Modeling Florentine Banking: Part I. Deposits and Loans

John F. Padgett
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In this memo, I model the domestic banking business of Renaissance Florentine bankers. “Domestic bankers,” in the sense of “those doing banking business within Florence,” is an elastic term, including as it does anyone from pawnbrokers to local deposit bankers to international merchant-bankers with a local branch in Florence.1 Practically speaking, my partnership data come from the records of the Arte del Cambio guild, and hence it encompass the second and third of these three categories. But in fact, the model applies to anyone who took deposits from Florentine citizens and made investment loans to Florentine firms (e.g., wool, silk, shoemaking, etc.).

I intend this model of deposits and loans to be of interest in its own right, but the reader should be aware that a second purpose of this “part I” memo is to provide a market-model foundation to a subsequent “part II” memo on banking firm formation. In other words, the actors in this memo are fixed, the issue being variable and evolving trading relations among them. The next memo deals with population dynamics and with speciation among the banks themselves.

I complete this introduction with a diagram:

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1 Indeed, Richard Goldthwaite (personal communication) makes the sound point that private citizens also made loans to each other, well outside of the banking system per se. He calls this practice “functional banking.” In terms of my diagram, these are loans directly from i to k, not mediated through l.
Investors made deposits\(^2\) in banks in one time period \((t-s)\) and received “interest” in return, spread over many time periods \((t)\). “Interest” is in quotes, because the usury doctrine forbade guaranteed rates of return. “Interest,” as a result, was framed culturally as uncertain-rate gifts. Bankers took these deposits, plus their own money, and made loans to profit-making firms. For the same religious reason, banks could not receive guaranteed rates of return for their loans. Rather they too received variable-return “gifts” in exchange, the magnitude of which depended on how the firms did.

I will model this system of exchange in three parts: (1) investment behavior, both of depositors and of bankers, (2) profit and loss of companies, and (3) repayment or “gift” behavior, both of companies and of banks. It will quickly become apparent that my model is in the family of adaptive-learning models of markets (e.g., Sargent 1993). As such, it will dynamically grope its way toward equilibria, rather than jump there immediately (and mysteriously).

**INVESTMENT BEHAVIOR**

Similar to the learning setup I have explored in previous memos, I presume that both investors (vis-a-vis banks) and banks (vis-a-vis companies) decide in whom to invest through a mixture of two procedures:

1. When there is little or no past experience with a potential investee, investors are more likely to explore those new-investment targets who are attributionally similar to themselves.\(^3\)
2. As experience accumulates, however, investors more and more go with those investees who have given good returns in the past.

A number of reasons could be advanced to justify my no-experience homophily hypothesis: interaction frequency, psychological comfort and trust, likelihood of third-party sanctions. Such a search rule, in the presence of little or no performance data, certainly seems more realistic than random search.\(^4\) As such data on relative returns accumulates, however, any rational investor (boundedly rational or otherwise) will respond.

Let me first write the learning rules that operationalize investment rules (1) and (2), and then explain them:

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\(^2\) Later I will simplify and make these fixed-term deposits (renewable), although that need not necessarily be the case.

\(^3\) Readers interested in the relationship between this economic model and my recent political model of Florentine republicanism should note that, in its “weak tie” focus on attributes, economic trading here is analogous to “neutral” voting there (with customers « voters). In the next part II memo, in its “strong tie” focus on multiple networks, firm formation will be analogous to partisan voting (with partners « partisans).

\(^4\) Financial performance of Florentine firms, companies or banks, was not public information. These were private partnerships, not publicly traded corporations. Company reputations no doubt were constructed through private gossip networks. While I do not model reputation explicitly (perhaps I will do so in the future), such effects are consistent with homophily, which I do model. Gossip networks are communication networks linking attributionally similar persons.
For investors \( i \) giving to banks \( l \),
\[
p(w_{ilt}) = \xi_1 \left( \frac{d_{il}}{\sum_l d_{il}} \right) + (1 - \xi_1) \left[ \sum_{s=1}^{\tau_1} x_{fit-s} / \sum_l \sum_{s=1}^{\tau_1} x_{fit-s} \right]
\]
\[
= \xi_1 \delta_{il} + (1 - \xi_1) \left[ \sum_{s=1}^{\tau_1} x_{fit-s} / \sum_l \sum_{s=1}^{\tau_1} x_{fit-s} \right].
\]

