The Predictive Power of Zero Intelligence in Financial Markets

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Abstract

Standard models in economics are based on intelligent agents that maximize utility. However, there may be situations where constraints imposed by market institutions are more important than intelligent agent behavior. We use data from the London Stock Exchange to test a simple model in which zero intelligence agents place orders to trade at random. The model treats the statistical mechanics of the interaction of order placement, price formation, and the accumulation of stored supply and demand, and makes predictions that can be stated as simple expressions in terms of measurable quantities such as order arrival rates. The agreement between model and theory is excellent, explaining 96% of the variance of the bid-ask spread across stocks and 76% of the price diffusion rate. We also study the market impact function, describing the response of prices to orders. The nondimensional coordinates dictated by the model collapse data from different stocks onto a single curve, suggesting a corresponding understanding of supply and demand. Thus, it appears that the price formation mechanism strongly constrains the statistical properties of the market, playing a more important role than the strategic behavior of agents.

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1 Introduction

Since the nineteenth century one of the classic questions in economics has been, “What determines supply and demand?” Similarly, since Bachelier \[1\] introduced the random walk model for prices in 1900, another important question has been, “What determines the price diffusion rate?”. Standard models in economics, which are based on rational utility maximizing agents, have had only limited success in addressing these questions. In this paper we demonstrate that a model built on the opposite approach – that agents are of zero intelligence, and simply place orders to trade at random – can successfully address these questions and others, providing one properly models the statistical mechanics of price formation.

Traditionally economics has devoted considerable effort to modeling the strategic behavior and expectations of agents. While no one would dispute that this is important, it has also been pointed out that some aspects of economics are independent of the agent model. For example, Becker \[2\] showed that a budget constraint is sufficient to guarantee the proper slope of supply and demand curves, and Gode and Sunder \[3\] demonstrated that if one replaces the students in a standard classroom economics experiment by zero-intelligence agents, price setting and other properties match better than one might expect. In this paper we show how this principle can be dramatically more powerful, and can make surprisingly accurate quantitative predictions. In particular, we test a zero-intelligence statistical mechanics model due to Daniels et al. \[4,5\], which builds on earlier work in financial economics \[6, 7, 8, 9\] and physics \[10, 11, 12, 13, 14\]. This added to the prior literature by constructing and approximately solving a simple model for price setting that makes quantitative, testable predictions about fundamental market properties, many of which can be expressed as simple algebraic formulas.

The model of Daniels et al. \[4\] assumes a continuous double auction, which is the most widely used method of price formation in modern financial markets \[5\]. There are two fundamental kinds of trading orders: Impatient traders submit market orders, which are requests to buy or sell a desired number of shares immediately at the best available price. More patient traders submit limit orders, which include the worst allowable price for the transaction. Limit orders may fail to result in an immediate transaction, in which case they are stored in a queue called the limit order book, illustrated in Fig. 1. As each buy order arrives it is transacted against accumulated sell limit orders that have a lower selling price, in priority of price and arrival time. Similarly for sell orders. The lowest selling price offered in the book at any point in time is called the best ask, \(a(t)\), and the highest buying price the best bid, \(b(t)\).

The model assumes that two types of zero intelligence agents place and cancel orders randomly (see Fig. 1). Impatient agents place market orders of size \(\sigma\), which arrive at a rate \(\mu\) shares per time. Patient agents place limit orders of the same size \(\sigma\), which arrive with a constant rate density \(\alpha \text{ shares per price per time}\), and queued limit orders are canceled at a rate \(\delta\), with dimensions of \(1/\text{time}\). Prices change in discrete increments called ticks, of size \(dp\). To keep the model as simple as possible, there are equal rates for buying and selling, and order placement and cancellation are Poisson processes. All of these processes are independent except for coupling through their boundary conditions: Buy limit orders arrive with a constant density over the semi-infinite interval \(-\infty < p < a(t)\), where \(p\) is the logarithm of the price, and sell limit orders arrive with constant density on the semi-infinite interval \(b(t) < p < \infty\). As new orders arrive they may alter the best prices \(a\) and \(b\), which in turn changes the boundary conditions for subsequent limit order placement. As a result \(a(t)\) and \(b(t)\) each make random walks, but because of coupling of the buying and selling processes the bid-ask spread \(s(t) \equiv a(t) - b(t)\) is a stationary random variable. It is this feedback between order placement and price diffusion that makes this model interesting, and despite its apparent simplicity, quite difficult to understand analytically \[5\].

2 Main results

In this section we test the model using data from the London Stock Exchange (LSE), and show that the for the three market properties we study, i.e. the bid-ask spread, the price diffusion rate, and the average market impact, the predictions match the data surprisingly well. We test the model using data
Figure 1: A random process model of the continuous double auction. Stored limit orders are shown stacked along the price axis, with sell orders (supply) stacked above the axis at higher prices and buy orders (demand) stacked below the axis at lower prices. New sell limit orders are visualized as randomly falling down, and new buy orders as randomly “falling up”. New sell orders can be placed anywhere above the best buying price, and new buy orders anywhere below the best selling price. Limit orders can be removed spontaneously (e.g. because the agent changes her mind or the order expires) or they can be removed by market orders of the opposite type. This can result in changes in the best prices, which in turn alters the boundaries of the order placement process. It is this feedback between order placement and price formation that makes this model interesting, and its predictions non-trivial.

from the London Stock Exchange (LSE) during the period August 1st 1998 - April 30th 2000, which includes a total of 434 trading days and roughly six million events. This data set shows all orders and cancellations, making it possible to measure the parameters of the model. For a more detailed description of the LSE and the dataset see Appendix section A.1. We chose 11 stocks with at least 300,000 events in the sample and at least 80 events on any given day. We measure the average value of the five above-defined parameters $\mu$, $\alpha$, $\delta$, $\sigma$, and $dp$ for each day, making the assumption that the parameters of the model are stationary within each day, but change from day to day. For example, $\mu$ is just the total number of shares of market orders placed in a given day, divided by the length of the day measured in terms of number of events (for details see Appendix A.2).

