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Abstract

We examine the relationship between population and settled area in a sample of 169 European cities from the early fourteenth century. We compare two behavioral models which make differing predictions regarding the quantitative form of this relationship. The *social reactor model* is based on movement and interaction within the urban built environment and has been applied successfully to contemporary cities. This model predicts a sublinear scaling relationship; i.e., that cities should become increasingly dense as they grow. The alternative *structured interaction model*, derived for the first time in this paper, is based on the assumption that social interactions are strongly channeled by hierarchical social institutions. This model predicts that agglomeration effects should be attenuated in accordance with the strength of institutional constraints. Our results are more consistent with the social reactor model. Although social life in medieval cities was certainly influenced by hierarchical institutions (e.g., guilds, the church, municipal organizations), the effects of these institutions for agglomeration effects appear to have been decidedly secondary. Given the convergence of the medieval data with patterns observed in studies of contemporary urban systems, we suggest urban life in both settings can be modeled using a single theoretical framework. Our results support the hypothesis that cities throughout history share key social networking processes that generate scaling regularities.

Keywords: Urban Systems, Scaling, Medieval Europe, Economic Geography, Population, Population Density

1. Introduction

From a contemporary point of view, Medieval European cities seem both familiar and strange (*1-7*). On the one hand, they are recognizably "cities" because they were concentrations of population in space, often delimited by clear spatial boundaries such as walls. They were also centers of trade, manufacture and innovation. European Medieval cities were typically strongly integrated with their immediate agricultural hinterlands, but also possessed long-range trade networks, had recognizable divisions of labor, and were home to a broad range of people, rich and poor. On the other hand, medieval cities were much smaller than contemporary cities and were dirty, crowded and unhealthy places characterized by high mortality and considerable demographic turnover. Productivity was lower, technology simpler, the market economy much less developed. Church, crown, and guild had a far greater influence on the regulation of social and economic life than in both cotemporary and many earlier urban systems (*4-7*). The freedom and anonymity that we typically associate with modern cities does not seem to have characterized medieval urban centers, except perhaps in relation to their countryside. As a result, we tend to view medieval cities today as structured, corporate societies where class, occupation, ethnicity, religion and other social groupings severely limited social and economic opportunities for individuals and households. As such, medieval cities seem far removed from the contemporary urban experience.

Despite these differences, studying Medieval European cities is not merely a particularistic historical exercise. Many organizational, social, cultural and technological innovations that would usher in modernity, capitalism and the Industrial Revolution originated in these settlements (*8-16*). And from a larger perspective, understanding the similarities and differences between medieval and modern cities is necessary for building a general, historically-informed theory of urbanism that applies to both modernity and the past. Here, we utilize an emerging framework known as *settlement scaling theory* to investigate one dimension of potential similarity. Recent work has identified a number of statistical regularities in the properties of contemporary urban systems which reflect underlying socioeconomic processes of interaction and exchange. This research suggests that, at a fundamental level, cities consist of overlapping social and physical networks self-consistently bounded by settled physical space (*17-19*). In this paper, we investigate whether the relationships between settlement population and land area predicted by scaling theory—and observed in contemporary cities—also characterized medieval cities.

Scholars in a variety of disciplines have long recognized the crucial role of population size for the economic and social characteristics of human settlements (*20-33*). Recent work on urban scaling (*17, 34-38*) has extended this body of inquiry in two ways. First, it has identified a set of robust empirical regularities that relate settlement population size to a wide range of aggregate socioeconomic and infrastructural measures (34). Second, it has led to a formal, mathematical, integrated theory that derives these observed regularities from first principles and makes a number of predictions concerning additional properties of human settlements (*17-19, 39*).

One of the important regularities that have been identified empirically, and predicted by scaling theory, relates the expectation value of a settlement's built-up area to its population size. In contemporary urban systems, larger cities are denser, on average, than smaller ones. A similar pattern has been observed among Pre-Hispanic settlements in the Basin of Mexico (*18*). Settlement scaling theory (*17, 18*) elaborates on a tradition of models of urban agglomeration (*40, 41*) to propose an explanation for these patterns based on social interactions as the "force" that creates and sets the limits of a city's built environment. The idea that the built environment both reflects and influences social practices is a commonplace in the social sciences (*42-44*). The innovation of settlement scaling theory is to identify and explain quantitative regularities in key aspects of this relationship.

Here we examine the relationship between settlement population and land area using a new dataset we have compiled from historical literature and maps for Western European cities ca. 1300 CE. We first provide an overview of the shared properties and varied institutional contexts of Medieval European cities. Then, we introduce two models of settlements as social networks embedded in space to derive predictions for how the spatial organization of medieval cities and towns may vary with population size. Model predictions are then tested against the quantitative patterns of 169 Medieval European settlements within the modern nations of France, Belgium, England, Italy, Switzerland and Germany (Figure 1). We show that the scaling patterns for medieval and contemporary cities are indeed similar, and we discuss how this result reflects similarities in the basic social processes behind the growth patterns of medieval cities as well as modern ones.

2. Medieval European Urbanism

2.1 General Patterns

Medieval European cities were nucleated urban agglomerations that were qualitatively distinct from their rural hinterlands. These settlements generally coalesced around built-up city centers, made up of functionally important buildings like castles, churches, wealthy residences, marketplaces and shops. Towns were often located along roads and waterways and were typically surrounded by walls beyond which suburbs could extend for a kilometer or more (*4-7, 45-49*). Although the urban-rural transition was often gradual, these settlements possessed definable spatial boundaries that differentiated them from associated rural hinterlands – thus making it possible and relatively straightforward with sufficient data to measure the areal extent of urban spaces (see Methods and Materials).

Because of their spatial, political, economic and social differences from the countryside, Medieval European cities can also be said to have had discrete populations. Indeed, medieval writers recorded numbers of hearths, soldiers, taxpayers and citizens from distinct urban areas, and it is by extrapolating these figures that historians have estimated their populations (*50-55*). These urban populations could fluctuate rapidly and asynchronously, and extrapolation-based population estimates must be treated with caution. Nevertheless, the cross-referenced population estimates of medieval cities we have compiled from the literature appear adequate for the purposes of this paper (see Methods and Materials).

Medieval cities shared a number of important socioeconomic characteristics. Significant fractions of urban populations engaged in food production and other extractive rural pursuits—especially in smaller towns and villages (*3-7*, *12*, *56-61*). But as nodes of exchange, consumption and production, Medieval European cities also housed inter-dependent economic networks that facilitated a well-developed division of labor.

Many landed aristocrats, clergy, and wealthy merchants resided in cities and stimulated demand for the importation of luxury goods and services. Elites created networks of market-based patronage, connecting themselves to specialists like traders, middle men, artisans and skilled laborers, many of whom also lived in cities to service this demand. Elites often controlled urban craft industries, using urban labor to manufacture goods for export such as textiles, clothing, leather goods and metals. These sectors in turn stimulated considerable demand for agricultural produce and other basic goods and services, which were facilitated by market-based transport and construction, skilled craftsmen, and retail middlemen (*3-8, 11-14, 16, 57, 58, 62, 63*). The strong hierarchical interdependence of these networks both created and required frequent and intense social interaction.