For banks \( l \) giving to companies \( k \),
\[
p(c_{ikt}) = \xi_2 \left( \frac{d_{ik}}{\sum_k d_{ik}} \right) + (1 - \xi_2) \left[ \sum_{s=1}^{\tau_2} x_{ikt-s} / \sum_k \sum_{s=1}^{\tau_2} x_{ikt-s} \right]
\]
\[
= \xi_2 \delta_{ik} + (1 - \xi_2) \left[ \sum_{s=1}^{\tau_2} x_{ikt-s} / \sum_k \sum_{s=1}^{\tau_2} x_{ikt-s} \right].
\]

Notation here is as follows:

- \( w_{ilt} \) is the investment “wealth” that \( i \) deposits with \( l \) at \( t \);
- \( c_{ikt} \) is the capital that \( l \) loans to \( k \) at \( t \);
- \( p(w_{ilt}) \) and \( p(c_{ikt}) \) are the probabilities of such, given that investments are made;
- \( d_{il} \) and \( d_{ik} \) are the number of attributes in common between \( i \) and \( l \) or between \( l \) and \( k \), respectively, to be explained below;
- \( \delta_{il} \) and \( \delta_{ik} \) are the relative number of attributes in common;
- \( x_{fit-s} \) and \( x_{ikt-s} \) are the “gifts” that \( l \) gives back to \( i \) (“interest”) or that \( k \) gives back to \( l \) (“returns”), respectively, over time;
- \( \xi_1 \) and \( \xi_2 \) are the relative weights that investors and bankers, respectively, give to exploring new prospects, via homophily, relative to investing in those with whom they have a track record of returns;\(^5\)
- \( \tau_1 \) and \( \tau_2 \) are “memory” parameters, measuring the length of time over which investors evaluate return gifts. These will be equivalent to the term of the deposit or loan.

First I will explain the attributional search logics—\((d_{il} / \sum_l d_{il})\) and \((d_{ik} / \sum_k d_{ik})\). Then I will explain the rational response logics—\([\sum x_{fit-s} / \sum_l \sum x_{fit-s}]\) and \([\sum x_{ikt-s} / \sum_k \sum x_{ikt-s}]\).

For explaining attributional search, no doubt an example is best. Let us say there are only two attributes: social class (\( popolani \) versus new men) and neighborhood (San Giovanni versus Santa Croce). And let us also say that there are four investors and four companies, each of whom (for simplicity) are only solo individuals: \( (P,SG) \), \( (P,SC) \), \( (NM,SG) \), and \( (NM,SC) \). Let us make banking partnerships a bit more complicated, either solo individuals or two-person partnerships: \( (P,SG) \), \( (NM,SC) \), \( (P,SG)+(P,SG) \), \( (P,SG)+(P,SC) \), and \( (P,SG)+(P,SC) \). Then, in this example,

\(^5\) Jim March (1991) would call these \( \xi \) parameters “exploitation versus exploration.” Certainly \( \xi_1 \) and \( \xi_2 \) could be made heterogeneous (\( \xi_{1i} \) and \( \xi_{2i} \)), but I do not explore this here.
### Banks:

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>(P,SG)</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>(P,SC)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>(NM,SG)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>(NM,SC)</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

and \( \delta_{ii} \):

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>(P,SG)</td>
<td>2/11</td>
<td>0</td>
</tr>
<tr>
<td>(P,SC)</td>
<td>1/9</td>
<td>1/7</td>
</tr>
<tr>
<td>(NM,SG)</td>
<td>1/7</td>
<td>0</td>
</tr>
<tr>
<td>(NM,SC)</td>
<td>0</td>
<td>2/5</td>
</tr>
</tbody>
</table>

\( d_{ii} \) and \( \delta_{ik} \) proceed in exactly the same way, except that banks are on the rows and that companies are on the columns. Given no prior performance information (i.e., no \( x_{fit-s} \)'s), investor \( i \) will invest in or explore bank \( l \) with probability \( \delta_{iil} \). Investors start out, at least, with banks that are similar to themselves.