2.1 Bid-ask spread

The bid-ask spread is of central interest in financial markets because it is an important component of transaction costs. The mean value of the spread predicted based on a mean field theory analysis of the model is $\hat{s} = (\mu/\alpha) f(\sigma \delta / \mu, dp / p_c)$, where $f$ is a relatively slowly varying non-dimensional function. To test this relationship, we measure the actual average spread $\bar{s}$ across the full time period for each stock, and compare to the predicted average spread $\hat{s}$ based on order flows. To test our hypothesis that the two values coincide, we perform a regression of the form $\log \bar{s} = A \log \hat{s} + B$ (we used logarithms because the spread is positive and the log of the spread is approximately normally distributed). The regression, shown in Fig. 2(a), has $R^2 = 0.96$, with $A = 0.99 \pm 0.10$ and $B = 0.06 \pm 0.29$, in comparison to the model predictions $A = 1$ and $B = 0$. We thus very strongly reject the null hypothesis that $A = 0$, indicating that the predictions are far better than random. Even more surprising, we are unable to reject the null hypotheses that $A = 1$ and $B = 0$, indicating that we match the data extremely well even without fitting any free parameters. Note that because of long–memory effects and cross–correlations between stocks the error bars are much larger than they would be for IID, normally distributed data. So, even though in Fig. 2(a) the fitted intercept appears high, our analysis makes it clear that we cannot reject the hypothesis that $B = 0$. (See Ap-
2.2 Price diffusion rate

Another quantity of primary interest is the price diffusion rate, which drives the volatility of prices and is the primary determinant of financial risk. If we assume that prices make a random walk, then the diffusion rate measures the size and frequency of its increments. The variance $V$ of an uncorrelated normal random walk after time $t$ grows as $V(t) = Dt$, where $D$ is the diffusion rate. Numerical experiments indicate that the short term price diffusion rate predicted by the model is $\hat{D} = k \mu^{5/2} \delta^{3/2} \sigma^{-1/2} \alpha^{-2}$ where $k$ is a constant. As for the spread, we compare this to the actual price diffusion rate $\bar{D}_i$ for each stock averaged over the 21 month period, and regress the logarithm of the predicted vs. actual values, as shown in Fig. 2(b). This gives $R^2 = 0.76$, with $A = 1.33 \pm 0.25$ and $B = 2.43 \pm 1.75$. Thus, we again strongly reject the null hypothesis that $A = 0$. The results are not quite as good as they are for the spread, but we are still unable to reject the null hypothesis that $A = 1$ and $B = 0$; in each case the measured value is a little more than one standard deviation too high. We have accomplished something that is rather hard to achieve in economics, i.e. we have made testable predictions that are validated without any adjustment of free parameters. However, an important caveat is that problems in measuring the parameters $\alpha$ and $\delta$ introduce some arbitrariness into the intercept parameter $B$ (see Appendix A.2).

2.3 Market impact

Finally, the model makes a prediction about market impact, which is the dominant source of transaction costs for large traders, and is related to supply and demand. When a market order of size $\omega$ arrives it causes transactions which can cause a change in the midpoint price $m(t) \equiv (a(t) + b(t))/2$. The average market impact function $\phi$ is the average logarithmic midpoint price shift $\Delta p$ conditioned on order size, $\phi(\omega) = E[\Delta p|\omega]$. The nondimensional coordinates dictated by the model are very useful for understanding the average market impact function. There are five parameters of the model and three fundamental dimensional quantities ($\text{shares}$, $\text{price}$, and $\text{time}$), leading to only two independent degrees of freedom. Since the order flow rates $\mu$, $\alpha$, and $\delta$ are more important than the discreteness parameters $\sigma$ and $dp$, it is natural to construct nondimensional units based on the order flow parameters alone. There are unique combinations of the three order flow rates with units of $\text{shares}$, $\text{price}$, and $\text{time}$. These define a characteristic number $N_c = \mu/\delta$, a characteristic price interval $p_c = \mu/\alpha$, and a characteristic timescale $t_c = 1/\delta$. These characteristic values can be used to define...
nondimensional coordinates $\hat{p} = p/p_c$ for price, $\hat{N} = N/N_c$ for shares, and $\hat{t} = t/t_c$ for time. For more detail on non dimensional coordinates see Appendix A.6.

Each market order $\omega_i$ causes a possible price change $\Delta p_i$, defining an impact event $(\omega_i, \Delta p_i)$; if the mid price does not move, $\Delta p_i = 0$. If we bin together events with similar $\omega$ and plot the mean order size as a function of the mean price impact $\Delta p$, we typically see highly variable behavior for different stocks, as shown in Fig. 3(b). However, if we plot the data in nondimensional units, we see a collapse of the data onto roughly a single curve, as shown in Fig. 3(a). The variations from stock to stock are quite small; on average the corresponding bins for each stock deviate from each other by about 8%, roughly the size of the statistical sampling error. We have made an extensive analysis, but due to problems caused by the long-memory property of these time series and cross correlations between stocks, it remains unclear whether these differences are statistically significant (see discussion in Appendix A.7). In contrast, using standard coordinates the differences are highly statistically significant. This collapse illustrates that the non-dimensional coordinates dictated by the model provide substantial explanatory power, and that we understand how the market impact varies from stock to stock by a simple transformation of coordinates. This sheds light on empirical results for the average market impact for the New York Stock Exchange [17].

If we fit a function of the form $\phi(\omega) = K\omega^\beta$ to the market impact curve, we get $\beta = 0.26 \pm 0.02$ for buy orders and $\beta = 0.23 \pm 0.02$ for sell orders, as shown in Fig. 4. The functional form of the market impact we observe here is not in agreement with a recent theory by Gabaix et al. [18], which predicts $\beta = 0.5$. There is an interesting underlying debate: Their theory follows traditional thinking in economics, and postulates that agents optimize their behavior to maximize profits, while the theory we test here assumes that they behave randomly, and that the form of the average market impact function is dictated by the statistical mechanics of price formation.