The hierarchical networks that constituted the urban division of labor were organized into tightly-integrated sociopolitical institutions that controlled the flow of people, goods and information. Guilds integrated the sociopolitical networks of economically-defined trades, craft industries, merchant communities and civil governments (burgesses, burghers, bourgeoisie, etc.) to protect their locally defined common interests. Likewise, Catholic parishes integrated the sociopolitical networks of spatially-defined neighborhoods, providing a basal infrastructure for ecclesiastical governance. Both guilds and parishes were responsible for local public goods provisioning, thereby extending their reach into all facets of social life (*4-7, 63-67*). The extent to which guilds, the church and the crown affected social and economic networks in medieval cities is an important question that we address here using scaling analysis.

Medieval urban populations were often subject to unsanitary conditions, rampant disease, and very high mortality. Even though peasants were tied to the land, rural overpopulation c.1300 induced throngs of landless or under-employed peasants to migrate into cities in search of economic opportunity. Because mortality disproportionately impacted the poor living in cramped squalor, the demographic growth of fourteenth-century cities was largely fed by poor rural migrants seeking work (*2-6, 61, 63-65, 68-73*). Using the Early Modern period as an analogue, de Vries has estimated that cities with more than 5,000 inhabitants had negative rates of natural demographic growth (*74*). This suggests that larger cities required a continual influx of migrants to maintain or grow their populations.

2.2 Regional Diversity

Historians have noted important structural, institutional and other types of differences among the urban systems of Italy, Belgium, Germany, England and France. It is important to highlight these differences because they may have influenced the character and productivity of urban social networks, and thus the scaling relations we investigate.

At 1300 CE Northern and Central Italy was one of the most heavily urbanized regions of Europe (*9, 75*), Fig. 1. The Italian urban network was Europe's primary hub of international and long-distance trade, a major source of demand for goods of all kinds and the birthplace of sophisticated financial institutions that played a major role in the commercialization of Western

Europe (*4, 5, 10, 11, 14, 76, 77*). As a major center of textile manufacturing and craft production, Italian cities also marketed their wares across Europe and the Mediterranean (*4, 5, 9, 11-14, 69, 77, 78*). Larger Italian cities politically and militarily dominated their smaller neighbors. Regardless of whether they were directly controlled by landed aristocrats or merchant elites, political and economic power was concentrated in relatively few hands (*12, 76, 79-81*). With this hegemonic power, Italian city-state capitals dominated their political subordinates through redistribution, institutional privileges, and monopoly. This ensured that commodities, capital, and labor flowed into capitals at the expense of minor centers (*4-6, 12, 14, 75, 77, 79-81*).

The cities of Belgium and western and central Germany also developed a thriving urban economic network by this time. The highly-urbanized economies of Flanders and Hainault dominated northern Europe's cloth and craft industries, and their cities had well-developed commercial institutions as well as capital and labor markets that commanded long-distance trade flows (*4, 5, 16*). The major cities of Germany had similarly well-developed urban economies at this time (*4-6*). Although they were encompassed by various small principalities and the Holy Roman Empire (with the exception of Flanders), the large urban centers of Belgium and western Germany were autonomous (or, "free") cities. Political and economic power strongly overlapped in these cities. Whereas Belgian cities ca. 1300 were solidly controlled by merchant elites (*4, 5, 16*), the trend towards control by small groups of merchant elites was just beginning in Germany (*4-6*). Unlike the Italian city-states, larger cities in Belgium and Germany dominated regional urban economies through protective trade policies and industrial monopolies as opposed to direct political and military control. Membership in urban confederations like the Hanseatic League further reinforced the economic prowess of large commercial centers by securing long-distance trade routes, providing juridical enforcement, and enforcing mutually beneficial trade policies at an inter-regional scale (*4-6, 16*).

As a large and unified territorial state, England's urban political institutions were surprisingly similar to those of Germany and Belgium. The political institutions of English cities c.1300 were mostly controlled by the enfranchised merchant elites (burgesses) of independent municipalities and only indirectly connected to landed elites through economic ties (*13, 64, 65*). In contrast to the political privileges of Italian city-state capitals, the English urban network developed decentralized market-based interdependencies among both large and small towns (*12, 13, 57-59, 64, 65, 82*). Larger English cities became central nodes in a mature urban economic hierarchy (*12, 13, 57-59, 62, 82-84*) in which capital investment and the economic integration of gentry, clergy, and merchants were increasingly incentivized (*13, 82, 85-87*). This resulted in booming craft and woolens industries and commercial specialization (*13, 57, 58, 78, 82*). Demand for these grew (*13, 57, 58, 62, 64*), and rural migrants poured into growing urban labor markets (*12, 64, 71-73*).

Until about c.1290, the cities of the vast Capetian French polity were organized into autonomous municipalities (*bourgs*) that experienced growth in craft industries, trade, and commerce (*4, 8, 63, 88*). However, Phillip IV's centralizing reforms beginning c.1290 took an enormous toll on the French economy and its urban network. The plundering and prohibition of Flemish merchants, arrest and taxation of Italian merchants, and the barring of wool exports effectively canceled the French woolens trade, marred its urban industries, and sank the trade flows through the Champagne fairs (*8*). Repeated war with (and annexation of) Flanders c.1297-1330 further damaged the economy of Belgian and Northern French cities and disrupted the lucrative cloth trade and industry there (*4, 5, 8, 49, 88*). Moreover, Phillip IV's simultaneous removal of urban tax exemptions, retraction of city self-government charters, and centralization of urban administration under the royally-dominated landed aristocracy all prevented urban economic rebounds (*4-6, 8, 49, 63, 88*). As a result, the cities of France and, to a lesser extent Belgium, experienced a significant economic disruption c.1290-1340 that is directly attributable to war and institutional change. These disruptions did not occur in Italy, Germany or England. Thus, one might expect these disruptions to be reflected in the data for French and Belgian cities, relative to the other urban systems we consider.

Northern France, Flanders, and Wallonia formed a relatively continuous urban network in the early 14th century—tied together by both geographical proximity and a shared history. In addition to shared Carolingian backgrounds, their mutual 12th and 13th century urban efflorescence heralded institutional similarities (bourgeoisie self-governance) and regional economic integration (*4-6, 8, 16, 88, 89*). Due to their intertwined and mutually disruptive historical trajectory ca. 1300, we combine the data from France and Belgium in the analyses below.

3. Models of Social and Spatial Organization in Cities

Medieval cities provide us with an opportunity to discuss two seemingly disparate views of cities. On the one hand, we can imagine a city as a self-organized, non-centralized type of social and spatial arrangement where interactions between people are only impeded by four potential types of obstacles: the complementarities between individuals; negative interactions (e.g., crime, disease); separations among individuals caused by institutional, political, or cultural factors; and the costs of movement. Such networks, which are 'well-mixed' (in the language of epidemiology), characterize modern cities well. This is consonant with the idea of modern cities as social and physical spaces that facilitate the unimpeded flow of people, ideas and information. We will refer to this view as the *social reactor model*.

On the other hand, we may think of a city as a settlement controlled by a diverse set of hierarchically-organized social institutions such as political authorities, religious organizations, guilds, elite and kinship groups. We may hypothesize that such social and economic structures regulate and control the social and economic contacts of individuals, thus strongly shaping the nature of social interactions that generate economic relations, flows of ideas, and the joint creation of knowledge. We shall call this the *structured social interaction model*. Below, we introduce these two scenarios and formulate their expectations in terms of scaling relations—specifically the relationship between population size and areal extension—to investigate the nature of the medieval city via the empirical analysis carried out in subsequent sections.

