incumbent. RAP approximate a smoke filled room selection process. The party gathers immutable potential candidates and selects the highest vote getter to represent the party in the election. We do not dispute the contention that this underestimates the ability of parties or candidates to adjust to public opinion. RAP are intended as lower bounds on the ability of parties to position themselves.

In contrast to RAP, both CAP and GAP selectively refine their platforms to improve vote totals. CAP begin with their current platform and experiment, slightly changing positions on a few issues. If the new platform fares better against the incumbent than did the previous one, the party switches to the new platform. These platform tests will be called hill-climbing iterations. The campaign length, L, equals the total number of hill climbing iterations (including those resulting in no improvement) that a party performs before the actual election. CAP enter the election with their final and, therefore, best-to-date platform. CAP represent parties who select a candidate and then adapt the candidate's platform to the electorate's views by testing alterations with focus groups and speeches. After finitely many refinements, the improved challenger faces the incumbent.

GAP, the third type of parties we consider, employ a genetic algorithm to guide their search (see Holland 1975, Goldberg 1989). Instead of adapting a single position, genetic algorithms adapt a population of platforms, attempting to discover nonlinear (epistatic) interactions among variables. Genetic algorithms were designed to work well in "complex" environments - spaces with nonlinearities, discontinuities, noise, and high dimensionality. For our purposes, GAP provide an indication vis-a-vis the other algorithms of the inherent difficulty of the spatial problem.

GAP represent parties whose potential candidates shift positions both by borrowing from competitors and by testing their own alterations. The genetic algorithm generates new platforms using three procedures. It begins with the random creation of, say, twelve platforms. The first operator, reproduction, randomly selects (with replacement) twelve pairs of candidates from the list and reproduces only the preferred member of the pair. The resulting candidates are then randomly arranged in pairs to which the crossover operator is applied. During crossover, the candidates randomly decide (with probability 50%) whether or not to trade positions on a few issues. If they decide to switch, they exchange groups of positions. Finally, the mutation operator allows each candidate to alter positions randomly on an issue or two.

Following biological convention, each application of the reproduction, crossover, and mutation operators is called a generation. We count each generation of the genetic algorithm as two units of campaign length, since both crossover and mutation involve candidate platform alterations. At the completion of a campaign of length *L*, which consists of *L/2* generations of the genetic algorithm, the party chooses the best-to-date platform.

Results

Several measures are of interest in comparing the different parties and search procedures. Given that our primary concern is the extent to which the distribution of winning party platforms will be biased towards regions of high centrality, we record the centrality of winning platforms. We also want to measure the ability of ambitious and ideological parties to defeat the incumbent, and to know how far ideological parties must stray from their ideal platforms to do so. Finally, we are concerned with the effects of varying the length of campaigns. The length of campaigns corresponds roughly to the amount of information parties have about voters before an election. Campaigns of length forty ought to enable parties to compete more effectively against incumbents than campaigns of length five.

Before presenting our findings, we should note that the robustness of computer simulation results often hinges upon sensitivity to parameter values. The findings appear qualitatively invariant to reasonable changes in parameters. We chose parameter values that seem realistic in the study of democratic elections and for which a wide range of surrounding values produced similar results. For example, we chose twelve elections because most interesting phenomena were manifest by that time. The following parameters were used:

Voter types (V) *251*

Number of issues (n)	*15*
Positions per issue (k)	*7*
Strengths (s_{ji})	*3*
Elections	*12*

All of these fall safely within ranges for which we observed no significant changes in the conclusions.

Centrality values have greater significance when viewed with respect to the distribution of centrality. Figure 1 shows numerically estimated distribution and density functions for the centrality of platforms. We compare election outcomes to the cumulative distribution function (cdf). Note that a winning party with a platform having a centrality of 0.55 lies in the upper 17% of the distribution.

(Place Figure 1 About Here)

We ran 200 trials of a twelve-election sequence for each party and type of algorithm. We recorded the average centrality of the winning party, average probability that the incumbent is defeated, and, in the ideological case, the average distance to the party's ideal platform.

Ambitious Parties

For all three types of algorithms, ambitious parties moved in directions of higher centrality (see Figure 2). By the sixth election CAP and GAP had expected centralities above 0.9 and RAP above 0.8, which placed all three types of parties in the top 0.01% of all platforms! It appears that convergence to high centrality and the increase in centrality over time are invariant to the type of search algorithm. As might be expected, CAP and GAP had higher centrality than RAP. With respect to the distribution of centrality, however, the differences were not large.

(Place Figure 2 About Here)

The probability of a challenger winning decreased from almost 1 in the first election to near 0.4 by the twelfth election for all three algorithms (see Figure 3).[6] In our model, ambitious challengers have increasing difficulty in defeating the incumbent. Incumbency advantage can be attributed to challengers' lack of information and the fact that the information is not used optimally.

(Place Figure 3 About Here)

Ideological Parties

We would expect the centrality of outcomes in elections between ideological parties to be lower than if the parties were solely ambitious. As a consequence, the probability of winning should be much higher in ideological contests. Our results confirmed these expectations. Centralities were lower overall for ideological parties compared with ambitious parties, and there was less variation over elections (Figure 4). Still, all three algorithms were in the top 3% of platforms at the end of twelve elections. Moreover, the probability of winning was high throughout the sequence of elections, tapering off in the later elections only for RAP (Figure 5).