The market impact function is closely related to the more familiar notions of supply and demand. At any instant in time the stored queue of sell limit orders reveals the quantity available for sale at each price, thus showing the supply, and the stored buy orders similarly show the revealed demand. The price shift caused by a market order of a given size depends on the stored supply or demand through a moment expansion [5]. Thus, the collapse of the market impact function reflects a corresponding property of supply and demand. Normally one would assume that supply and demand are func-
Figure 4: The average market impact vs. order size plotted on log-log scale. The upper left and right panels show buy and sell orders in nondimensional coordinates; the fitted line has slope $\beta = 0.26 \pm 0.02$ for buy orders and $\beta = 0.23 \pm 0.02$ for sell orders (see Appendix A.7). In contrast, the lower panels show the same thing in dimensional units, using British pounds to measure order size. Though the exponents are similar, the scatter between different stocks is much greater.
tions of human production and desire; the results we have presented here suggest that their form is dictated by the dynamical interaction of order accumulation, removal by market orders and cancellation, and price diffusion.

3 Conclusion

These results have several practical implications. For market practitioners, understanding the spread and the market impact function is very useful for estimating transaction costs and for developing algorithms that minimize their effect. And for regulators they suggest that it may be possible to make prices less volatile and lower transaction costs by creating incentives for limit orders and disincentives for market orders.

The model we test here was constructed before looking at the data [4, 5], and was designed to be as simple as possible for analytic analysis. A more realistic (but necessarily more complicated) model would more closely mimic the properties of real order flows, which are strongly correlated, and would hopefully be able to capture even more features of the data, such as the power law tails of prices. Nonetheless, as we have shown above, this simple model does a remarkable job of explaining important fundamental properties of markets, such as transaction costs, price diffusion and supply and demand. The model captures the statistical mechanics of the market quite well, and in particular, the way order placement and price formation interact to alter the accumulation of stored supply and demand. For the phenomena studied here this appears to be the dominant effect. We do not mean to claim that market participants are unintelligent: Indeed, one of the virtues of this model is that it provides a benchmark to separate properties that are driven by the statistical mechanics of the market institution from those that are driven by conditional strategic behavior. It is surprising that such a simple model can explain so much about a system as complex as a market, and shed light on century-old questions about the rate of price diffusion and the form of supply and demand.
A Appendix

A.1 Description of the London Stock Exchange (LSE) and this data set

The London Stock Exchange is composed of two parts, the electronic open limit order book, and the non-electronic upstairs market, which is used to facilitate large block trades. During the time period of our dataset 40% to 50% of total volume was routed through the electronic order book while the rest through the upstairs market. It is believed that the limit order book is the dominant price formation mechanism of the London Stock Exchange: about 75% of upstairs trades happen between the current best prices in the order book [19]. Our analysis involves only the data from the electronic order book.

The London Stock Exchange data set was chosen because for the electronic exchange we have a complete record of every action taken by every participating institution allowing us to measure the order flows and cancellations and estimate all of the necessary parameters.

We used data from the time period August 1st 1998 - April 30th 2000, which includes a total of 434 trading days and roughly six million events. We chose 11 stocks having the property that the number of total number of events in the sample exceeds 300,000 and was never less than 80 on any given day. Some statistics about the order flow for each stock are given in table 1.

The trading day of the LSE starts at 7:50 with a 10 minute opening auction period (during the later part of the dataset the auction end time varies randomly by 30 seconds). During this time orders accumulate without transactions; then a clearing price for the opening auction is calculated, and all opening transactions take place at this price. Following the opening at 8:00 the market runs continuously, with orders matched according to price and time priority, until the market closes at 16:30. In the earlier part of the dataset, until September 22nd 1999, the market opening hour was 9:00. During our analyzed period there have been some minor modifications of the opening auction mechanism, but since we discard the data for this period anyway this is not relevant.

Some stocks in our sample (VOD for example) have stock price splits and tick price changes during the period of our sample. We take splits into account by transforming stock sizes and prices to pre-split values. In any case, since all measured quantities are in logarithmic units, of the form \( \log(p_1) - \log(p_2) \), the absolute price scale drops out. The only place that absolute price levels are relevant is Fig. 11d. Our theory predicts that the tick size should change some of the quantities of interest, such as the bid-ask spread, but the predicted changes are small enough in comparison with the effect of other parameters that we simply ignore them (and base our predictions on the limit where the tick size is zero).

A.2 Measurement of daily parameters

Reconstructing the limit order book throughout the day makes it clear that the properties of the market tend to be relatively stationary during each day, changing more dramatically at the beginning and at the end of day. It is therefore natural to measure each parameter for each stock on each day. Since the model does not take the opening auction into account, we simply neglect orders leading up to the opening auction, and base all our measurements on the remaining part of the trading day, when the auction is continuous.

In order to cope with diverse types of orders traders can submit in a real market (for example, crossing limit orders, market orders with limiting price, “fill-or-kill”, “execute & eliminate”) we use redefinitions based on whether an order results in an immediate transaction, in which case we call it an effective market order, or whether it leaves a limit order sitting in the book, in which case we call it an effective limit order. Marketable limit orders (also called crossing limit orders) are limit orders that cross the opposing best price, and so result in at least a partial transaction. The portion of the order that results in an immediate transaction is counted as an effective market order, while the non-transacted part (if any) is counted as an effective limit order. Orders that do not result in a transaction and do not leave a limit order in the book, such as for example, failed “fill-or-kill” orders, are ignored altogether. These have no affect on prices, and in any case, make up only a very small fraction
of the order flow, typically less than 1%.

The measure of time is based on the number of events, i.e., the time elapsed during a given period is just the total number of events, including market order placements, limit order placements, and cancellations. Price intervals are computed as the difference in the logarithm of prices, which is consistent with the model, in which all price intervals are assumed to be logarithmic in order to assure prices are always positive.

The parameter $\mu_t$, which characterizes the average market order arrival rate on day $t$, is straightforward to measure. It is just the ratio of the number of shares of effective market orders (for both buy and sell orders) to the number of events during the trading day. Similarly, $\sigma_t$ is the average limit order size in shares for that day, and $dp_t$ is just the tick size, which is fixed for each day but varies from stock to stock and for a given stock changes occasionally during the sample.