3.1 The Social Reactor Model

The starting point of all spatial models of cities, including very simple classical models of geography and urban planning (*90*), is the idea of a spatial equilibrium. The city as a spatial equilibrium results in a changing density of land use that is set by the balance between socioeconomic interactions, such as trade and social opportunities, and land and transportation costs (*91*). Expressed at a slightly more abstract level, socioeconomic activities create attractive ("centripetal") forces that create a city, but are kept in check by repulsive ("centrifugal") forces related to the costs of maintaining such dynamics operating over space and time.

From this perspective it is immediately apparent that such mechanisms are very general and could conceivably apply to any human settlement throughout history (*18*). The fundamental idea behind settlement scaling theory is that all human settlements—regardless of scale or social complexity—share fundamental quantitative similarities in terms of their overall form and function. This derives from the existence of general advantages of social agglomeration, whether it be for defense, shared infrastructure, religion or trade (*92*). Similarly, there will always be restrictions to land use and costs deriving from transportation even in societies with economic and political organizations very different from those of contemporary cities. In the following paragraphs we derive the basic expectations of this theory (More detailed derivations are given in Ref. (*32*)).

First, we derive the expected relationship between the population of a settlement and its overall land area. This can be done without assuming that the city is radially symmetric, as in older models (*93*). To do this we balance the average benefit that accrues to an individual from social interactions, $y = GN/A$, against the associated transportation costs, $c = \varepsilon A^{H/2}$. The first expression is derived by embedding a social network in space, where an individual meets others in a settlement at an average rate, y, proportional to its density (N/A). The parameter G is the net benefit per interaction in a context of agglomerated individuals (*94*). The nature of the social and economic interactions that an individual can experience is certainly context-dependent, and very different in a medieval vs. a modern city. However, such differences can be captured in a simple way on aggregate via changes in the magnitude of G. The per capita cost of interaction (c) is set by the expense of movement per unit length ε, which is a function of technology (e.g. walking vs. horse-riding) and also on how individuals explore the space of a city (A) parameterized by a fractal dimension, H. For H=1, individuals can explore the city through a line-like trajectory, whereas as $H \to 2$, they would explore the city exhaustively as an area, which typically becomes difficult in large places. In the limit, $H \to 0$, individuals stay limited to a single place and the city falls apart as a socioeconomic network of interaction. To see this we equate benefits to costs and solve for area as a function of population to obtain

$$A(N) = a\,N^{\alpha}, \text{ with } \alpha = \frac{2}{H+2}, \quad a = \left(\frac{G}{\varepsilon}\right)^{\alpha}. \tag{1}$$

This simple result already captures some of the most important characteristics of human settlements. First, if social benefits are small relative to transportation costs, which is likely true in medieval cities, then a will be small and all settlements will be quite dense (we refer to the parameter a as the baseline area per person). We also see that if H=0, as might occur in a segregated settlement, the total settled area becomes proportional to population and no agglomeration effects are present. For H=1, one obtains the special value for the exponent, $\alpha = 2/3$, a situation we described elsewhere (*18*) as the amorphous settlement model. In such settlements, population density, n, increases strongly with population size as, $n = \frac{N}{A(N)} = a^{-1}N^{1/3}$.

However, in deriving these properties we have not yet considered the fact that urban land use and built space become increasingly structured as settlements grow. Specifically, towns and cities become organized in terms of access networks (streets, canals, paths) and places, which include places of work and residence as well as public spaces (*95*). This leads to a characteristic topology of cities, set by the spatial relationship of places to their accesses that is general to all cities, regardless of their specific geometry (*96*). This means in particular that the relevant space for social interactions in cities is set by its *access network*. We designate the total area of this network A_n, and compute it via a decentralized infrastructure network model, which predicts that the area of network per individual is set by the length scale derived from the density *n*, as

$$A_n = l\, n^{-1/2} N = a_0 N^{1-\delta}, \text{with } \delta = \frac{H}{2(H+2)}, a_0 = l\, a^{1/2}, \tag{2}$$

where *l* is a length scale measuring the width of the network at each end-place, e.g. a door. This calculation can also be derived from a more detailed model of infrastructure from which it can be shown that the costs of transportation over these networks, *W*, scales superlinearly with population size, $W \sim Nc \sim N^{1+\delta}$, just like socioeconomic interactions in built space, Y=$yN \sim G\frac{N^2}{A_n} \sim N^{1+\delta}$ (*18*). This results in a new spatial equilibrium model for a *networked settlement*. The basic unit of settlement productivity is then the total number of socioeconomic interactions it sustains, $K(N) = k_0 N^{1+\delta} \sim Y(N)$ (*18*). Such a model predicts many of the characteristics of modern cities (*34*), in broad agreement with empirical observations. It also applies to at least some large settlements in antiquity, such as those of the Pre-Columbian Basin of Mexico (*18*).

Relative to models from urban economics, scaling theory has the advantage of not requiring the specification of production functions or utility functions accounting for the behavior of firms or consumers. Instead, increasing returns to scale in the productivity of cities -- the fact that Y increases per capita with city size -- derives from network effects (a spatial version of Metcalfe's law). A production function for settlements, showing increasing returns to scale, can then be derived as an emergent property of these socioeconomic networks embedded in the structured spaces of each settlement (*39*).

For the analysis carried out in the remainder of this paper, two properties of the social reactor model are especially important. First, this model predicts the settled area of a city or town in a given urban system should increase with its population, on average, with an exponent $\frac{2}{3} \leq \alpha \leq \frac{5}{6}$, where the lower range would correspond to amorphous settlements, typically small sites, and the upper value to the fine measurement of all built up surfaces in structured settlements (This is not possible to do directly, but see below). The second property derives from the extent to which social interactions are structured. This is not to say that the social reactor model is at all incommensurate with hierarchical institutional structures— only that the number and degree of individual socioeconomic interactions are not strongly determined by them. A graphical depiction of the social reactor model shows multiple connections between all individuals in a city (Figure 2A). Since this may not always be the case, we next develop an alternative model that incorporates the effects of social groups in shaping the socioeconomic networks of cities.

3.2 The Structured Social Interaction Model

We now show how social groups and institutions can mediate social interactions and introduce further constraints to urban networks that change basic scaling relations quantitatively. There are many ways in which social groups can condition individual social interactions. Our aim here is to build a simple model which will allow us to discuss the problem and modulate the predictions of the previous section away from the simplistic assumption of full social mixing.

Consider, then, a simple model for an urban social network where social interactions beyond the household are determined by a hierarchy of "institutions" (Fig. 2B). Such institutions may be formal, in the sense of guilds or the church, or they may be more informal, such as family groups, ethnicities, and social class. The structured interaction model supposes that most or all social interactions are strongly constrained or restricted to these hierarchical institutional structures, with little possibility of outside interaction. While this scenario is simplistic, it gives us a general sense of the limitations that such an arrangement can place on socioeconomic opportunities within a larger population.