(Place Figures 4 and 5 About Here)

Note that CAP performed better than RAP and GAP in terms of winning elections, but won with lower centralities. CAP basically were more successful than the others at fine tuning their platforms towards their ideal platforms.

Another noteworthy result came from the simulation of ideological parties. We observed that, for all three algorithms, the distance to party ideals increased by small amounts, while the distance to the median decreased over time (Figure 6). We refer to this positioning behavior as the dumbbell waltz. The challenging party dances in the neighborhood of its ideal platform until it finds a winning platform. This neighborhood slowly converges to areas of high centrality. A chart of winning platforms would consist of two disjoint neighborhoods, one near each of the ideal platforms, and resemble a dumbbell. The ends of the dumbbell slowly converge as the number of elections increases (Figure 7).

(Place Figure 6 and 7 About Here)

Campaign Length

Increasing the length of the campaign increases the ability of parties to learn about and adapt to voters' collective preferences. Figure 8 shows centralities and probabilities of winning for ambitious and ideological CAP as the length of the campaign increases from 5 to 40. As expected, both centrality and probability of winning tend to increase with campaign length. To give a complete 12 election example, Figure 9 reveals how increasing campaign length for ambitious CAP qualitatively increases centrality. For ideological parties, though, centrality decreases for lengths greater than about 30 (see Figure 8) because more informed and intelligent ideological parties will be able to locate winning platforms nearer their ideal platforms.

(Place Figure 8 and Figure 9 About Here)

Our results tend to support the idea that boundedly-rational parties will converge to central regions of the issue space in a Downsian fashion.[7] Since centrality measures the closeness of parties to voters' ideal points, winning parties with very high centralities can be said to give voters high utility. In this sense, our results suggest that even with fairly simple adaptive parties (e.g., RAP), a two-party system should lead to normatively appealing outcomes. Not surprisingly, ambitious parties reach higher centrality than ideological parties. Finally, all three search procedures for both types of parties produced similar outcomes, indicating that there may exist large equivalence classes of adaptive behavior by parties that may allow researchers to undertake a unified analysis. Substantively, our use of three algorithms bolsters the conclusion that departures from the strict informational and rationality assumptions of earlier formal models lead us to convergence results.[8]

Discussion

Rational choice theories of electoral competition have led to two incompatible conclusions. On the one hand, convergence results offer an intuitively appealing explanation for the responsiveness and stability of two-party systems. On the other, chaotic results raise doubts about the legitimacy of democratic outcomes. Both sets of results have relied on strict assumptions of complete information and global optimization. We have relaxed these assumptions for parties, and have observed the parties in our model, even ideological ones, converging to central regions of the issue space. Although some mathematical models indicate that democracy might yield arbitrary results with respect to voters' preferences, our model suggests that this typically will not happen with adaptive parties. Moreover, boundedly-rational parties may not be able to defeat incumbents. Mathematical proofs of the existence of winning strategies should not be equated with the ability of human actors or organizations to locate them.[9]

Our results, supported by their invariance to parameter choices and search procedures, also help confirm basic intuitions about the role of information and ideologies in elections. As the length of campaigns increases, or as parties have more information about voters, parties tend to converge toward centrist outcomes. This observation seems to be consistent with general comparisons of both national and local elections. Extreme candidates rarely emerge as national candidates, and when they do, they lose by a wide margin. Yet at the local level, extremists can thrive. The lack of information among voters has received a great deal of theoretical attention. The lack of information among parties and candidates deserves to be explored as well.

Our techniques are designed to analyze the behavior of a complex adaptive system, a system in which endogenous aggregate behavior emerges from the knowledge and rule-based behavior of individual agents. Our model can be extended in many directions, incorporating a variety of components known to exist in the real world.[10] It is possible to include interest groups, issue polling, term limitations for incumbents, voters with incomplete information, voters who are influenced by party platforms, and multiple parties in future adaptive models. Moreover, we can model elections with co-adaptive parties, although doing so might require problematic assumptions about levels of strategic complexity. Finally, with all of these extensions, there is the opportunity to calibrate a simulation to data.

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1. The top cycle set is the smallest set of points whose members defeat all points not in the set. For example, if A defeats B, B defeats C, C defeats A, and A, B, & C each defeats all other points in the space, then {A,B,C} is the top cycle set.

2. By logical consistency we mean in the narrow sense of adherence to a rule and not in the broader sense of satisfying mathematical axioms.

3. Central limit theorems and the like are not appropriate given the relatively small number of voters and the size of the space. With 15 issues, 7 positions and 3 strengths, there are 21^{15} ($\approx 10^{20}$) possible voter types.

4. This construction allows an ideological party to choose a platform which is further from its ideal platform than the incumbent's platform is. However, this only occurs when the distance between party ideal points is improbably small.

