Time and Money

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ABSTRACT: General equilibrium is timeless, and without outside money, the price system is homogeneous of order zero. Some finite horizon strategic market game models are considered with an initial issue of fiat money held as an asset. For any arbitrary finite horizon, the solution is time-dependent. In the infinite horizon, time disappears with the initial issue of fiat money present as circulating capital in the fully stationary state and the price level is determined.

1 AN APPROACH TO THE THEORY OF MONEY AND FINANCIAL INSTITUTIONS

In the past twenty-five years my basic research has been directed primarily toward the development of a theory of money and financial institutions. This paper is devoted to an essentially nontechnical presentation of the key elements of the approach together with some of the key results. I also indicate the basic shift in paradigm away from equilibrium towards the study of process, learning and evolution, which is transforming economic analysis.

The basic approach involves a combination of game theory and experimental gaming. The economy must be modeled and considered as a "playable game," i.e., a fully defined process model whose rules can be sufficiently comprehended that the game can be played.

The central feature of a theory of money and financial institutions is the minimization of the need for trust. The key role of financial institution is information processing, aggregation and disaggregation, and evaluation. However, the interpretation of data is critical. It is not what the numbers are, but what they mean. Thus heuristic phrases such as the old banking maxim of "character, competence and collateral" can be operationalized.

In this paper I deal only with exchange economies with completely stationary or cyclical inputs. I believe that the basic points made here hold for growth models with some adjustments concerning the boundedness conditions on the money supply being related to the size of growth.

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* Much of the work noted here has been the result of joint collaborations over many years. My main collaborators have been Lloyd Shapley, Pradeep Dubey, John Geanakoplos, Yannis Karatzas, William Sudderth as well as several others. Specific acknowledgements are given in my forthcoming volume (Shubik, 1996).

overlapping generations (OLG) games with government, other legal persons of indefinite lives, and real persons.[1] A player in a game of strategy is characterized by a strategy set and a payoff function. A solution of a game involves the specification of some sort of choice rule usually based on some operation by the player on his strategy set. This operation may involve the player's attempt to maximize his payoff function. But to describe a game of strategy it is not necessary that an individual who is a strategic player has a utility function. Instead any player, such as government or an institution, could be equipped with a *choice rule* that instructs the player as to what to do in all circumstances. Even though the player may have an infinite life the decision rule whereby it advances into the future may be based on a finite set of arguments. An appropriate way to incorporate government and corporations into an overlapping generations model is to introduce them as infinitely lived strategic players without utility functions but with a decision or choice rule determined by the real persons. An alternative way of modeling government is as a mechanism whose move is determined as the outcome of a game played between a set of politicians and bureaucrats where the politicians are elected every so often by the overlapping generations of real persons and the politicians appoint the bureaucrats, but for longer terms than themselves.

Figure 1 suggests the scheme for an overlapping generations economy containing three types of agent. They are: (1) the young, (2) the old, and (3) the government. The first two are the real or natural persons, who can be regarded as many in number and possibly are most conveniently represented by a continuum of agents. The third is the government and may be represented by a single large atom.

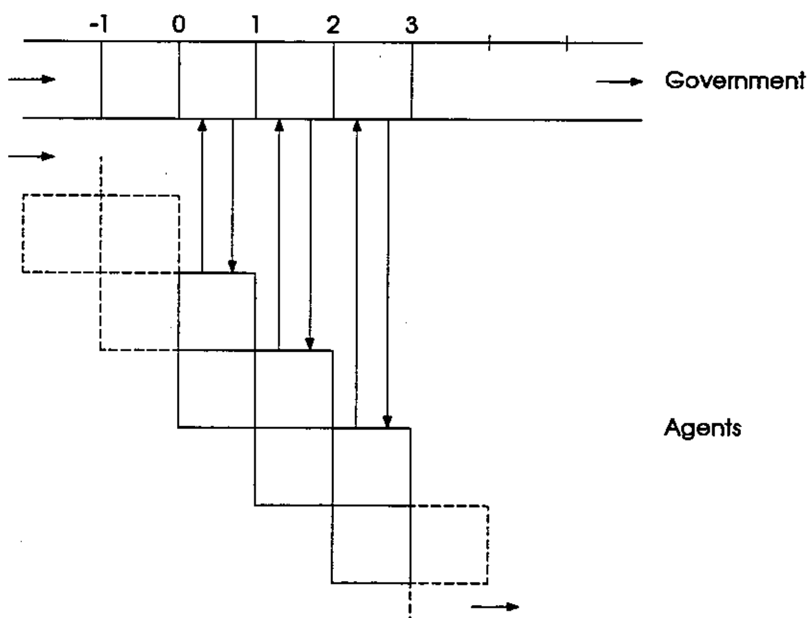

FIGURE 1: The Games within the Game

[1]See Woodford and Muller (1988) for a treatment of an economic model with both finitely and infinitely lived agents.

reasonable to accept as a comfortable upper bound on the length of any announced plan the lifetime of the youngest strategically active individual.