As above, consider a settlement with a population of size, N. We assume that each individual in the settlement belongs primarily to a single group at the lowest level, and that through this group's connections may indirectly belong to other groups at higher levels of the hierarchy. Beyond the local level -- the household -- we assume that social interactions are mediated by these groups only. The hierarchy of groups in Figure 2B is parameterized by h levels and at each level we assume that b connections are possible. Thus, b is the average branching ratio in this hierarchical network. This is similar to an organizational chart for a company, and thus, suggests conceptualizing the city in analogy to a classical firm (*97*). The relationship between N and b is familiar for any balanced tree graph, as $h(N) = \log_{\mathrm{b}} N$.

Clearly, even a hierarchy such as that depicted in Figure 2B allows every individual to make contact with any other, in principle, through higher nodes (groups) in this structure. So, a final ingredient is that institutions can "dampen" or restrict such interactions. This results in a more divided social network, with institutions as the gatekeepers for contact between individuals. We assume that there is a damping factor, $0 \leq s \leq 1$, that affects each link controlled by a group. This number, like b, is in general specific to each relationship, an elaboration of the model that we do not pursue here.

It is now straightforward to compute the number of contacts an individual can have as higher and higher groups mediate his/her interactions. The crucial parameter is the *social horizon*, $r = sb$. If r>1 (s>1/b) then the city persists as an integrated socioeconomic system. For r<1 (s<1/b) it falls apart into a number of disconnected small groups. To see this, consider the number of interactions for a typical individual. At the first level, there are b interactions, at the second there are b+ sb, at the third b+sb+(sb*b) and so on. The total number of connections of an individual, at a given level of dampening, is then the sum of the finite geometric series

$$k_s(N) = b[1 + sb + (sb)^2 + \cdots] = b\,\frac{1-r^h}{1-r}. \quad (3)$$

For very small s, r << 1 and $k_s(N) = b/(1 - sb) \cong b$, that is, the interactions stay essentially circumscribed to the household. For s close to 1, r>1 and we can write

$$k_s(N) = b\,\frac{(sb)^{h-1}-1}{sb-1} \approx s^{h-1}k(N) = N^{-\theta}k(N), \text{ with } \theta = \left|\frac{\ln s}{\ln b}\right|, \tag{4}$$

where $k_s(N)$ is the average connectivity per individual in the structured interactions model and where we assumed that the terms that do not involve *s* correspond to the connectivity computed in the previous subsection, $k(N)$, since in that case the groups do not exert any effect on connections.

Thus, in this limit as $s \to 1$, the exponent θ vanishes. However, while θ is non-zero, it produces a small negative correction to standard agglomeration exponents, making them effectively weaker. If we write the productivity of a city as proportional to its connectivity [36], and use the expressions from subsection 2.1, we conclude that the built up area of the city then scales as

$$A_n \sim \frac{N}{k(N)} \sim N^{1-\delta+\theta}, \tag{5}$$

in the case when the effect of institutions is small but non-zero. In the opposite limit, when institutions are very restrictive, $A_n \sim N$, leading to no population densification (*n*=constant) with settlement size. This shows how hierarchical institutions that limit social opportunities weaken and can even destroy socioeconomic agglomeration effects, spatial densification and ultimately cities themselves.

The hierarchy depicted in Figure 2B is a simple instance of a more general situation long studied by social scientists, namely the relationship between social position, power and control (*98, 99*). Again, we emphasize that this model does not equate to the mere existence of hierarchical institutions, a pervasive situation among both pre-modern and modern cities. Rather, the structured social interaction model supposes that there are few social interactions that occur outside of these hierarchical networks. Here, by construction, top institutions have high network *centrality* (various centrality measures apply) and, as such, control the largest number of possible connections and all connections between groups that are very different from each other. It is also important to note that, if such restriction of interactions would be circumscribed to only negative interactions, such as violence, or if these institutions could *reduce* the cost of interactions by acting as central places or clearing-houses, they could instead contribute to *stronger* agglomeration effects. This can be modeled by considering the values of *s*>1 and flipping the sign of the exponent θ. Many modern institutions, such as universities, are built with the intention of promoting positive social interactions, related to innovation and economic growth, beyond what is possible by chance within a city (*100*).

4. Results

Settlement scaling theory suggests that many aspects of social and economic complexity can be considered as emergent properties of scale in social networks that are embedded in space. Since we can independently measure the population and areal extent of medieval cities, the aggregate-level outcomes (e.g. population density) of micro-level social processes are observable. This makes it possible to analyze the relationship among these quantities across settlement systems of markedly different times and places.

The theoretical framework of settlement scaling should apply to pre-modern settlements insofar as they can be characterized as (structured) social networks co-located in space and time. If so, our framework leads to the following expectations which can be tested by examining the relationship between population and settled area across the cities of a given urban system. First, the land area encompassed by a Medieval European city should increase more slowly than its population, which is to say, with a scaling exponent $\alpha < 1$, in accordance with the social reactor model. Second, this exponent should lie somewhere between 2/3 and 5/6 but may be relatively larger (closer to 1 and potentially >5/6) in contexts where social interactions were relatively constrained by hierarchical social institutions, in accordance with the structured social interaction model. Third, the scaling prefactor a should vary in accordance with differences in the relationship of interaction benefits to intra-settlement transportation costs across urban systems. Since it appears reasonable to assume that transportation technology was constant across Medieval Europe, any differences in the scaling prefactor should derive from differences in the productivity of social interaction. So for example, based on the discussion in section 2.2 (above), we might expect the scaling prefactor to be lower (and the exponent higher) for the French and Belgian urban system than was the case for the other systems.

We estimate scaling exponents and prefactors through linear regression of the natural logarithm of areal extent against the natural logarithm of population size:

$$\ln(areal\ extension_i) = \beta_0 + \beta \ln(population_i) + \varepsilon, \tag{6}$$

where i indexes a city within a specified urban system and ε denotes an i.i.d. Gaussian white noise. Equation (6) was estimated using OLS with the Huber/White correction for heteroscedasticity. Scatter plots for the dependent versus independent variables (Figure 3) show linear relationships indicating that the model is not mis-specified (Estimations were done using the Stata v11 software package). The regression results for four regional European urban settlement systems, and the pooled dataset, are given in Table 1. Note that equation (6) is merely the log-transformed version of equation (1), and thus β, the scaling coefficient in the log-transformed case, is also the scaling exponent α in equation (1). In the same way, β_0, the intercept in equation (6), is related to the prefactor a in equation (1) by $a = e^{\beta_0}$.

The results, given in Table 1, reveal that all estimated scaling coefficients are significantly less than one and fall within the range $2/3 \leq \alpha \leq 5/6$. These results are generally in line with those generated using data for modern urban systems, although the values for the medieval cities are somewhat closer to 2/3 whereas in modern cities they are typically closer to 5/6. This suggests that medieval cities grew partly by in-filling of available space, with streets and paths emerging less formally than is the case in contemporary cities.

It is notable that the actual values of the scaling coefficients are quite similar across systems, with point estimates varying between .72 and .84. All of these are within the range $(2/3 \leq \alpha \leq 5/6)$ as suggested by the social reactor model. It is also notable that the scaling coefficients and intercepts are even closer for three of the four systems (and in the pooled data), with point estimates for the coefficients varying only between .72 and .76 and intercepts varying only between 0.08 and 0.12 ha. This uniformity is quite remarkable given the varied cultural, political and economic institutions that characterized medieval England, Germany and Italy. Apparently, these differences did not affect overall patterns of social and economic networking to an appreciable degree. This finding provides striking support for both the social reactor model and for the general applicability of settlement scaling theory to pre-modern as well as contemporary systems.