5. As an example, in a five issue, seven position space, where the current party platform is (6,2,5,4,1), the party will be much more likely to test the platform (5,2,5,4,2) than the platform (1,6,1,3,6), given their relative distances to the current platform.

6. The dip in the second election results from our random assignment of an incumbent prior to the first election. Half of the time, the challenger defeats the incumbent prior to adapting a new platform. After adaptation, the election is a landslide. Subsequently, the ex-incumbent (now the challenger) has a difficult time winning the second election against such a strong opponent.

7. Kramer (1977), relying on stricter assumptions, has shown that parties will converge towards the minmax set--the set of positions that lose by a minimal amount--but that the minmax set is not a stable attractor. In other words, parties can leave the minmax set to win once they converge to it. Our results suggest that if movements out of the minmax set occur, on average they do not lower centrality.

8. The equivalent performances of GAP and the other less sophisticated algorithms suggests that the underlying search problem is relatively easy.

9. Consider chess. A winning strategy exists, yet no one has found it to date.

10. We have some preliminary findings which suggest that convergence is robust to changes in

the distribution of voter preferences.

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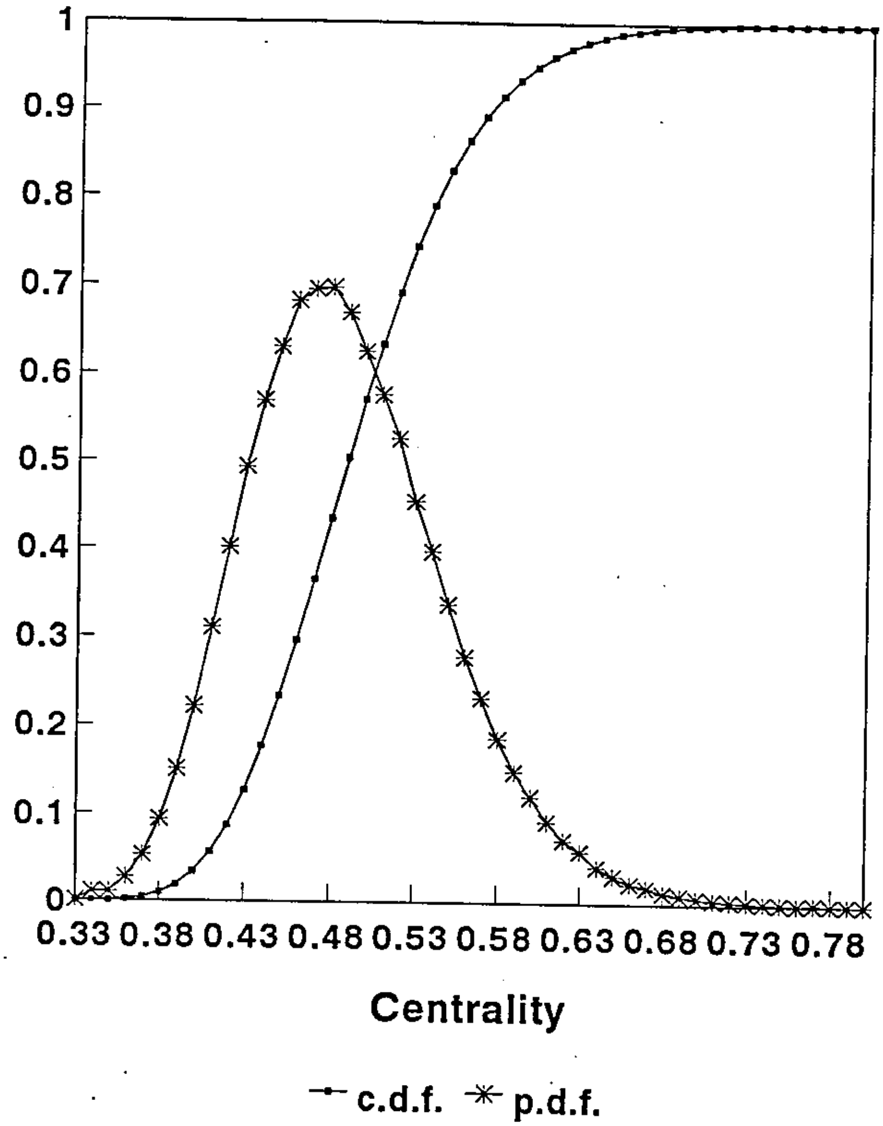
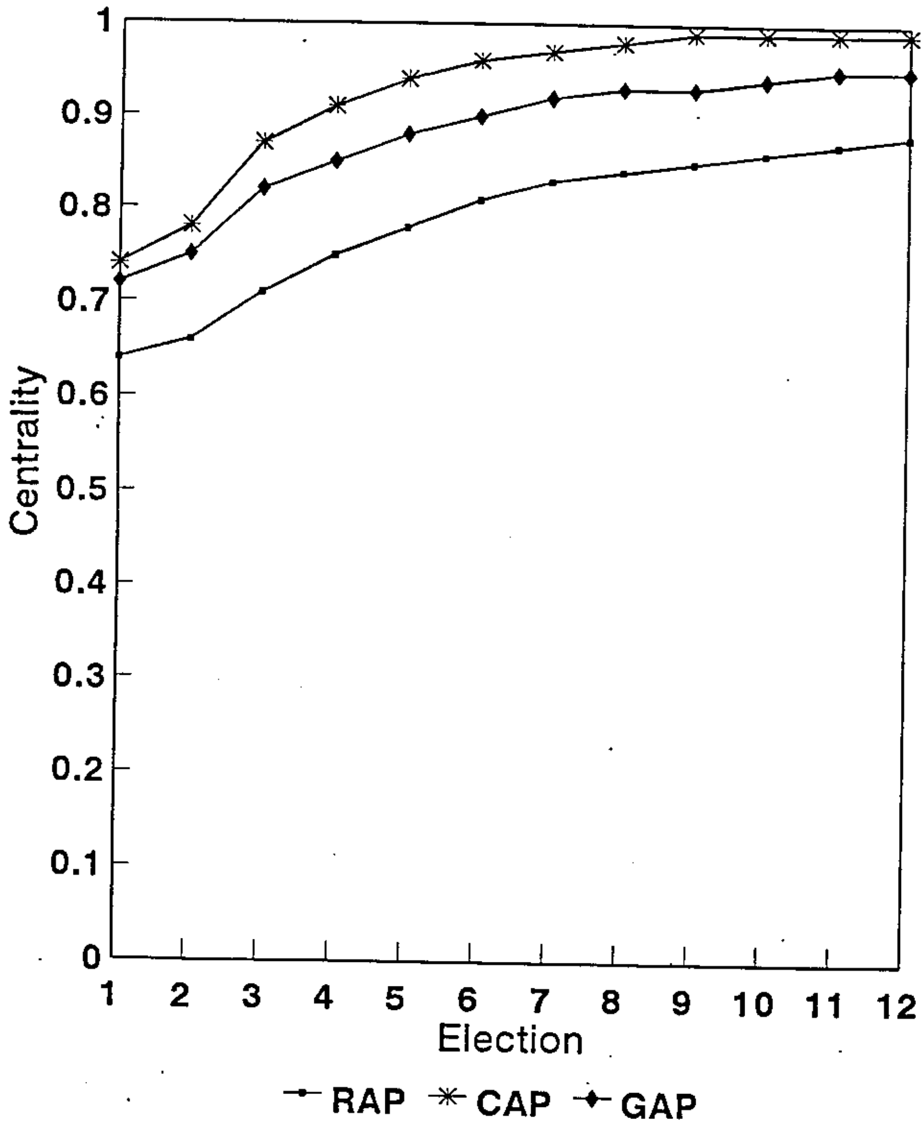
Figure 1
Centrality Distribution/Density



