No Provable Limits to 'Scientific Knowledge'

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In an article concerning problems that are impossible to solve exactly, but which can be computed in an average sense, J. F. Traub and H. Wozniakowski raised a challenge: "We believe it is time to up the ante and try to prove there are unanswerable scientific questions. In other words, we would like to establish a physical Gödel's theorem."[1]. Out of this idea grew a workshop at the Santa Fe Institute (May 24-26, 1994), chaired by J. L. Casti, and J. F. Traub, entitled "Limits to Scientific Knowledge". This workshop involved biologists, physicists, economists, philosophers, psychologists, mathematicians, and computer scientists, all with various visions of 'limitations', involving the natural sciences or otherwise. Some of the ruminations of this group, and their subsequent email exchanges, have been collected in a Santa Fe Institute Report [2]. The title of this report, "On Limits", omits reference to 'scientific knowledge', which was largely unaddressed at this workshop. Likewise, published commentaries in the media concerning this workshop did not take notice of this basic concept [3]. In this article one concept of 'scientific knowledge' will be outlined, and the issue of limits of this scientific knowledge will be addressed.

Clearly, any tentative definition of 'scientific knowledge' places a limitation on the range of knowledge that will be considered to be 'scientific'. To be quite clear on these points, in keeping with the Traub-Wozniakowski "physical" ante, the sciences that I am referring to are the physical, or natural sciences, and 'knowledge' is understood here in the context of some 'understanding', which is a multi-faceted concept, requiring careful clarification. As Traub rephrased the issue of the workshop, "Can we up the ante and prove that there are unanswerable questions in science that is, limits to scientific knowledge? A key word in that sentence is prove. Can we turn philosophical inquiry into proof?"[4]. It is this issue of proof that I particularly want to address, and also to note that a lack of a logical proof does not imply that the activity of understanding technical 'limitations' in science needs to revert to merely a 'philosophical inquiry'.

To clarify the possible meaning(s) of 'scientific knowledge' requires discussing several issues: (i) the sources of scientific information and the processes of obtaining 'scientific information' ('observables') from these sources, (ii) the discovery or prediction of correlations within sets of these observables, and (iii) the implementation of some 'scientific method', involving formal reasoning which has historically been the generally-agreed basis for our 'understanding' of these correlations in a given physical phenomenon. This definition of 'scientific knowledge' relates to an understanding involving formal systems of reasoning as well as observations, and needs to be distinguished from other scientific perceptions about natural phenomena, related to (ii), to be discussed shortly.

The first step in the search for 'scientific knowledge' is to obtain 'scientific information', which can be obtained from several sources, the most fundamental being from observations of natural phenomena. To be scientific, it does not suffice to make a "Yogi-Berra-style" observation (who noted that "You can observe a lot just by watching"). Rather, it is necessary to establish 'scientific observables', or simply 'observables'. An observable is some symbolic representation of a natural phenomenon (a 'projection' process) which has the properties that (a) it has a communal character, in the sense that the physical phenomenon can be observed by other interested persons, and is generally agreed to be related to the symbolic characterization, and (b) the observable can be recorded and preserved for future reference and comparisons. For example, the phenomenon of qualia, mentioned by Crick [5] and Penrose [6] does not fall within this framework of an observable (since no one else can observe my qualia), and hence lies forever outside of the present conception of science. It is an inherent 'limitation' simply imposed by the present definition, which is based on the communal observables, traditionally required by the natural sciences.

At present there are three sources of scientific information: (i) the physical observations of natural phenomena just noted, (PO), (ii) some formal mathematical model, which can be associated with these observations, (MM), and (iii) the implementation of digital computer algorithm(s), which generate 'computer experiments', or perhaps better, 'computer explorations', (CE).

These are only the sources of information. To obtain information that is comprehensible to us ('observables') from these sources, requires that methods of 'projecting' be found, which differ for each source, and yield distinct 'observables' in each case. Very briefly stated, in the case of PO we obtain observables by using a physical 'instrument' (including humans), whereas the 'observables' from MM are obtained by logical methods of analysis that can yield conclusions from the original models. These results are rarely described as observables in mathematics, but they are in fact the only information we can comprehend from the original equations. Thus Poincaré's genius supplied us with a number of methods of projecting (deducible) information from differential equations into new nonanalytic mathematical observables. In the case of CE, the methods for obtaining 'projectors' (dynamic-data compressors) to obtain 'observables' from vast amounts of computer data is, generally speaking, in a very formative stage of development [7]. Frequently we are left only with Yogi-Berra observations of our CE, be it data or screen graphics, which stymies the process of generating scientific knowledge.

Each of these forms of information is fundamentally distinct from the others, as discussed more thoroughly in [8]. Briefly, whereas MM are often viewed as being associated with the infinitely precise real numbers, PO generally represents information in the form of a finite set (due to the duration of observations) of domains of real numbers (established by the precision of the instruments), or in some symbolic form. On the other hand, the information obtained from a CE is always equivalent to a finite set of integers (note that CE refers to the finite operations with real computers, which involve many features that are quite distinct from those considered in Computer Science).