Finally, we note that the scaling coefficient is notably larger, and the intercept notably smaller, for the French and Belgian system than it is for the other systems. Although its coefficient is well within the normal range for modern cities, the French and Belgian system is markedly closer to the predictions of the structured social interaction model than its European counterparts. Once again, this pattern appears consistent with historical records which suggest institutional changes that dampened social mixing occurred in France and Belgium in the decades immediately prior to the period of interest.

5. Discussion

It appears that, despite their many structural differences, and a temporal distance of 700 years, medieval urban centers share at least one basic similarity with modern settlements: larger cities have higher population densities than their smaller counterparts in a given region. Overall, these data conform in a striking way to expectations of the social reactor model for the quantitative form of the area-population relation. Even though medieval cities were certainly structured by hierarchical institutions, the aggregate outcomes of their socioeconomic interactions expressed in the area-population scaling relation suggest that such restrictions did not exert much effect on overall social and economic mixing, such that the free flow of people, ideas, and information was possible even in systems characterized by large and powerful institutions. In other words, the structured social interaction model fails to capture the dominant dynamics of movement and interaction in medieval cities. We take these findings as an indication that the underlying micro-level social dynamics of medieval cities were fundamentally similar to those of contemporary cities. Despite their many structural and functional differences, and contrasting macro-level processes that influenced urbanization in each context, both medieval and modern cities appear to be characterized by social networks that become increasingly spatially-dense, at specific predictable rates, as they grow.

In the context of settlement scaling theory, our results have important theoretical implications for medieval urbanization. For example, they imply that rates of social interaction increased along with settlement density in larger medieval cities, and this in turn enabled greater organizational efficiency, productivity and functional diversity. This capacity made cities central nodes in the development of socioeconomic and political institutions that expanded the division and coordination of labor, centralized networks of commodity flows, and intensified political and economic organization (*17, 18, 34-38, 101*). These processes are well documented in historical sources, and our analysis has now provided a quantitative model for their emergence. Our results further suggest that modern and medieval cities had similar underlying functions, and on a deeper level, that urbanization itself may have general micro-level root causes. In this way, theories of contemporary urban processes (e.g. from urban economics or economic geography) may be applicable to the past not because of similarities in macro-historical structures (e.g. political and economic institutions), but because of fundamental similarities in micro-level behaviors (e.g. agglomeration and interaction networks) and their emergent system-level outcomes (urban land use, division of labor and economic productivity in cities). Rather than assuming that modern theories either do or do not apply to the past, this approach forces us to consider *how* and *why* past social, economic, and political conditions impacted the structure and dynamics urban systems.

It is also interesting that our empirical results suggest regional historical processes impacted the scaling relationship for France and Belgium in accordance with predictions of the structured social interaction model. As discussed in Section 2.2, we suspect the observed differences are the result of the policies introduced by Phillip IV (c.1290-1337), which negatively impacted French and Belgian cities relative to those in the other regions. In particular, warfare and centralizing institutional change under Phillip IV damaged national economies, degraded the social fabric and disrupted flows of labor migration into cities. Since our data directly correspond to the impacted period (c.1290-1330), and quantitative metrics of French urban trade during the post-1290 period reflect these disruptions (*8*), it seems reasonable to expect that our data should reflect them as well. This would imply that the higher scaling exponent and lower prefactor observed in the French and Belgian data are the result of contextual factors that simultaneously restricted social mixing (i.e. increased the s term from the structured social interaction model, thus increasing the scaling exponent) and reduced the productivity of individual interactions (thus reducing the G term from the social reactor model and reducing the scaling prefactor). If this correlation is genuine, this finding is an initial indication that scaling analysis can identify differences in the functioning of urban systems that derives from variation in social, political and economic conditions.

If analyses of additional urban systems are also successful in identifying patterned variation associated with specific historical contingencies, the theoretical framework of settlement scaling may prove to be a useful addition to the toolkit of historians who study of pre-modern urbanization. Indeed, settlement scaling measures variables common to human settlements regardless of time and place, similar to methods like central place theory and rank-size analysis. Because the quantitative expressions of these scaling relationships emerge from particular historical contexts, urban scaling theory provides an analytical framework for comparing urban systems over time and space in terms of specific parameters, the values of which reflect key systemic properties. As such, these metrics can be expanded into the early modern period and the industrial revolution to investigate fundamental patterns associated with major economic and political transformations. By providing an analytical framework and a set of theoretical expectations, scaling analysis can make significant contributions to a wide variety of topics in social and economic history.

6. Data and Methods

Although our data represent only a sample of the settlements known to have been inhabited during the early fourteenth century, we refer to these data as collectively characterizing “Medieval Europe”. Similarly, we use the term “city” to describe settlements that medieval scholars have often referred to as “towns,” even though a considerable number of our cases had fewer than 10,000 inhabitants. This is because our aim is to investigate how settlement space responds to population at multiple scales.

We collected population and settled area data on medieval settlements in Western and Central Europe from secondary and tertiary sources. Some areas were measured from published historical and archaeological maps (using ImageJ software), and others were taken from settled area estimates in the secondary literature. Population estimates were drawn from the secondary literature and tertiary databases. Each case comprises both population and area data derived from independent evidence; we did not use population estimates based on settled area extrapolations. All population and area estimates were focused on the early fourteenth century, roughly spanning a 50 year period from c.1290-1340. All cases had populations of >2,000, and most cases had populations >5,000. Our dataset includes 169 cities located in modern-day England, France, Belgium, Germany, Austria, the Czech Republic, Switzerland and Italy. These were divided into four "national" groupings: England (n=40), central and northern Italy (n=31), France and Belgium (n=59), and western and central Germany (n=39). Whereas England was characterized by a unified state, and both Italy and Germany were characterized by small city-states, and the France and Belgium dataset is more heterogeneous; see Figure 1.

Quantitative data on medieval settlements are inherently contingent on philological interpretations, secondary reconstructions, and historiographical paradigms. Moreover, the sources to which we had access were of variable quality and abundance across regions. In order to deal with these problems, we developed a data collection protocol that systematically compared and contextualized different quantitative estimates when possible. Rather than blindly assembling data through uncritical use of available sources, our method enabled us to evaluate the degree of confidence in each estimate. The core of this method involved the following steps:

1. We outlined the major trends in the urban historiography (formal historical methods and assumptions) of medieval regions from the secondary literature;
2. We collected population and area estimates from works representing the most recent school of thought about each place, whenever possible;
3. We noted population and area estimates from major tertiary compilations of population and area data (*53, 102, 103*); and
4. We compared the available evidence for each case to arrive at a single provisional estimate suitable for scaling analysis.

While these data can be seen as ballpark estimates by leading experts on the basis of imperfect evidence, they nevertheless represent a major improvement over tertiary databases (~30-40 years old) that are still regularly used in an uncritical fashion (*53, 102, 103*).

Conflicting population estimates of medieval settlements are common, and this makes careful source criticism especially important. In cases where recent secondary sources were available, we analyzed the context, methods and reasoning behind each population estimate in order to select the most up-to-date figure. This was possible for all English cases because the evidence exists on a national scale (nationwide tax rolls), as well as many cases in France, Belgium and Italy. Secondary literature was available for only the largest German cases. Although population data were systematically collected for both England and Italy, we decided to use the well-researched datasets of Campbell (*83*) and Malanima (*75, 104*), respectively, because of their high quality. These datasets matched our own independent estimates very closely for both regions.