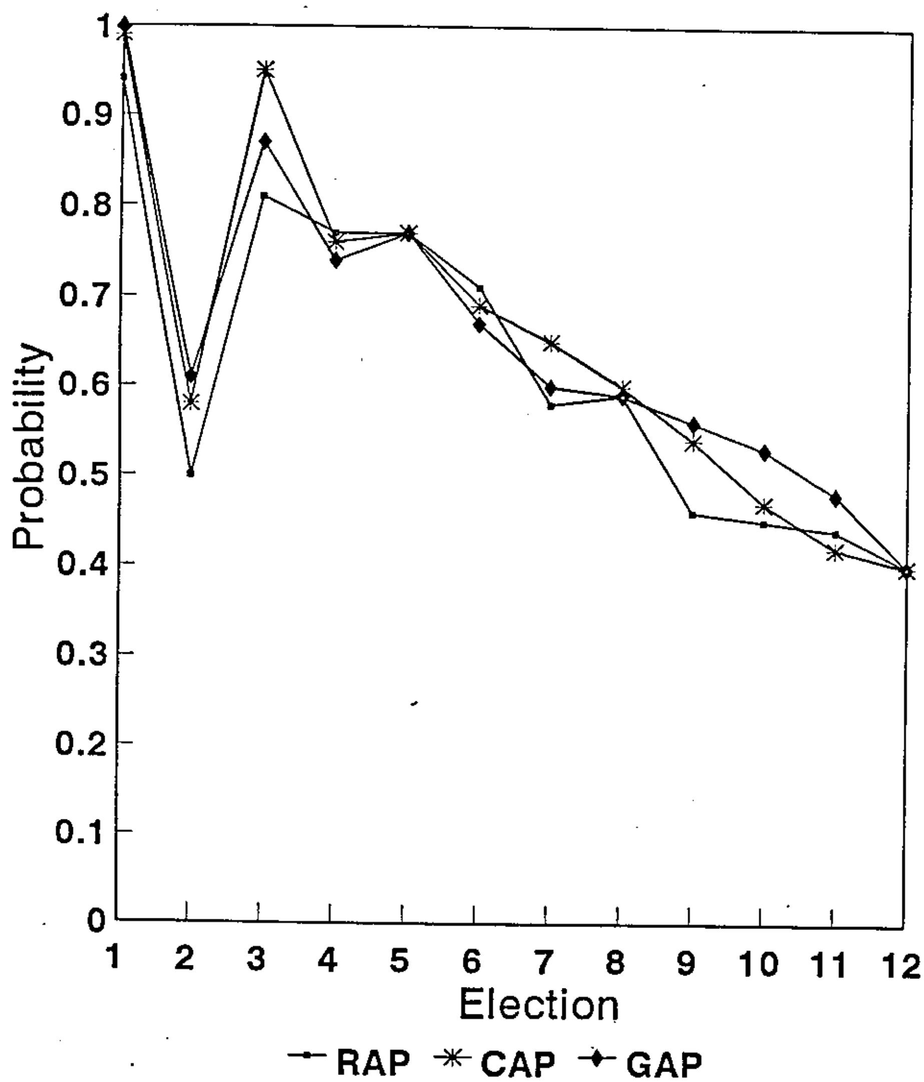
10,000 Monte Carlo trials

Figure 2
Centrality of Ambitious Parties



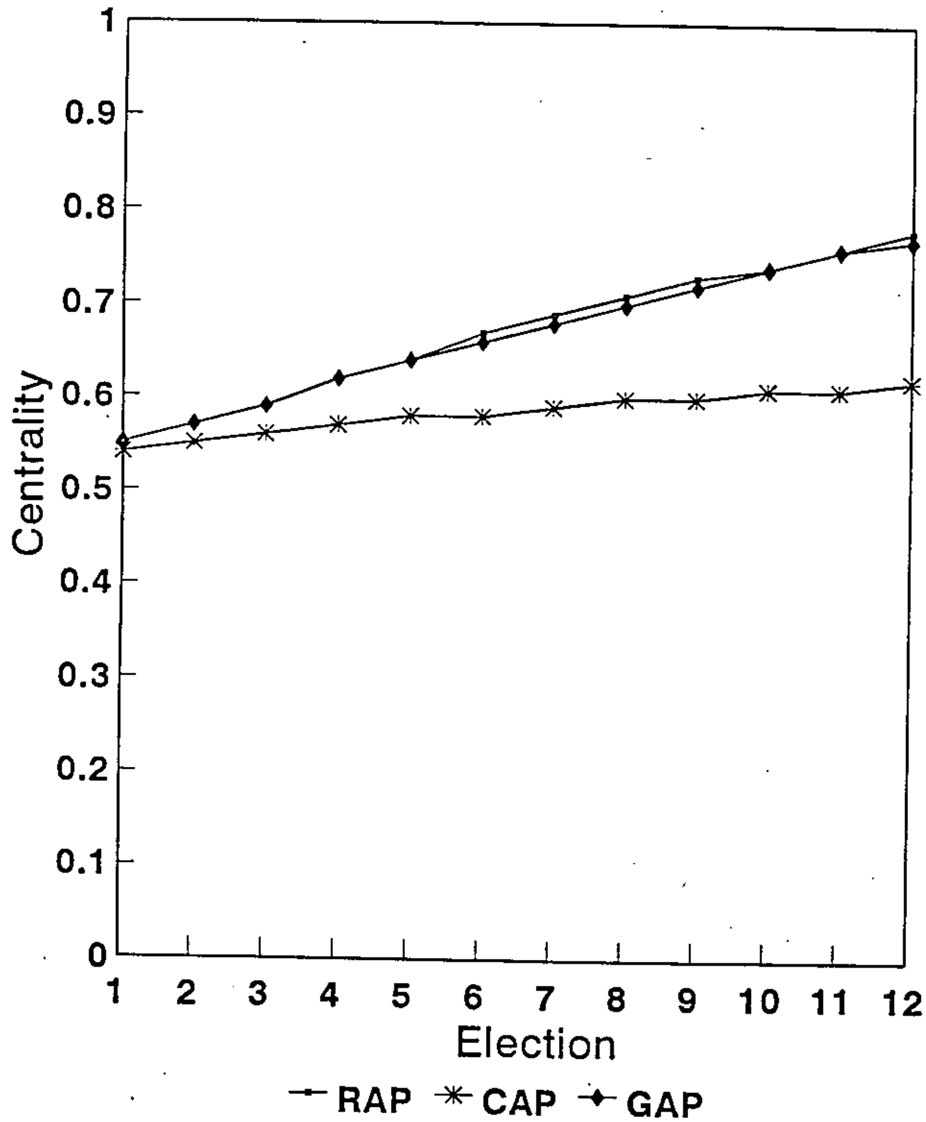
200 trials

Figure 3
Probability of Winning for Ambitious Parties