Measuring the cancellation rate $\delta_t$ and the limit order rate density $\alpha_t$ is more complicated, due to the fact that the highly simplified assumptions we have made for the model do not match the data very well. In contrast to our assumption of a constant density for placement of limit orders across the entire logarithmic price axis, real limit order placement is highly concentrated near the best prices (roughly 2/3 of all orders are placed at the best prices), with a density that falls off as a power law as a function of the distance $\Delta$ from the best prices. We cope with this problem as described below.

In order to estimate the limit order rate for day $t$, $\alpha_t$, we make an empirical estimate of the distribution of the relative price for effective limit orders placement. For buy orders we define the relative price as $\Delta = a - p$, where $p$ is the logarithm of the limit price and $a$ is the logarithm of the best selling price. Similarly for sell orders, $\Delta = p - b$, where $b$ is the best buying price. (By using this convention we can include all effective limit orders and guarantee that $\Delta$ is always positive). We then somewhat arbitrarily choose $Q_{t\text{lower}}$ as the 2 percentile of the density of $\Delta$ corresponding to the limit orders arriving on day $t$, and $Q_{t\text{upper}}$ as the 60 percentile.

\begin{table}
\centering
\begin{tabular}{|l|ccccc|cc|c|}
\hline
stock ticker & num. events & average & limit & market & deletions & eff. limit & eff. market & no. days \\
& (1000s) & (per day) & (1000s) & (1000s) & (1000s) & (shares/day) & (shares/day) & \\
\hline
AZN & 608 & 1405 & 292 & 128 & 188 & 4,967 & 4,921 & 429 \\
BARC & 571 & 1318 & 271 & 128 & 172 & 7,370 & 6,406 & 433 \\
CW. & 511 & 1184 & 244 & 134 & 134 & 12,671 & 11,151 & 432 \\
GLXO & 814 & 1885 & 390 & 200 & 225 & 8,927 & 6,573 & 434 \\
LLOY & 644 & 1485 & 302 & 184 & 159 & 13,846 & 11,376 & 434 \\
ORA & 314 & 884 & 153 & 57 & 104 & 12,097 & 11,690 & 432 \\
PRU & 422 & 978 & 201 & 94 & 127 & 9,502 & 8,597 & 354 \\
RTR & 408 & 951 & 195 & 100 & 112 & 16,433 & 9,965 & 431 \\
SB. & 665 & 1526 & 319 & 176 & 170 & 13,589 & 12,157 & 426 \\
SHEL & 592 & 1367 & 277 & 159 & 156 & 44,165 & 30,133 & 429 \\
VOD & 940 & 2161 & 437 & 296 & 207 & 89,550 & 71,121 & 434 \\
\hline
\end{tabular}
\caption{Summary statistics for stocks in the dataset. Fields from left to right: stock ticker symbol, total number of events (effective market orders + effective limit orders + order cancellations) in thousands, average number of events in a trading day, number of effective limit orders in thousands, number of effective market orders in thousands, number of order deletions in thousands, average limit order size in shares, average market order size in shares number of trading days in the sample.}
\end{table}

\footnote{The model assumes that the average size of limit orders and market orders is the same. For the real data this is not strictly true, though as seen in Table 1 it is a good approximation to within about 20%. For the purposes of the analysis we use the limit order size as the measure because for theoretical reasons we think this is more important than the market order size, but because the two are approximately the same, this will not make a significant difference in the results.}
Assuming constant density within this range, we calculate \( a_t \) as \( a_t = L/(Q_t^{\text{upper}} - Q_t^{\text{lower}}) \) where \( L \) is the total number of shares of effective limit orders on day \( t \).

Similarly, to cope with the fact that in reality the average cancellation rate \( \delta \) decreases \([10]\) with the relative price \( \Delta \), whereas in the model \( \delta \) is assumed to be constant, we base our estimate for \( \delta \) only on canceled limit orders within the range of the same relative price boundaries \((Q_t^{\text{lower}}, Q_t^{\text{upper}})\) defined above. We do this to be consistent in our choice of which orders are assumed to contribute significantly to price formation (orders closer to the best prices contribute more than orders that are further away). We then measure \( \delta_t \), the cancellation rate on day \( t \), as the inverse of the average lifetime of a canceled limit order in the above mentioned price range. Lifetime is measured in terms of number of events happening between the introduction of the order and its subsequent cancellation. (Cancellation refers to the spontaneous removal of an order without it being executed, either because the trader changes his/her mind or because the order expires.) Some simple diagnostics of the parameter estimates are presented in Fig. A.2

The procedures above are obviously crude simplifications that allow us to use a model that for purposes of simplicity makes unrealistic uniformity assumptions. The arbitrary choices involved in choosing price intervals effectively introduce some uncertainty into the measurement of the intercept parameter \( B \). However, from our experiments with this choice, it seems that our results are not sensitive to the particular choice as long as they are in reasonable ranges, i.e. the first limit is close to zero (counting all or almost all orders that are near the best price) and the second limit is large enough to include a reasonable amount of data, but not so large as to include orders that are unlikely to ever be executed, and therefore unlikely to have any effect on prices.

Alternatively, it would also be useful to create a model based on the best available information about the real order flows, a problem that members of our group are actively working on. However, the model introduced in reference \([4]\) has the important advantage of simplicity. In particular, this model naturally gives rise to non-dimensional coordinates that are essential for understanding the behavior of the market impact function, as discussed in section A.6.

A.3 Measurement of spread and the price diffusion rate

Spread is measured as the daily average of \( \log(\text{bid}) - \log(\text{ask}) \). The spread is measured after each event, with each event given equal weight. Measuring the price diffusion rate is a bit more complicated. The variance of mid-point returns at time scale \( \tau \) is defined as \( \sigma^2(\tau) = \langle (m(t+\tau) - m(t))^2 \rangle \), where \( \langle \cdot \rangle \) indicates an (event weighted) time average. For a random walk with stationary increments the variance varies with \( \tau \) as

\[
\sigma^2 = D\tau^{2H},
\]

where \( D \) is the diffusion rate, and \( H \) the Hurst exponent, which for an i.i.d. Gaussian random walk is \( H = 1/2 \). By measuring this variance at different intra-day time scales \( \tau \) we estimate \( D \) using expression (1) by a linear regression weighted by the square root of the number of independent observations, and assuming \( H = 1/2 \). An example of the measurement of price diffusion at different time scales \( \tau \) is given in Fig. A.3. The slope is the diffusion rate \( D_t \) for day \( t \). We show an example of how the diffusion rate varies on a daily basis and its daily autocorrelation function in Fig. A.4. Similarly, we estimate the diffusion rate across the full 434 day sample as a weighted average of the daily diffusion rates. Weighting is done based on the number of events in a day.