For individual settlements where secondary treatments were scarce or unavailable, we assembled all of the estimates and selected among divergent/contradictory data through internal coherence and crude historiographical criticism. This involved giving preference to estimates made by recent and reliable secondary sources that were written by experts on a particular urban system (e.g. Brabant). The reliability of sources was gauged by how many well-cited research publications an author had on the particular subject and whether the cited reference was an in-depth analysis or a passing comment based on preliminary evidence or assumptions. If detailed scholarly discussions on the case were unavailable, we began to narrow down the set of possible estimates by removing estimates from older sources that stood apart from more recent sources. In the case of tertiary sources, source criticism was based on the estimate's citation—not the tertiary source itself. After identifying the most reliable, recent and coherent cluster of estimates, we took the mode (when one existed) or the average of all available estimates as our point estimate for the population of a given city ca. 1300 CE.

The same procedure was followed for the estimation of settled areas when conflicting estimates were encountered. Topographical map reconstructions of medieval European cities produced by archaeologists and historians are largely embedded in case-specific studies of particular locales—making the compilation and measurement of settled area data a very time consuming endeavor. While it would have been relatively simple to only measure the walled (or fortified) areas of medieval towns, many settlements had shifting extramural suburbs that extended beyond their walls, while others never filled their walled areas completely. Either way, these intramural and extramural areas are reflected in contemporary population counts (*53, 105-107*). Accordingly, our method involved 1) conducting a survey of the archaeological and historiographical literature on medieval urban regions, 2) collecting maps, area estimates, and contextual specifications on each case, and 3) comparing the available evidence of each case to arrive at a single provisional estimate suitable for analysis.

When measuring maps for their settled areas, it was necessary to contextualize the map to understand the chronology of its features and weigh alternative area estimates. For example, we found a number of conflicting area estimates for Bristol c.1300. Bristol's outer wall was constructed first during the late 12th century, extended southward during the 13th century, and *murage* grants were issued contiguously from 1232 through the 15th century (*48, 108*). Kermode indicates that its walled area was 55 ha (*109*), but Russell estimated that the settled area had to have been over 80 ha due to suburban sprawl ca. 1300 (*53*). Most maps of Bristol's walled area c.1300 match Kermode's 55 ha (*45, 110*), but leave out their extramural suburban areas. Nevertheless, Keene's map of Bristol c.1300 (see Figure 5) duly indicates a walled area of 55ha, and a total settled area of 130ha when suburban areas are included (*111*). This measurement is further corroborated by Russell's earlier suspicion that Bristol must have encompassed more than 80 ha ca. 1300. We therefore used the estimate of 130ha derived from Keene's map in our analysis. This example embodies the general process of estimation for each of our cases.

Despite careful data collection, the match between settled area estimates and population estimates is not always precise. The walls of European towns were increasingly expanded over the course of the later middle ages, and it is well attested that the suburbs of medieval settlements fluctuated greatly, rapidly and asynchronously in accordance with economic and demographic trends (*4, 5, 7, 45-48*). We deliberately collected estimates pertaining to the early fourteenth century (c.1300-1350) because of the relative abundance and quality of the data for this period. Nevertheless, our estimates for each case should be understood as inferences of the approximate size and extent of settlements over a period measured in decades. Not unlike settled area measurements from nineteenth- and early twentieth-century sources, this level of resolution is simply the reality of the available evidence.

While there may be multiple reasons to doubt the accuracy of the individual data points in our analyses, we believe these data are adequate for an initial investigation of population-area scaling relationships for the following reasons: (1) the errors are relatively unstructured across settlement sizes; (2) the cases in each analysis encompass the full range of settlement sizes in the given system; (3) the magnitudes of the population and settled area estimates vary by several orders of magnitude; and (4) we have a sufficient number of cases to satisfy the central limit theorem at the heart of OLS estimation.

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Figures

Table 1. Estimation results for *ln(areal extent) = β₀ + βln(population)*.

Greater France and Belgium	*N = 59*		**England**	*N = 40*	
		95% CI			*95% CI*
intercept	-3.356		*intercept*	-2.124	
scaling coefficient	0.837	[0.744, 0.931]	*scaling coefficient*	0.739	[0.581, 0.879]
R^2	0.85		R^2	0.79	
Northern and Central Italy	*N = 31*		**Germany**	*N = 39*	
		95% CI			*95% CI*
intercept	-2.284		*intercept*	-2.502	
scaling coefficient	0.725	[0.559, 0.892]	*scaling coefficient*	0.763	[0.616, 0.909]
R^2	0.72		R^2	0.79	
Pooled Data	*N = 169*		**Notes**		
		95% CI	Estimations done using OLS with corrections made for heteroeskedasticity. Confidence intervals are in brackets (all scaling coefficients are significant at the .05 level).		
intercept	-2.286				
scaling coefficient	0.733	[0.675, 0.790]			
R^2	0.82				

Figure Captions

Figure 1. Western European medieval cities ca. 1300 CE (n = 169) examined in this paper, from England (red; n = 40), France and Belgium (blue; n = 59), Northern and Central Italy (green; n = 31) and Germany (yellow; n = 39). All cities examined had populations of >2,000, and most cases had populations >5,000.

Figure 2. Schematic Social Network of Towns and Cities. A. An unstructured network where anyone can in principle connect with anyone else, subject to limitations deriving from cost of movement. Such a network is characterized by increasing connectivity with city population size, with mean degree $k(N) = k_0 \, N^{\delta}$, $\delta \sim 1/6$. B. A structured socioeconomic network. In this case, interactions between individuals are regulated by social groups and institutions (black squares) and may be damped by a factor s<1, for each level of institutions involved. If the parameter s<1, the net effect of institutions is to weaken social possibilities and thus reduce agglomeration effects, taking the exponent of the scaling of area with population for settlements closer to unity.

Figure 3. Estimation of Area – Population scaling relations for: A. England (red); B. France and Belgium (blue); C. Northern and Central Italy (green); and D. Germany (yellow). In all charts the black line represents proportionate (linear) scaling; the yellow line the theoretical prediction where $\alpha = 5/6$; and the red line the best-fit line from OLS regression of the log-transformed data.

Figure 4. The Area – Population scaling relation for the entire data set after (A) pooling and (B) centering and pooling. In B, the data series have been centered by subtracting the average scaling relation in logarithmic variables, $\langle \ln A \rangle = \ln a + \alpha \langle \ln N \rangle$, from both datasets, so that the data for each nation share the same average coordinate on both axes.

Figure 5. Bristol's built-up areas in the later middle ages (late 13th – early 14th centuries), including built-up suburban areas shaded in grey (redrawn from Keene, 1975: 101). The red line indicates the 130 ha settled area we measured for the city, whereas the inner area circumscribed by walls and rivers measures only 55 ha. Even our relatively conservative outline of the city's built up area more than doubles Bristol's settled area.

Figure 1.