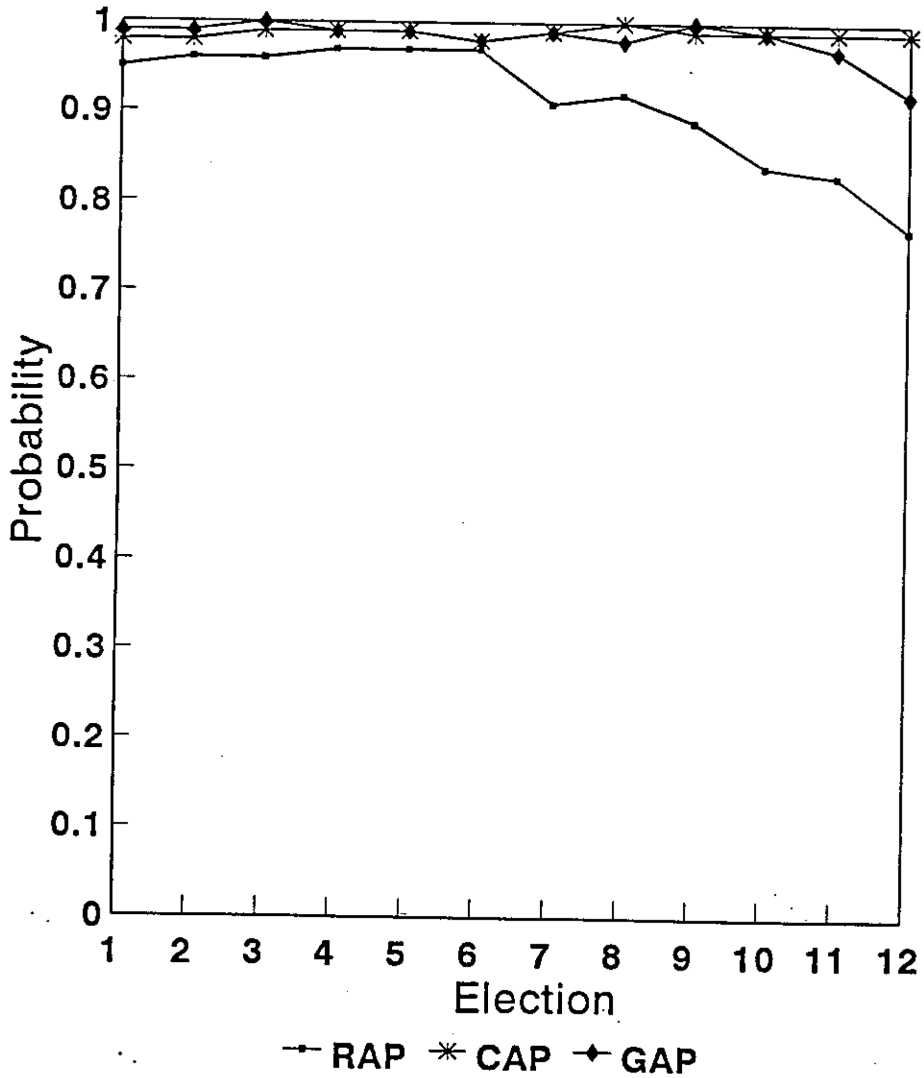
200 trials

Figure 4
Centrality of Ideological Parties



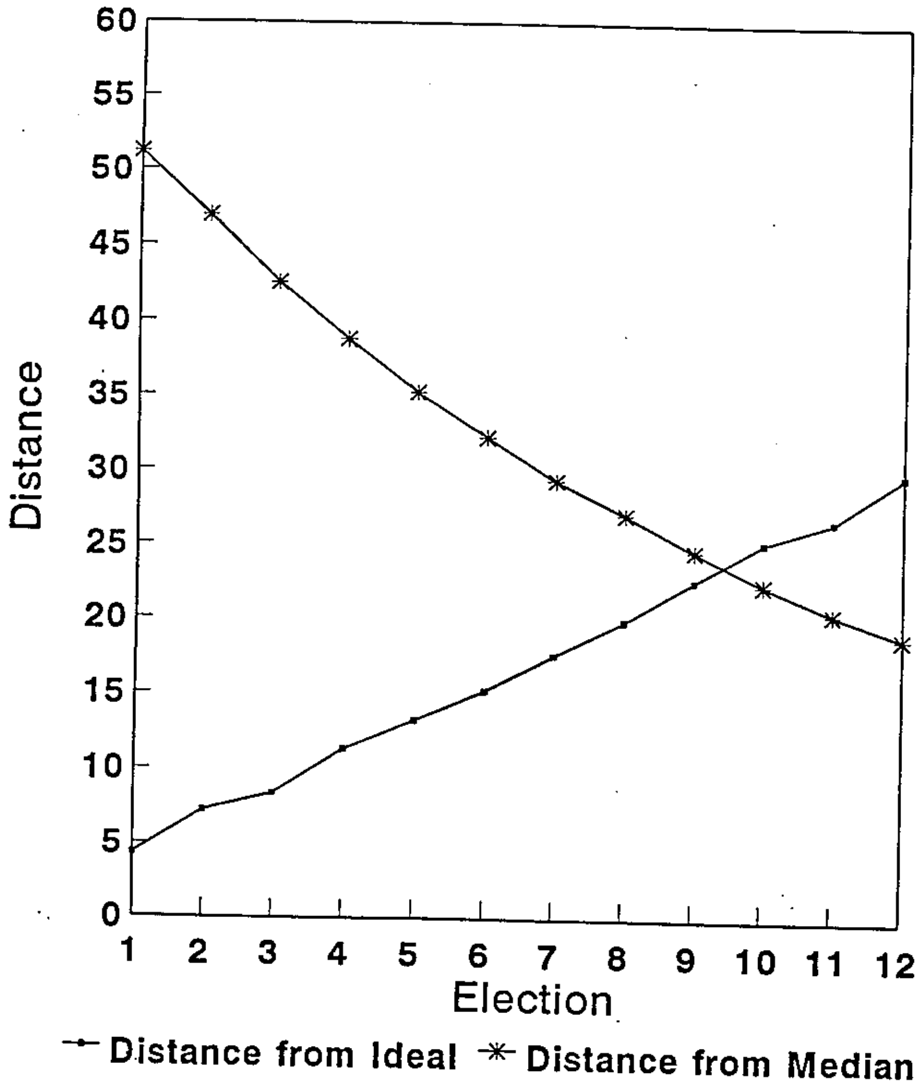
200 trials

Figure 5
Probability of Winning for Ideological Parties



200 trials