It is common in the financial literature to speak about the variability of prices in terms of “volatility”. This is a somewhat vague term with variety of definitions. A commonly used one is the standard deviation of logarithmic prices within a given time interval, such as a day. We instead use the price diffusion for several reasons: It avoids specifying a time interval, it is more statistically stable to measure, and as a description of the generating process of volatility, it is more fundamental to the price process.

A.4 Longitudinal vs. cross-sectional tests

It is possible to test this model either longitudinally (across different time intervals for a given stock)
Figure 5: Density estimations and cross correlations for Vodafone between the four model parameter measures. On the diagonal we present the histogram of the corresponding parameter. Upper off-diagonal plots are the time cross correlation. We see that $\delta$ is uncorrelated with other measures, while the other three are quite correlated although without any noticeable lead-lag effects. The lower off-diagonal plots are scatter plots between the parameters. $\mu$ and $\alpha$ are particularly strongly correlated; fortunately, for the prediction of the spread their ratio is the most important quantity, so this correlation cancels out.
Figure 6: Example illustrating the procedure for measuring the price diffusion rate for Vodafone (VOD) on August 4th, 1998. On the $x$ axis we plot the time $\tau$ in units of ticks, and on the $y$ axis the variance of mid-price diffusion $\sigma^2(\tau)$. According to the hypothesis that mid-price diffusion is an uncorrelated Gaussian random walk, the plot should be linear with the slope equal to the diffusion constant (see equation (1)). To cope with the fact that points with larger values of $\tau$ have fewer independent intervals and are less statistically significant, we use a weighted average to compute the slope.

Figure 7: Time series (top) and autocorrelation function (bottom) for daily price diffusion rate $D_t$. As is evident from the figure, the price diffusion rate is highly variable, and there are very long range correlations in its daily movements. Note though that the ACF coefficients for larger lags are poorly determined due to the short length of the series (430).
market orders, and $1/\delta$, which is the characteristic time for spontaneous removal of limit orders. For the data here it appears that $\sigma/\mu$ is typically less than a minute, whereas $1/\delta$ ranges from a few minutes to a few hours. Thus, $1/\delta$ is the slowest relaxation time, and in some cases at least it is potentially problematic for a daily analysis. In addition, there is the very significant problem that real order flows are strongly autocorrelated, discussed below.

Cross-sectionally, in contrast, we expect a priori that different stocks should have different parameters. There are likely to be larger variations in the parameters between stocks than in the parameters for a given stock at different times. In addition, for a cross-sectional analysis there are no problems with relaxation times, and in any case averaging over longer periods of time reduces the sampling error. Thus cross-sectional analysis is expected to be more promising and more reliable.

As noted, for the daily analysis, and even for cross-sectional analysis over long periods of time, there are problems caused by the the long range autocorrelations of real order flow, spreads, and price diffusion rates. Autocorrelations can remain strongly positive on the order of 50 days; indeed, we know that order flow signs are a long-memory process [15, 21] and we strongly suspect that this is true for the spread and the price diffusion rate as well. This creates problems in performing the regression, and can result in a systematic bias in the estimated parameters. It causes severe systematic biases and interpretation problems for a daily analysis.

To produce estimates of the average values of the parameters and of the price diffusion and spread across the full 21 month period for the cross-sectional regressions, we have used the event-weighted average of daily values to compute values for each of the order flow parameters, and then make predictions for each stock based on the average values. The weighting is done by the number of events in a day, which for simple quantities such as the the market impact rate reduces to something that is equivalent to applying the analysis over the full period. Similarly, to get the 21 month average of the spread and price diffusion we simply compute an event-weighted average of their daily values. We have tried several variations on this procedure and the differences appear to be inconsequential.

When we perform longitudinal regressions at a daily time-scale we get values for the slope coefficient of the regressions that are less than one, often by a statistically significant amount. We believe this is caused by the strong autocorrelation. For example, consider a time series process of the form

$$ y_t = ax_t + \rho y_{t-1} + n_t $$

where $n_t$ is an IID noise process. In case $x_t$ are i.i.d., regressing $y_t$ against $x_t$ will result in coefficients that are systematically too small, due to the fact that the $y_{t-1}$ term damps the response of $y_t$ to changes in $x_t$. Of course, one can fix this in the simple example above by simply including $y_{t-1}$ in the regression. For the real data, however, the autocorrelation structure is more complicated – indeed we believe it is a long-memory process – which is not well modeled by an AR process in the above form. Without finding a proper characterization of the autocorrelation structure, we are likely to make errors in estimating the dependence of the predicted and actual values. This is borne out in the error analysis presented in Section (A.5), where we see that as we break the data into shorter subsamples, the estimated slope coefficients systematically decrease. Thus we view that the cross-sectional results are more reliable, and that it is not surprising that they match the model better than the daily results (which already match pretty well).

A.5 Estimating the errors for the regressions

The error bars presented in the text are based on a bootstrapping method. We are driven to use this method because order flow variables, spread, and price diffusion rates all have slowly decaying posi-
tive autocorrelation functions, and indeed we suspect that they are long memory processes (i.e. the autocorrelation function decays as a power law with an exponent less than one, which implies that it is non-integrable). In addition the spread, price diffusion rates, and parameters are highly cross-correlated between stocks. This complicates the statistical analysis, and makes the assignment of error bars difficult.

The method we use is inspired by the variance plot method described in Beran [22], Section 4.4. The basic idea is to divide the sample into blocks, apply the regression to each block, and then study the scaling of the deviation in the results as the blocks are made longer to coincide with the full sample. We divide the $N$ daily data points for each stock into $m$ disjoint blocks, each containing $n$ adjacent days, so that $n \approx N/m$. We use the same partition for each stock, so that corresponding blocks for each stock are contemporaneous. We perform an independent regression on each of the $m$ blocks, and calculate the mean $M_m$ and standard deviation $\sigma_m$ of the $m$ slope parameters $A_i$ and intercept parameters $B_i$. We then vary $m$ and study the scaling as shown in Figures 8(a) and 8(b).