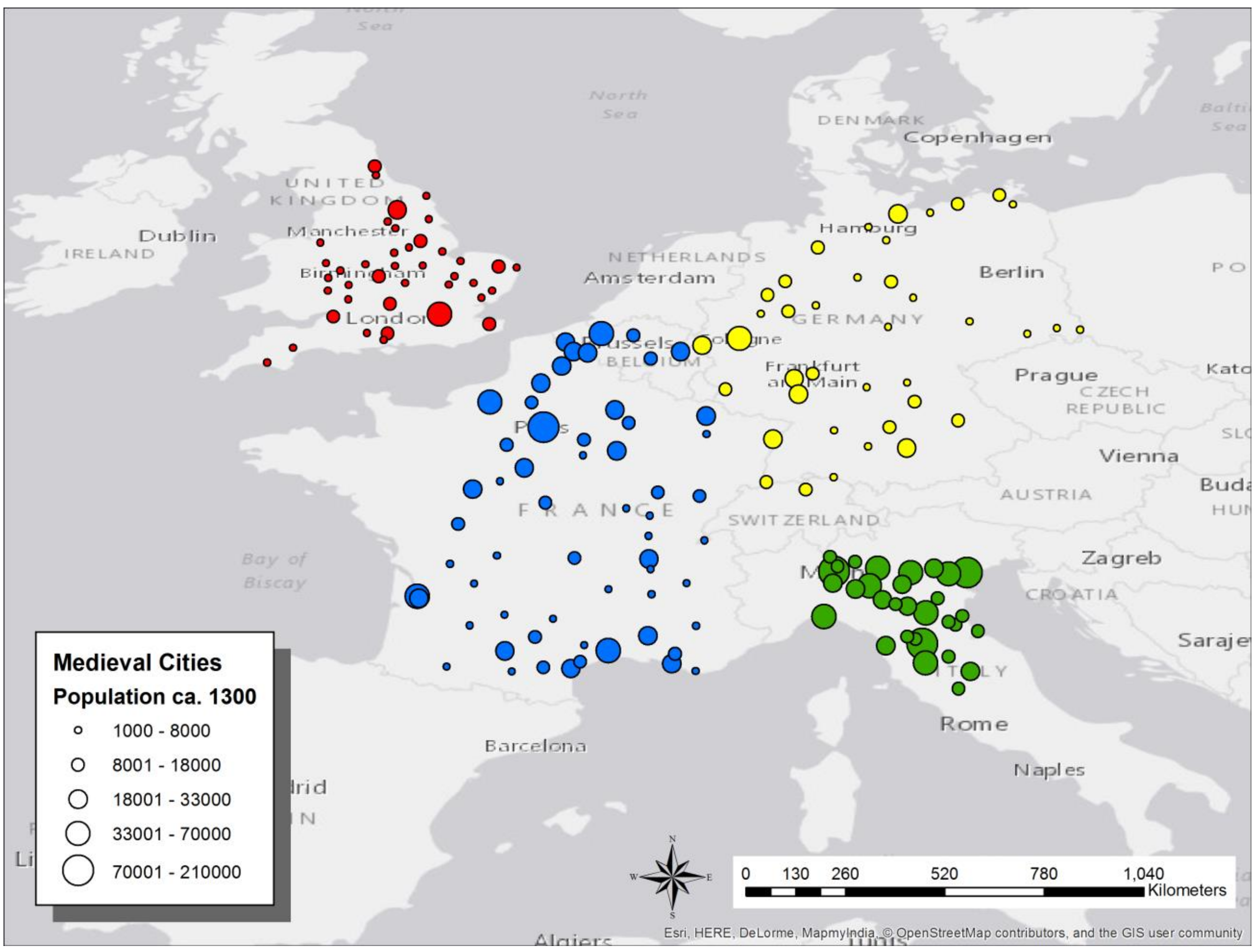


Figure 2.

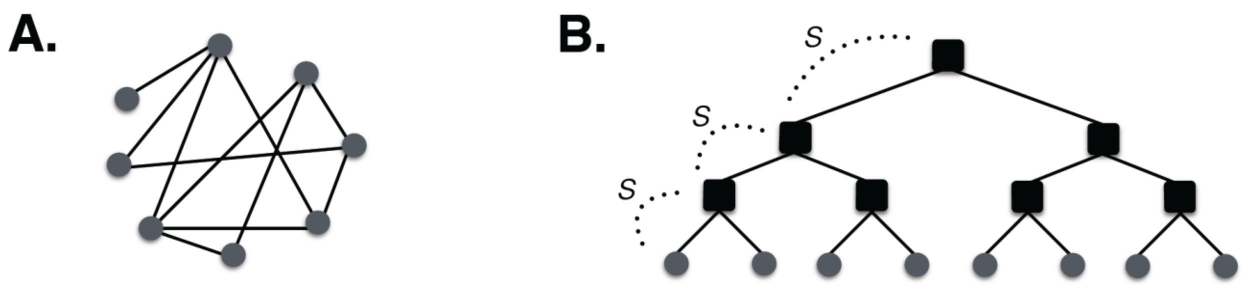


Figure 3.