Figure 6
Distance from Ideal and Median for Ideological GAP



200 trials

Figure 7

Dumbbell Waltz for Ideological Parties

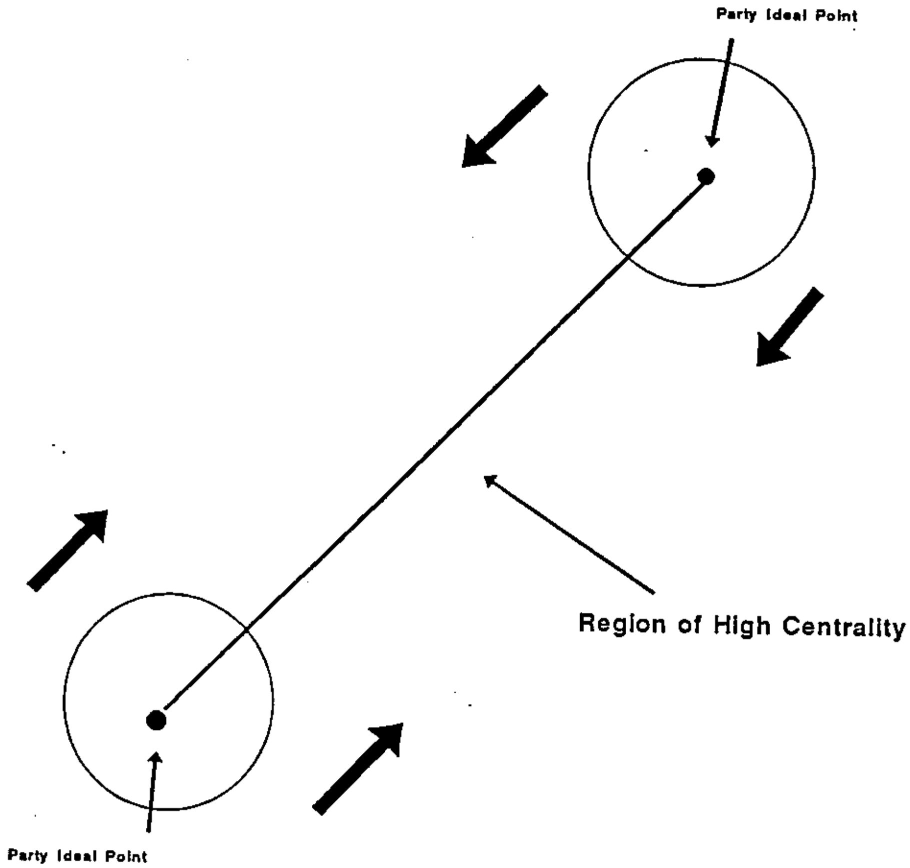
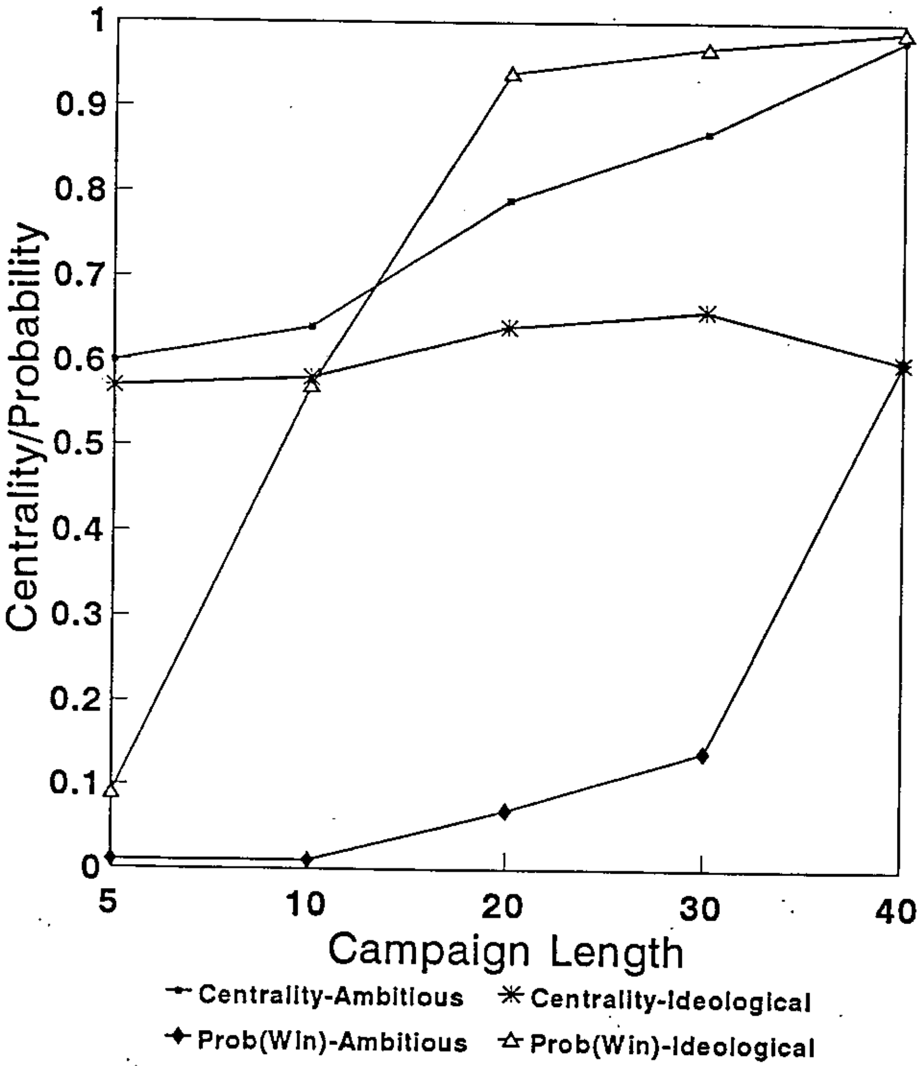