Figure 8(a) and (b) illustrates the values obtained for the spread, and Figure 9(a) and (b) illustrates this for the price diffusion rate. Similarly, panels (c) and (d) in each figure show the mean and standard deviation for the intercept and slope as a function of the number of bins. As expected, the standard deviations of the estimates decreases as $n$ increases. The logarithm of the standard deviation for the intercept and slope as a function of $\log n$ is shown in panels (e) and (f). For IID normally distributed data we expect a line with slope $\gamma = -1/2$; instead we observe $\gamma > -1/2$. For example for the spread $\gamma \approx -0.19$. The smaller $\gamma$ is an indication that this is a long memory process; see the discussion in Section A.7.

This method can be used to extrapolate the error for $m = 1$, i.e the full sample. This is illustrated in panels (e) and (f) in each figure. The inaccuracy in these error bars is evident in the unevenness of the scaling. This is particularly true for the price diffusion rate. To get a feeling for the accuracy of the error bars, we estimate the standard deviation for the scaling regression assuming standard error, and repeat the extrapolation for the one standard deviation positive and negative deviations of the regression lines, as shown in panels (e) and (f) of Figures 8 and 9. The results are summarized in Table A.1.

One of the effects that is evident in Figures 8(c-d) and 9(c-d) is that the slope coefficients tend to decrease as $m$ increases. We believe this is due to the autocorrelation bias discussed in Section A.4.

### A.6 More on non-dimensional units

As mentioned in the text, because the properties of the market are more sensitive to the three order flow rates $\mu$, $\alpha$, and $\delta$ than to the discreteness parameters $\sigma$ and $dp$, we can define non-dimensional coordinates based on the order flow rates. This gives characteristic scales for price, shares, and time, that are unique up to a constant, as described in the text. This procedure has the great advantage that it reduces the number of degrees of freedom from five to two. The remaining two degrees of freedom are naturally characterized in terms of non-dimensional versions of the discreteness parameters. A non-dimensional scale parameter based on order size is constructed by dividing the typical order size $\sigma$ (with dimensions of shares) by the characteristic number of shares $N_c$. This gives the non-dimensional parameter $\epsilon \equiv \sigma / N_c = \sigma / \mu$, which characterizes the granularity of the orders stored in the limit order book. A non-dimensional scale parameter based on tick size is constructed by dividing the tick size $dp$ by the characteristic price, i.e. $dp / p_c = \alpha dp / \mu$. The usefulness of this is that all the properties of the model only depend on the two nondimensional parameters, $\epsilon$ and $dp / p_c$. Furthermore, because the order flow parameters are more important, most of the variation in the properties of the market under variation of parameters is captured by simply using non-dimensional coordinates, which are defined so they already take the variation of the order flow parameters into account. For a more detailed discussion, see reference.

### A.7 Mean average market impact and its error bars

The market impact of an effective market order is the logarithmic change in price that it causes. For example, we measure the market impact for a sell market order of size $\omega(t)$ as $r(t) = \log(m(t + \epsilon))$ —
Figure 8: Subsample analysis of regression of predicted vs. actual spread (see Fig. 1a of main paper). To get a better feeling for the true errors in this estimation (as opposed to standard errors which are certainly too small), we divide the data into subsamples (using the same temporal period for each stock) and apply the regression to each subsample. (a) (top left) shows the results for the intercept, and (b) (top right) shows the results for the slope. In both cases we see that progressing from right to left, as the subsamples increase in size, the estimates become tighter. (c) and (d) (next row) shows the mean and standard deviation for the intercept and slope. We observe a systematic tendency for the mean to increase as the number of bins decreases. (e) and (f) show the logarithm of the standard deviations of the estimates against log \( n \), the number of each points in the subsample. The line is a regression based on binnings ranging from \( m = N \) to \( m = 10 \) (lower values of \( m \) tend to produce unreliable standard deviations). The estimated error bar is obtained by extrapolating to \( n = N \). To test the accuracy of the error bar, the dashed lines are one standard deviation variations on the regression, whose intercepts with the \( n = N \) vertical line produce high and low estimates.
Figure 9: Subsample analysis of regression of predicted vs. actual price diffusion (see Fig. 1b of main paper), similar to the previous figure for the spread. The scaling of the errors is much less regular than it is for the spread, so the error bars are less accurate.
Table 2: A summary of the bootstrap error analysis described in the text. The columns are (left to right) the estimated value of the parameter, the standard error from the cross sectional regression in Figure 1 of the main paper, one standard deviation error bar estimated by the bootstrapping method, and one standard deviation low and high values for the extrapolation, as shown in Figures 8(e-f) and 9(e-f).

<table>
<thead>
<tr>
<th>regression</th>
<th>estimated</th>
<th>standard</th>
<th>bootstrap</th>
<th>low</th>
<th>high</th>
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<td>0.25</td>
<td>0.33</td>
</tr>
<tr>
<td>spread slope</td>
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<td>0.08</td>
<td>0.10</td>
<td>0.09</td>
<td>0.11</td>
</tr>
<tr>
<td>diffusion intercept</td>
<td>2.43</td>
<td>1.22</td>
<td>1.76</td>
<td>1.57</td>
<td>1.97</td>
</tr>
<tr>
<td>diffusion slope</td>
<td>1.33</td>
<td>0.19</td>
<td>0.25</td>
<td>0.23</td>
<td>0.29</td>
</tr>
</tbody>
</table>

log(m(t)) where \( m(t) \) is the midpoint price at time \( t \) just before the order is placed and \( m(t + \epsilon) \) is the midpoint price immediately after the order has been executed (and before the next order is executed).

There are several potential problems with this measurement. First, we lump together all effective market orders, irrespectively of whether they were induced by a pure market order, a market order with a limiting price or a crossing limit order. Tests segregating out different order types seem to indicate that this is not a serious problem. Second, and more important, both the absolute value of price changes and the size of effective market orders are long memory processes, with positive autocorrelations that decay as a power law \[15, 21\]. This means that the errors tend to be large, and the assignment of accurate error bars is difficult.