The device of the "game within the game" allows for a reconciliation of local optimization with global evolution. The small live players behave by trying to optimize locally and taking, as a first order approximation, their environment as given; their actions change the environment, but usually on a time scale too slow to influence their daily concerns.

1.3 Quantitative and qualitative results

In my forthcoming book (Shubik, 1996) the detailed description of strategic market games together with the considerable notation and many examples and calculations are presented. The mathematical proofs are to be found in a series of papers with colleagues and are due primarily to them.

As the purpose of the exposition here is to sketch the ideas without delving into the notation, proofs or details, we concentrate on the qualitative aspects of the work rather than becoming enmeshed in the quantitative detail.

1.4 Four basic modeling principles

The basic approach adopted here has been dominated by four basic methodological rules, which merit discussion. They are:

[1] The insistence on the construction of well defined process models.

[2] The specification of laws of conservation.

[3] The understanding of the role of symmetry.

[4] The understanding of the invariant properties of the system.

A brief elaboration of these points is now given.

[1] Economic dynamics are notoriously difficult. Broad intuitive insights have enabled economists such as Marx and Keynes broadly to sketch global grand dynamics, which have caught the imagination of governments and the masses and have had long term consequences on policy. When the more cautious attempts have been made to measure phenomena, such as the propensity to consume or the Phillips curve, however, the grand dynamics appear to melt into a welter of special cases and context dependent incidents. In contrast, the highly mathematical and apparently precise formulation of general equilibrium has no dynamics as it avoids process modeling. A first step in trying to overcome the gap between the two sets of theory is to provide a process basis for the general equilibrium models. But in attempting to do so, phenomena such as thin markets, inactive markets, hyperinflation and panics appear to be intimately connected with mathematical details such as division by zero. An insistence on well-defining

1.5 The key models

There are 12 ($3\times2\times2$) basic models that must be examined to appreciate the basic features of an economy utilizing fiat money as a means of exchange. These basic models involve the elementary descriptions of the trading and finance structure of the economy treated to study their behavior when there is no exogenous uncertainty present and then when there are stochastic elements. All models must be also considered in a finite horizon version and then when there is an infinite horizon. The three basic models consist of: (1) an economy that uses only fiat money for transactions; all agents have a nonzero supply and there are no credit markets; (2) an economy with an initial issue of fiat, but with an outside bank, which stands ready to borrow and lend at an exogenously set rate of interest; (3) an economy with an initial issue of fiat and an inside money market for borrowing and lending where the endogenous rate of interest is formed by the competitive forces of supply of fiat and the demand for loans.

Figure 2 shows the 12 basic models that are considered:

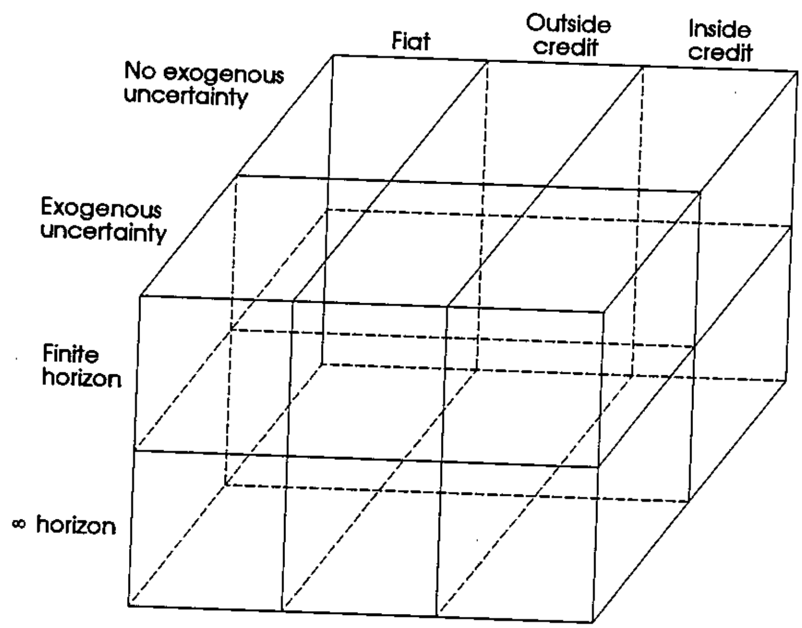

FIGURE 2: 12 Basic Models

The results noted here have been obtained for players who conceivably could live forever; but with little difficulty they can be replaced by overlapping generations, and the results are substantially similar for some settings. But with OLG one has to become specific about the birth-death processes assumed and the rules of inheritance.

The first of the two terms involving a minimum is the period by period penalty for going bankrupt. If we were to introduce assets directly there would also be information concerning the loss of assets and garnishing of income — but at the highest level of simplicity this term indicates that going bankrupt has unpleasant consequences. The last two terms indicate the cost of bankruptcy at the end of the game and the value of assets left over. The μs can be regarded as simple parameters or complex expressions. For simplicity they can be regarded as simple scalars.

A term involving β^T is in front of this last bankruptcy term indicating that for a large T it will be extremely weak verging on insignificant. This enables us to make a distinction between new fiat or outside money that might be introduced as a short term loan to be paid back after a single period and the initial supply of fiat which can be regarded either as an asset, or as a loan which need only be paid back at the end of the game. But for a game with a long duration the distinction between the two last alternatives can be made arbitrarily small. A bankruptcy penalty discounted far into the future is hardly a penalty. An immediate corollary that follows from our addition of a bankruptcy penalty to the payoff function is that *a debt which never has to be paid back is an asset.*

The term showing the value attached to the assets left over at the end is sometimes referred to in dynamic programming as the salvage value term. An alternative economic interpretation is that it portrays expectations.

We now are confronted with a paradox. If we consider the infinite horizon we do not see the extra terminal conditions involving bankruptcy and the worth of left over assets. The approach which can cover this difficulty is to study the limiting behavior of the finite horizon problem and ask whether the limiting behavior appears to approach the behavior at the limit. Fortunately this appears to be the case.

2.1 The finite horizon

We now consider an extremely simple model for purposes of illustration. Suppose that we consider a simple T period economy where each individual tries to maximize his utility function as noted above. Each individual begins with one unit[2] of fiat money and has an ownership claim each period to the income from the sale of one unit of the only consumer perishable, all of which is put up for sale each period.

A strategy of an individual α is a vector of the form

$$b^\alpha = (b_1^\alpha, b_2^\alpha, \dots)$$

[2]If I were being precise in the notation I would distinguish between a density and a finite measure. The agents are assumed to be nonatomic hence their resources are in terms of densities. The central bank is atomic. As the examples are extremely simple the meaning should be clear from context.

The solution to these for Δ and p yield

$$\Delta_t = \frac{(1+\rho)^{t-1}}{(1+\rho)^T - 1} \quad \text{and} \quad p = 1 - \frac{(1+\rho)^T}{(1+\rho)^T - 1} .$$

We observe that as $T \to \infty$, $\Delta_T \to \beta$ and $p \to 1$. The system approaches a full stationary state and the need for bank reserves diminishes. For any finite first sequence of periods the amount borrowed can be made arbitrarily close to zero and the two solutions without and with the outside bank approach each other.

2.2 The dual solutions: Fiat with or without the outside bank

In the examples given above for a simple trading economy, the amount of good traded each period is the same. The introduction of the outside bank seems to be an economically irrelevant artifact. Suppose instead the economy were cyclical. For illustration we select a periodicity of 3. Instead of having $Q = 1$ be the amount of good for sale each period we may consider an economy where the amount of good for sale is :

$$Q_{3t+1} = Q_1;\ Q_{3t+2} = Q_2;\ Q_{3t+3} = Q_3 \quad for \quad t = 1, 2, \ldots .$$