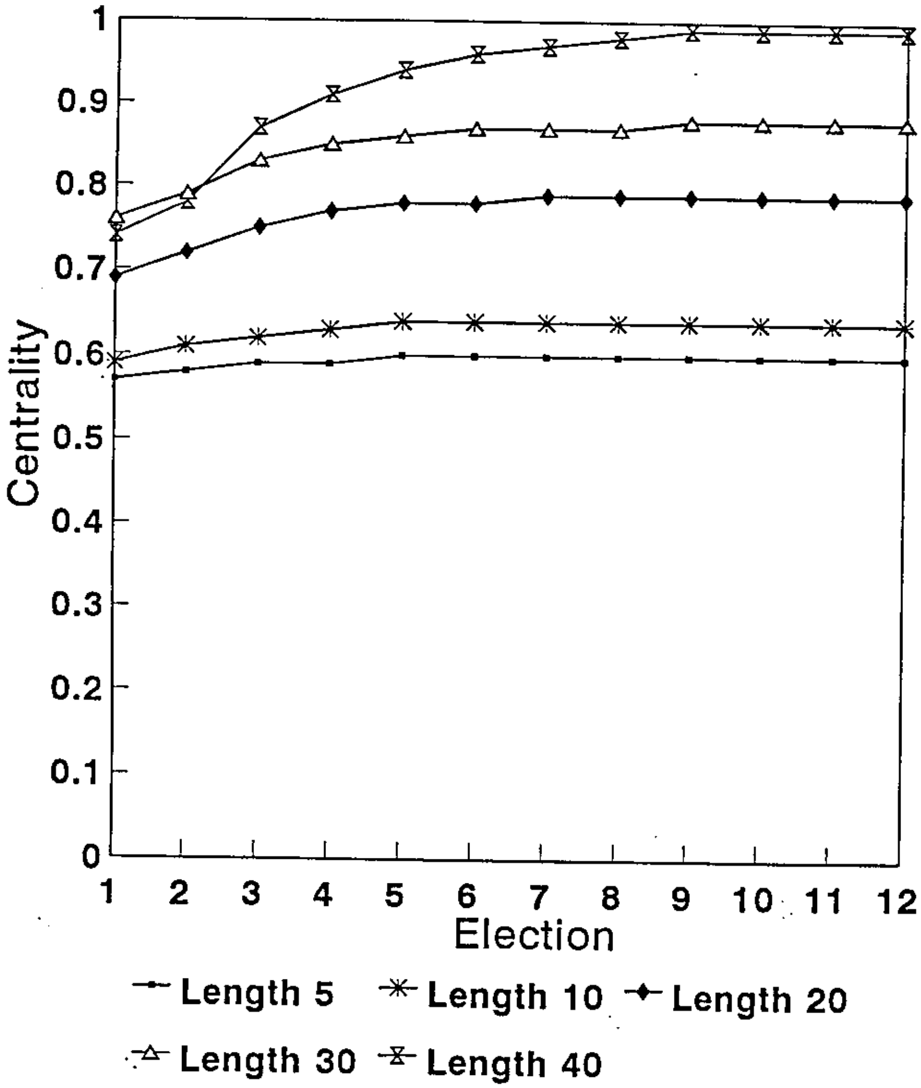


Figure 8
Centrality and Probability of Winning for CAP by Length of Campaign



200 trials
Values for the 8th election

Figure 9
Centrality of Ambitious CAP by Length of Campaign



200 trials