Hence, there is an unavoidable difference in the character of the information that can be obtained from these three sources, so no logical one-to-one relationship can be established - indeed the mapping is generally many-to-one in both directions, so there is no logical connection between the observables obtained from different sources of information. Moreover there are many questions, such as "What will be the average world temperature in 2001?", and "When will the universe stop expanding?" [4], which are not scientifically meaningful, because they require information that can never be obtained by any PO (which can only yield finite detectable information). Indeed the field of cosmology is subject to many such questions, and, to that degree, lies outside the present definition of science [9]. Of course metaphysical questions can have provocative value, and as Schrödinger so nicely expressed it, "metaphysics does not form part of the house of knowledge but is the scaffolding, without which further construction is impossible" [10]. Thus, while of value, such questions do not test the limits of 'scientific knowledge', as constrained by these observables.

But, even scientific information does not constitute knowledge, at least in the sense of understanding. The next step requires a 'discovery phase', in which correlations between a set of observables are experimentally found or theoretically predicted to exist. Classic examples are the discovery that, under suitable conditions, pressure, volume, temperature, and the number of moles of a gas are approximately related to each other, or Kepler's and Mendel's laws, which involve temporal correlations. Clearly at this stage some real foothold to 'scientific knowledge' has been established. There is a scientific perception at this stage, in the sense that insights, specific ideas, concepts, and awareness of specific correlations exist. Something is certainly 'understood', and it has a scientific (observable) basis. To be clear, I will call this discovery phase a 'scientific perception', and simply note that whatever it is called (possibly 'scientific knowledge') it bears no relationship to formal logic, and therefore is not susceptible to any logical proof concerning the limitations of such correlations. To come to a kind of understanding of these correlations that incorporates logical reasoning requires showing that there is some relationship, or conjunction, between the correlations found from PO information and one or both of the formal sources of information, MM and CE.

Even prior to the time of Francis Bacon the methodology (/mythology) of science had begun to develop around the idea that our 'understanding' of natural phenomena, and hence the bedrock of 'scientific knowledge', should come from obtaining some form of agreement between the PO information and the MM information (i.e., observables) - the so-called 'scientific method' [11]. The issue here is not whether such a method is always applied in an unbiased fashion, but rather that any 'understanding' requires that most scientists agree that some conjunction has been achieved between these different forms of observables. Today, the above triad of information sources affords a variety of 'scientific methods', depending on the new cyclic connections that are developed between these sources, always involving PO, each yielding new forms of 'scientific knowledge' [8]. Because the MM, CE and PO forms of information are inherently distinct, it is impossible to establish a unique logical relationship between them, without a precise consensual definition of some aspect of the 'conjunction' between these forms of information. Space does not permit an adequate discussion of some implications of this mismatched information. However some features of 'conjunctions' between MM, CE, and PO information can in fact be rationally discussed, and if these become consensual concepts, then, and only then, could formal limitations of the 'scientific method', hence 'scientific knowledge', be established. Until such time, the natural sciences are not subject to any logical limitations.

Even physicists, the historically-proclaimed 'hard scientists', have never rigorously deduced general features of any nontrivial physical phenomenon from 'fundamental equations' - a fact which has obtained growing recognition in the latter half of this century [12]. Some of the reasons for this were uncovered by Poincaré a century ago [13], and subsequently extended by other mathematicians [14]. Moreover these studies also showed that some of those facts that can be deduced from deterministic and realistic MMs seem to imply (as commonly interpreted) that a physical systems could have nondeterministic observable features - that is, either never observable or ambiguously observable. This is related to the sensitivity of real dynamics, and was partially appreciated early this century by Duhem and Borel [15]. Because of the finite operations, integer representations, and round-off errors of any real-world CE, it is not not as easy to make precise statements concerning such difficulties associating CE with PO. However, recent theorems, computations, and experiments are beginning to probe what this all means about basic limitations to scientific knowledge [16]. A very interesting issue is whether only some of the deductions obtained from the formal sources of information may be relevant to scientific knowledge. Thus it may be that algorithms and mathematical equations are not only tentatively related to phenomena in the real world, but that only portions of their deducible content will ever be related to physical phenomena.

Concerning the tentative character of MMs, what some mathematicians and computer scientists may not appreciate is that logical limitations within their formal fields (á la Gödel, Turing, Church, et. al), do not establish a limitation to scientific knowledge. What such limitations imply is that some mathematical 'observables' - deduced results that scientists can comprehend - can not be obtained. This simply means that this excluded information from tentative MMs is irrelevant to science, since it can not contribute to any scientific method. To view this deducible limitation otherwise is to consecrate the mathematical equations with a physical reality that they have no right to claim. In this context one might recall Einstein's remark that "as far as the propositions of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality." [17].

Many physical scientists at least partially recognize that scientific knowledge is inherently limited by the character of scientific information. This in turn is limited by the various 'source-projectors' available - not the least being the human mind - all of which have inherent characteristics, and in this sense 'limitations'. Some of these characteristics are clear - even 'provable' in a nonlogical sense, whereas others have been less explored and understood. As already noted, this is the case in trying to obtain CE-information [7]. In addition there is the unknown character of our Darwinian wiring and preconceptions about the world, which may direct us down particular PO-roads of discovery. As has been discussed and illustrated by Shepard [18], these evolutionary influences may have internalized within our minds universal regularities of Nature that psychology can aspire to uncover. Such internalizations may not speak as much about limitations as they do of the possible potential of our minds to be directed to look for existing universal correlations within Nature. As Needham has beautifully discussed [19], this 'correlative view' characterized the ancient cultures of the East, in contrast with the causal views so prevalent in Western thought. Perhaps science is evolving a form of understanding that relies technically, and not simply philosophically, on both of these views of natural phenomena. The search for these technical, nonprovable characteristics of science (rather than limitations) is certainly a worthy challenge.

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