The time series of price changes \( \Delta p \) have the problem of long memory, i.e. their absolute value has a slowly decaying positive autocorrelation function. The signed price changes \( \Delta p \) have an autocorrelation function that rapidly decays to zero, but to compute market impact we sort the values into bins, and all the values in the bin have the same sign. One might have supposed that because the points entering a given bin are not sequential in time, the correlation would be sufficiently low that this might not be a problem. However, the long memory effect is sufficiently strong that its effect is still significant, and must be taken into account. Even with twenty bins, the correlations are sufficiently long lasting that there are still strong correlations between successive points entering the same bin, which create problems that must be addressed to get good error bars.

To cope with this we assign error bars to each bin using the variance plot method described in, for example, Beran \[22\], Section 4.4, which was already discussed in Section \[A.5\]. The sample of size \( N = 434 \) is divided into \( m \) subsamples of \( n \) points adjacent in time. We compute the mean for each subsample. We then vary \( n \) and compute the standard deviation of the means across the \( m = N/n \) subsamples. We then make use of theorem 2.2 from Beran \[22\] that states that the error in the \( n \) sample mean of a long-memory process is \( \hat{e} = \sigma n^{-\gamma} \), where \( \gamma \) is a positive coefficient related to the Hurst exponent and \( \sigma \) is the standard deviation. By plotting the standard deviation of the \( m \) estimated intercepts as a function of \( n \) we estimate \( \gamma \) and extrapolate to \( n = \text{sample length} \) to get an estimate of the error in the full sample mean. An example of an error scaling plot for one of the bins of the market impact is given in Figure 10.

When all said is done, the interesting question is whether the data for different stocks collapse onto a single curve, or whether there are statistically significant idiosyncratic variations from stock to stock. From the results presented in Figure 2b of the main paper this is not completely clear. Most of the stocks collapse onto the curve for the pooled data (or the pooled data set with themselves removed). There are a few that appear to make statistically significant variations, at least if we assume that the mean value of the bins for different order size levels are independent. However, they are most definitely not independent, and this non-independence is difficult to model. In any case, the variations are always fairly small, not much larger than the error bars. Thus the collapse gives at least a good approximate understanding of the market impact, even if there are some small idiosyncratic variations it does not capture.
Figure 10: The variance plot procedure used to determine error bars for mean market impact conditional on order size. The horizontal axis \( n \) denotes the number of points in the \( m \) different samples, and the vertical axis is the standard deviation of the \( m \) sample means. We estimate the error of the full sample mean by extrapolating \( n \) to the full sample length.

A.8 Alternative market impact collapse plots

We have demonstrated a good collapse of the market impact using nondimensional units. However, in deciding what “good” means, one should compare this to the best alternatives available. We compare to three such alternatives. In Figure 11 the top left pane shows the collapse when using nondimensional units derived from the model (repeated from the main text). The top right plot shows the average market impact when we instead normalize the order size by its sample mean. Order size is measured in units of shares and market impact is in log price difference. The bottom left attempts to take into account daily variations of trading volume, normalizing the order size by the average order size for that stock on that day. In the bottom right we use trade price to normalize the order sizes which are now in monetary units (British Pounds). We visually see that none of the alternative rescalings comes close to the collapse we obtain when using non-dimensional units; because of the much greater dispersion, the error bars in each case are much larger.

A.9 Functional form of market impact and its relation to power law scalings

The functional form of the average market impact function has recently been a matter of some debate, which also relates to the underlying explanation of the power law tails seen for large price fluctuations. We briefly review some of the issues here.

Gabaix et al. [18] have recently proposed a theory to explain the power law fluctuations in the tails of prices. A keystone of their theory is that the average market impact function should scale as 
\[ E[\Delta \log(p)] = A \omega^\beta, \]
with \( \beta = 1/2 \). They present arguments for this based on the ability of agents to trade in such a way as to minimize their transaction costs. This has been challenged by Farmer and Lillo [23], who argue that there are flaws in the statistical analysis of Gabaix et al., and that a proper analysis gives much smaller exponents. The evidence here supports this. When plotted in log-log scale, the collapsed market impact function appears to approximately obey a power law, but with \( \beta < 0.5 \), as shown in Figure 11. A best fit to pooled data set gives \( \beta = 0.26 \pm 0.02 \) for buy orders and \( \beta = 0.23 \pm 0.02 \) for sell orders. Similarly, a best fit using the unnormalized pooled data set gives similar values. While the above error estimates are standard errors, and are surely much too optimistic, it is nonetheless quite clear that these data are inconsistent with \( \beta = 1/2 \).

However, there are still important open questions about modeling the market impact function with the approach we present here, that relate to the problem of power law tails in prices. The model we present here captures the *scale-dependent* properties, such as mean values of bid-ask spread and price volatility, quite well. However, it does not capture the *scale-independent* properties underlying power law fluctuations of extreme values. We believe this is primarily due to the Poisson order flow assumptions (see discussion in Section A.11).

Models based on random order flow produce very plausible average market impact functions. However, the functional form depends on the order flow...
Figure 11: Market impact collapse under 4 kinds of axis rescaling. In each case we plot a normalized version of the order size on the horizontal axis vs. a (possibly normalized) average market impact $\log(p_{t+1}) - \log(p_t)$ on the vertical axis. (a) (top left) collapse using non-dimensional units based on the model; (b) (top right) order size is normalized by its mean value for the sample. (c) (bottom right) order size is normalized the average daily volume. (d) (bottom right) Order size is multiplied by the current best midpoint price, making the horizontal axis the monetary value of the trade.
The collapse that occurs for the data using nondimensional coordinates is in some sense actually better than that expected according to the theory. That is, with Poisson order flow the market impact function should vary significantly with the parameters $\epsilon$ and $dp/p_c$ (though most of the variation occurs with $\epsilon$ rather than $dp/p_c$). The variations in the real data are much less than we would predict with the Poisson model, i.e. based on the variation in $\epsilon$ in the real data, the model predicts a substantial variation in market impact, which we do not observe. This is very interesting, as it indicates that the nondimensional coordinates dictated by the model are actually doing a better job than one would expect. We believe this is related to deeper regularities in the order flow, but at this point this is a speculation. Nonetheless, the fact that the collapse occurs speaks for itself, and makes it clear that we have uncovered a very interesting regularity of the price formation process. The model led us to this regularity, even if it does not fully explain it.