If we now consider an example analogous to the economy in Section 2.1 without a bank, but where all have an initial issue of fiat, prices can be adjusted from period to period only by hoarding. For example suppose the amounts of good were 1, 2, 3 repeated cyclically and the utility functions were:

$$\sum_{t=1}^{3T} \beta^{t-1} x_t^{\alpha} .$$

Spot prices in general equilibrium should be the same each period, but as there is a fixed amount of money in the system, this can only be achieved by hoarding. In general for a k-cycle with arbitrary equilibrium points we must consider 2^k Lagrangian multipliers to determine when the individual should hoard or spend all (Quint and Shubik, 1995a,b,c). In this simple example, if β is small enough the present will always dominate the future, thus all will always spend all and the spot prices will be 1, 1/2, 1/3. At the other extreme if $\beta = 1$ the spot prices would be 1/3, 1/3, 1/3 and in the second and third periods 1/3 and then 2/3 of the money supply would be hoarded. In general, prices will be max[$1/3\beta^2$, 1], max[$1/3\beta$, 2/3], 1/3. If, instead of an economy with fiat and no borrowing and lending, we introduce an outside bank and an exogenous rate of interest the solution changes. In particular, there the bank reserves are depleted to zero, which is the equivalent to the no hoarding in the third period without the bank.

Empirically we do not see the lining up of relative prices brought about by massive hoarding or intertemporal loans achieved by A staying out of the market while B buys. Instead we see an economy where there is borrowing and lending and a rate of interest. Although in a stationary analysis the two possibilites appear to be available as substitutes, in an actual dynamic economy the economy with borrowing and lending and an interest rate appears to require far less coodrination and information than the other.

2.3 The disappearance of time and money

There are two basic points to be gleaned from the discussion in Sections 2.1 and 2.2. They are the relationship between time and the presence of fiat money, and the duality between hoarding and borrowing and lending.

OBSERVATION 1: The presence of an initial issue of fiat which is an asset (or the equivalent, an indefinitely long debt) creates an overall system which is slightly nonsymmetric in the sense that this money is the only financial asset without a counterclaim against it. When there is an outside bank present, the inside economy over time consumes this initial input of fiat and it flows into the bank as a net interest payment. If we limit the investigation to any finite horizon, no matter how long, by backward induction the terminal constraint is tight, when there is no borrowing or lending and no salvage value for left over assets. This, in essence, fixes a price level.[4] When there is borrowing and lending, the bankruptcy conditions must be introduced to complete the definition of the model.

If the initial issue of fiat is regarded as a debt to be paid back to the government at the end of the game, then the possibility for the existence of a continuum of prices will depend on the magnitude of the bankruptcy penalty.

As long as the system is finite, it remains slightly nonsymmetric and the borrowing and lending is not perfectly stationary. When the system is infinite, the system becomes completely symmetric and the amount of fiat money held jointly by the economic agents and the outside bank is conserved. This amount is the circulating capital of the economy. In a world with no transactions needs whatsoever there is no need for circulating capital.

The system becomes fully stationary or timeless in the sense that optimal strategies are based only upon a non-time indexed state of the system.

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[4]There may still be some indeterminacy caused by constraints in the form of inequalities and by alternative trading and inventory possibilities. See Qunt and Shubik (1995a,b,c).

and hoarding will take place. The money market alone cannot adjust the money supply seamlessly to the variations needed for the appropriate relative prices.

OBSERVATON 3: The efficient equating of the supply and demand for money in a competitive loan market via a well defined positive rate of interest is in general not possible. The belief that it is possible can only be based on the extension of trade in money substitutes which one might wish to consider as money. An inside money market trades fiat for debt, it does not create fiat.

3 INCOMPLETE MARKETS AND STOCHASTIC ELEMENTS

In Section 2 we observed a duality between nonstochastic economies with and without an outside bank. When there is a stochastic element present the duality is by no means as clear. In particular in a nonstochastic economy with loans the role of bankruptcy is to prevent strategic default. At least around equilibrium there should be no active bankruptcy if the default penalty is sufficiently harsh. This is no longer true when there is stochastic uncertainty present

3.1 Fiat money as insurance

The occurrence of hoarding in the model of an exchange economy utilizing money, but without loans illustrated in Section 2 serves to enable the individuals to vary relative prices. A new reason to hold fiat appears when there is exogenous uncertainty present in the market. The fiat saved serves as insurance against a low income. It provides precautionary reserves against disaster. Alternatively it provides immediate liquidity to take advantage of opportunities, KSS (1994) provides an example where in equilibrium the wealth distribution of a population of agents with the same preferences is such that a part of the monetary wealth of the economy is always in hoard.