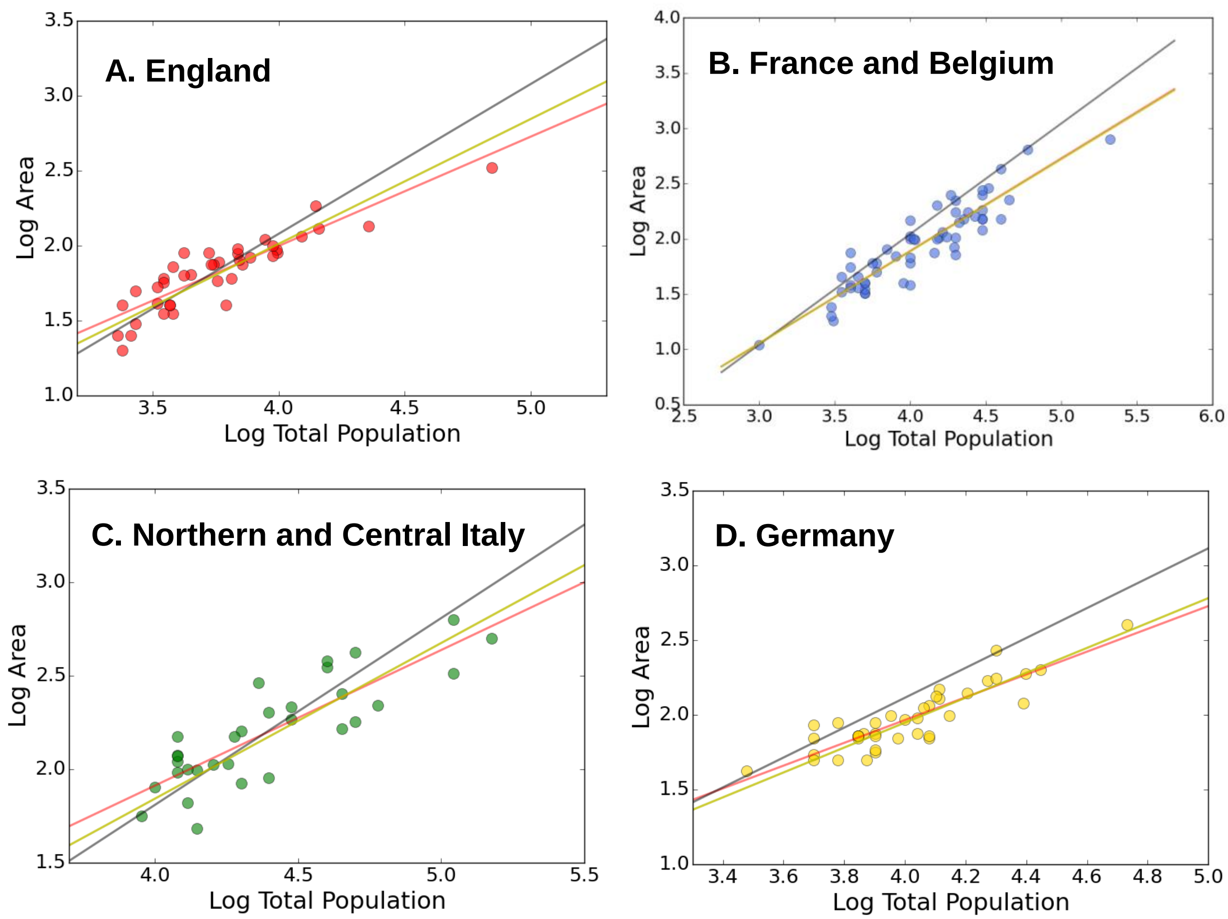
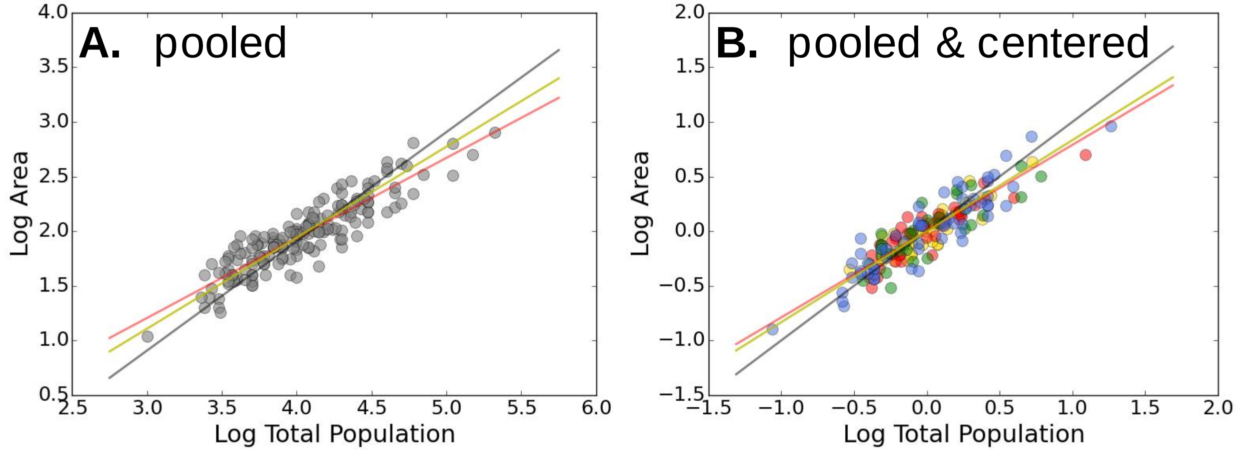
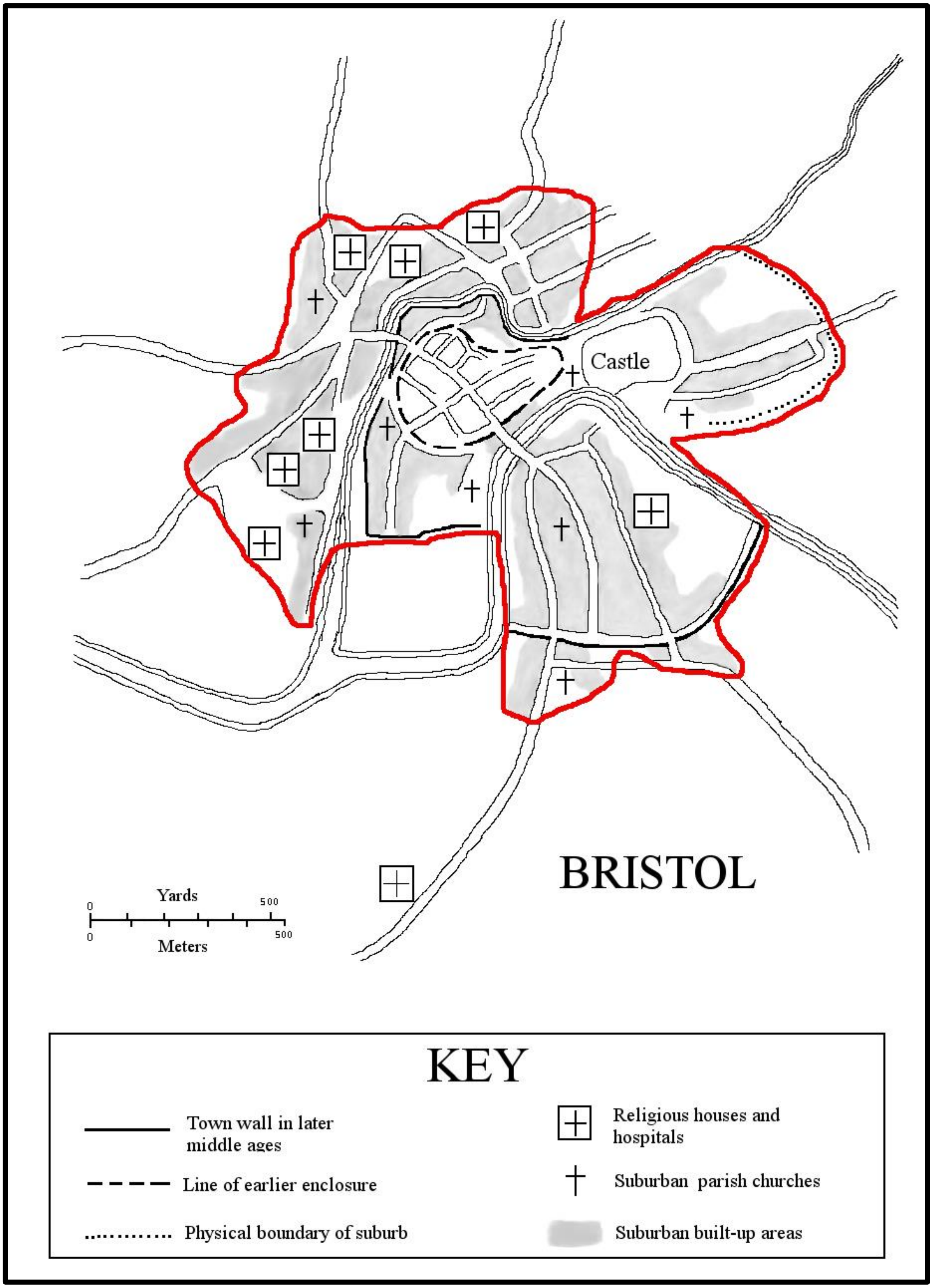


Figure 4.



A. pooled
Log Area
Log Total Population
B. pooled & centered
Log Area
Log Total Population

Figure 5.


Castle
BRISTOL
Yards
0
500
0
500
Meters
KEY
Town wall in later middle ages
Line of earlier enclosure
Physical boundary of suburb
Religious houses and hospitals
Suburban parish churches
Suburban built-up areas