However, we want to stress that the Poisson order flow model does not reproduce the distribution of fluctuations in market impact, which for the real data are described by a power law. Doing this will require a more sophisticated order flow model.

A.10 The relationship between market impact and supply and demand

The market impact function is closely related to supply and demand. At each moment in time, the limit order book contains all orders offered for buying or selling. We can consider the volume $V_i(p)$ stored at each price level as the instantaneous revealed supply and demand functions. The stored buy limit orders are the revealed demand, and the stored sell limit orders are the supply. Similarly, the market impact function for buy orders probes the revealed supply, and the market impact function for sell orders probes the revealed demand. We use the prefix revealed since there may be traders at the market who are willing to trade more, but for strategic reasons have not revealed their intentions by posting them in the book.

When an effective market order of size $\omega$ is placed, assuming there is at least $\omega$ stored limit orders of the opposite sign, there are a set of transactions at the prices where matches occur. This may result in a price shift $\delta b$ in the best price, which can be determined from the relation

$$\omega = \sum_{p=b}^{b+\delta b} V_i(p). \quad (3)$$

An order of size $\omega$ will transact with orders in the book starting from the best price $b$ to the final price $b + \delta b$. The instantaneous market impact $\delta b$ is determined by the volume of the order and the current state of the book $V_i(p)$. The instantaneous market impact functions (which in principle can be different for buying and selling) and the supply and demand functions are in one-to-one correspondence, i.e. there is a one-to-one map relating them.

The average market impact function is the expectation of market impact conditional on order size $E(\delta b|\omega)$, but the relation to supply and demand is no longer quite so simple. By going into co-moving coordinates (e.g. that use the mid-price as a reference point), it is also possible to define the relative average revealed supply and demand functions $\langle V_i(p - m(t)) \rangle$, where $m$ is the midpoint price. In the absence of any fluctuations in supply and demand, the average market impact functions and the supply and demand functions would be in one-to-one correspondence. However, in the presence of fluctuations the relationship is more complicated, and it becomes necessary to characterize their relationship in probabilistic terms. The average market impact can be computed in terms of a moment expansion of the probability distribution of relative revealed supply or demand, as explained by Smith et al. [4]. Thus the two are closely related, and universal behavior of the market impact function strongly suggests universal behavior of the revealed supply-demand function.

We have chosen to measure average market impact in this paper rather than average relative supply and demand for reasons of convenience. Measuring the average relative supply and demand requires reconstructing the limit order book at each instant, which is both time consuming and error prone. The average market impact function, in contrast, can be measured based on a time series of orders and best bid and ask prices.
A.11 What the model does and does not do

This model was intended to describe average properties of quantities such as supply and demand, which as we have stressed, it does very well. What it does not do is describe the scale-free power law properties. This would require a more sophisticated model of order flow, taking into account both the underlying power law behaviors and the relationship of the different components of the order flow to each other. This is a much harder problem, and is likely to require a more complicated model. While this would have some advantages, it would also have some disadvantages.

In the interest of full disclosure, and as a stimulus for future work, in this section we detail the ways in which the current model does not accurately match the data. As we have already said, we believe the biggest cause of this is the simplified assumptions about order arrival and cancellation. The model assumes that these are described by independent Poisson processes, but the real order flows are more complicated long-memory processes, with significant cross-correlations, e.g. between order placement and cancellation.

• Price diffusion. The variance of real prices obeys the relationship $\sigma^2(\tau) = D\tau^{2H}$ to a good approximation for all values of $\tau$, with $H$ close to and typically a little greater than 0.5. In contrast, under Poisson order flow, due to the dynamics of the double continuous auction price formation process, prices make a strongly anti-correlated random walk, so that the function $\sigma^2(\tau)$ is nonlinear. Asymptotically $H = 0.5$, but for shorter times $H < 0.5$. Alternatively, one can characterize this in terms of a timescale-dependent diffusion rate $D(\tau)$, so that the variance of prices increases as $\sigma^2(\tau) = D(\tau)\tau$. Refs. [3][4] showed that the limits $\tau \to 0$ and $\tau \to \infty$ obey well-defined scaling relationships in terms of the parameters of the model. In particular, $D(0) \sim \mu^2\delta/\alpha^2\epsilon^{-1/2}$, and $D(\infty) \sim \mu^2\delta/\alpha^2\epsilon^{1/2}$. Interestingly, and for reasons we do not fully understand, the prediction $D(0)$ does a good job of matching the real data, and $D(\infty)$ does a poor job. Note that it is very interesting that the double continuous auction produces anti-correlations in prices, even with no correlation in order flow.

One can turn this around: Given that prices are uncorrelated, there must be correlations in order flow. And indeed this is observed to be the case. The observation of correlations in real order flows can be regarded as a validation of the predictions of the model.

• Correlations in spread and price diffusion. We have already discussed in Section (A.4) the problems that the autocorrelations in spread and price diffusion create for comparing the theory to the model on a daily scale.

• Lack of dependence on granularity parameter. In Section (A.9) we discuss the fact that the model predicts more variation with the granularity parameter than we observe. Apparently the Poisson-based nondimensional coordinates work even better than one would expect. This suggests that there is some underlying simplicity that we have not fully captured with the Poisson model.

Although in this paper we are stressing the fact that we can make a useful theory out of zero-intelligence agents, we are certainly not trying to claim that intelligence doesn’t play an important role in what financial agents do. Indeed, one of the virtues of this model is that it provides a benchmark to separate properties that are driven by the statistical mechanics of the market institution from those that are driven by conditional intelligent behavior. Despite the problems detailed above, we think it is rather surprising that such a crude model works so well – at any rate, this surprised us.

References and acknowledgments


[19] *SETS four years on - October 2001*, published by the London Stock Exchange


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