3.2 Institutions and equilibrium

In attempting to establish the existence of equilibrium in strategic market games a persistent set of difficulties keep appearing. At first glance they appear to be minor and seem to represent some form of casual extra institutional observation which should be omitted in early modeling by the theorist trained in abstraction and desiring high levels of generality. Then can be added later after the broad outlines of a general theory have been established. In the development of an understanding of the properties of a price system this was the approach adopted. A rigorous, frictionless and timeless theory of price was developed in a series of models abstracting out the role of process.

In attempting to build a complete closed process model of an exchange economy utilizing fiat money the annoying details which could be avoided in the nonprocess analysis appear to be of more importance

3.3 Panics and power laws

In Section 3.2 a sketch of the reasons for the need for institutions and appropriate rules such as credit constraints and bankruptcy laws has been given. Our original concern has been with the existence of equilibrium in the financial and product markets. But it is reasonable to ask the reverse question along with several other obvious companion questions.

The basic question is what happens to the system if the regulatory rules are not sufficient to guarantee the existence of equilibrium? An immediate companion question is empirical. Are the institutions and laws of society such that they are always sufficient to guarantee the existence of equilibrium?

I conjecture that under fairly reasonable and general circumstances the type of power law suggested by Mandelbrot (1963, 1967) associated with financial panics may arise in the models postulated here when the system starts in disequilibrium and the laws and regulatory agencies are not adequate.

I further suggest that there are more or less natural reasons to suspect that the institutional safeguards against the possibilities for the development of "economic earthquakes" are rarely if ever perfect. The observations here are, for the most part, consonant with the discussion given by Minsky (1986) on the financial system as well as the approach of Arthur (1994), Ijiri and Simon (1977), and others in their approach to simulating economic dynamics. In essence, many of the credit conditions have a crawling peg built into them in the way that expectations are permitted to influence the granting of credit. The presence of leverage combined with the possibility for chain-reaction bankruptcy can trigger disasters of any magnitude if the credit limits are functions of inflated or deflated recent asset prices and expected prices. It is often when the economy has heated up that the credit restrictions become lax and in the downward spiral when more credit is needed to prevent the slide, the credit tightens.

The physical scientist acquainted with a power law such as that for the probability of the occurrence of an earthquake of some given magnitude knows that there is no physical way known to humans to truncate the fat tail of the distribution. The situation with the economic earthquake is somewhat different. Beyond a certain force of the economic Richter scale the stock markets close and bank holidays are declared. The economy reorganizes and the law suits stretch out for many years.

4 INSTITUTIONS AND EVOLUTION

The time for going beyond the essentially pre-institutional timeless models of general equilibrium has clearly arrived in economic theory. The ability to compute and to simulate has changed by many orders of magnitude in the last few years. These new abilities permit an exploration of dynamics that was recently impossible. But the suggestion made here is that the natural first careful extensions beyond general equilibrium theory can be made by formulating well defined, even though highly simplified process models. In the act of fully defining these models the institutions are invented as a matter of

4.3 Institutions: The self-correcting system?

In Section 3 I have suggested the special role for institutions and laws in serving not only as carriers of process, but in providing the appropriate bounds to guarantee the existence of equilibrium. In Section 3.3 the possibility was conjectured that with imperfect institutions panics, best illustrated by power laws, could be present. But, as observed above, unlike natural power laws such as that which may govern earthquakes, individuals do have some control over financial power laws. When the game becomes too disastrous for all, they change the game. This is tantamount to postulating an elemental structure to the financial games-within-the-game. All participants take the rules of the game as given and act as context-constrained local optimizers, where the context is the set of laws and institutions of their society. When a disaster of a sufficiently large magnitude strikes enough of the economic agents are hurt that legal, political and social change modifies the economic environment.

I have entitled this section "The self-correcting system?" with a stress on the "?". The evidence from 1700 (when fiat money, stockmarkets and modern financial institutions started to proliferate) unto this present day is that there has been a series of panics and crashes of various magnitudes, but that after each the rules have changed and (at least with Candide bifocals) it appears that the laws and institutions, by some measures may have improved.

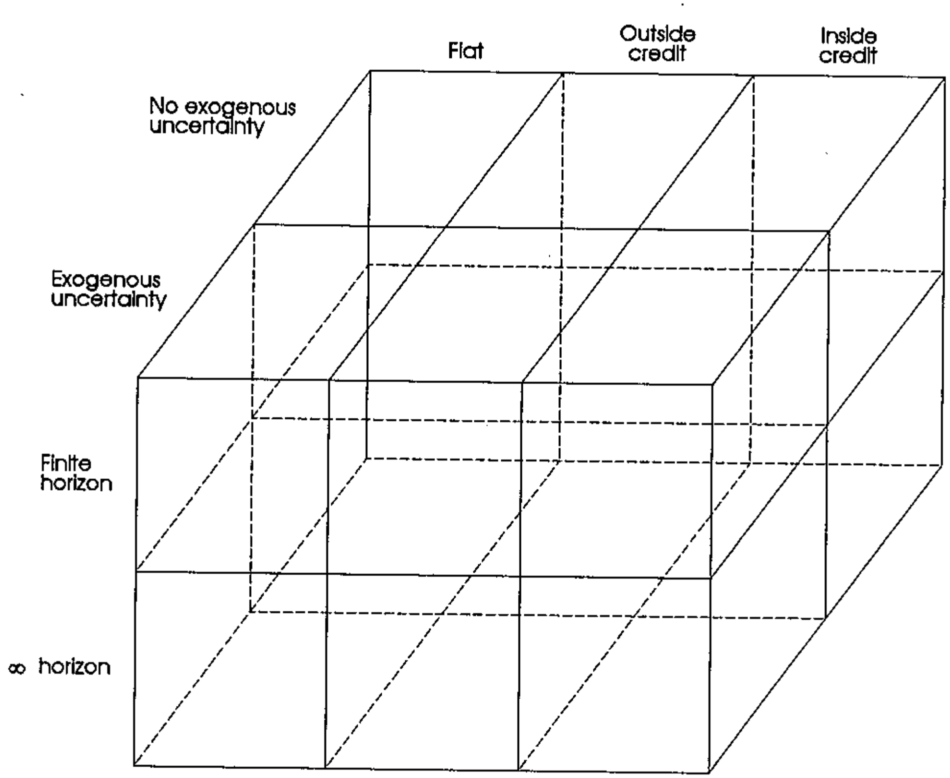

FIGURE 2: 12 Basic Models

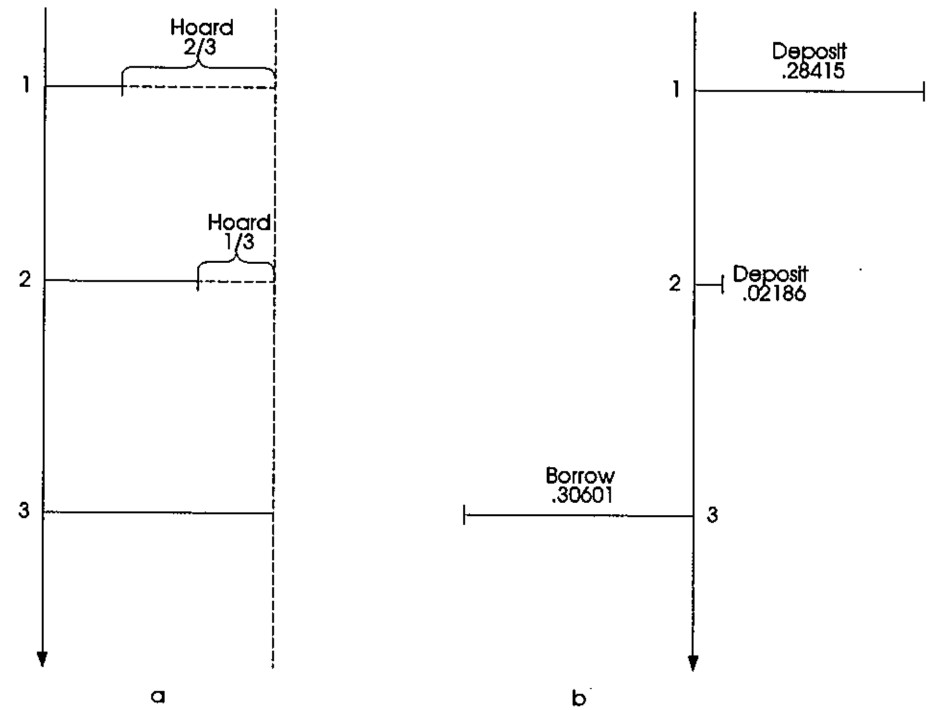

FIGURE 3: Hoarding or Depositing and Borrowing