Again assuming that experience has not yet developed, this table also assigns deposit capital to banks. Indexing the total investment wealth of our four investors by \( w_1, w_2, w_3 \) and \( w_4 \), then the deposit capital\(^6\) of our first bank at \( t=0 \) is

\[
c_1 = (2/11) w_1 + (1/9) w_2 + (1/7) w_3,
\]

or in general,

\[
c_l = \sum_i \delta_{il} w_i.
\]

In actuality, in all of my memos, I will dealing with four group attributes on which people compare themselves: Lineage, Social Class, Neighborhood, and Guild/Occupation. In this particular banking-market model, guild/occupation washes out, because bankers are all constant on that dimension. But the other three group attributes remain, as bases for initial investment selection.

Once the system has been running for a while,\(^7\) however, investors learn about which banks or companies give them returns and which do not. Investors in this model incrementally modify their investment behavior accordingly.

\(^6\) Complete bank capital is investors’ deposit capital, plus the bankers’ own personal investment commitment, called \( \text{corpo} \).

\(^7\) “System running for a while” refers to each investor individually. Investor newcomers all start over again from scratch in this model. One can modify this feature if one wants—most obviously by permitting information retention between fathers and sons.
In particular, I hypothesize the simple rule of reciprocity—namely, “invest in those in proportion to the gifts they have given you.” The formula \[ \left( \frac{\sum_{t} x_{lt-s}}{\sum_{l} \sum_{t} x_{lt-s}} \right) \] specifies this quite literally. In words, it says:

(a) over the relevant period \( \tau_{1} \), compare the sum total of the gifts you have received from \( l \) to that received from all of your other \( l \)'s; and then
(b) give investments back to \( l \) at new time \( t \) in proportion to how \( l \) treated you over \( t-\tau_{1} \), relative to how the other \( l \)’s treated you.

Tit-for-Tat, if you will.

As gifts accumulate, \( \sum_{t} x_{lt-s} \) and \( \sum_{l} \sum_{t} x_{lt-s} \) grow over time, while \( \delta_{t} \) stays fixed. Thus the relative importance of attributional “attractiveness,” so important at the outset of an investment relation, diminishes. In the long run, performance dominates over homophily, as long as the exploration parameter \( \xi_{1} \) is not too high.\(^8\) Since newcomers keep coming into the market, however, a cross-section of the market will always include both those wizened investors driven primarily by known performance and those experience-poor investors driven primarily by attributional attractiveness. All these statements, moreover, apply equally to investors (vis-à-vis banks) and to banks (vis-à-vis companies).

PROFIT/LOSS

The next step in the model is to specify profit and loss for Florentine companies, and hence to define what sorts of gifts are capable of flowing back, first to bankers and then to individual investors.

The main choice I make in this step is to deny any “fundamental values”—that is, any inherent differences in quality among firms. Profits and losses, thus, are basically stochastic. One could make the plausible empirical case for Florence that such quality differences were muted by guild standardization. But actually, I make this modeling choice not because I truly believe this to be so, but rather because I am primarily interested in exploring the internal dynamic logic of the model—not in finding out whether or not learning can converge on “true” value. (I assume it can.) In particular, I want to discover whether or not the model can “tip” from one attributional dimensional to another (“regimes”), as a result of internal lock-in mechanisms—located primarily in the part II memo about population dynamics.

With this modeling choice, representing stochastic profits and losses to companies becomes simple:

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\(^{8}\) There is one exception to this statement: in the model, if performance is abysmal, but attributional attractiveness is high, then investors still come back to that “loser” with some non-zero probability. That probability keeps shrinking, as others’ relative gift-giving grows, but as long as \( \xi_{i} \) remains there, this shrinking probability will never vanish altogether. This bounded-rationality scenario is of course inconsistent with pure maximization, but I believe it to be more realistic than pure maximization.
(1) Represent the macro-economy as $C_t$: the total GNP of the Florentine economy. Thus $(C_t - C_{t-1})$ is the yearly growth or decline in the macro-economy, for whatever reason (wars, international monetary crises, trade disputes between city-states, papal excommunications, etc.).

(2) Define a Normal distribution of company profit and loss rates, whose mean is controlled by the macro-economy: $\mu = (C_t - C_{t-1}) / C_{t-1}$, and whose variance is an exogenous “riskiness” parameter: $\sigma^2$. For whatever reason, the Renaissance Florentine economy was well known for being risky indeed (although the degree of risk varied by industry); personal fortunes were made and lost in Renaissance Florence at dizzying rates. In other words,

$$r_{kt} \sim \text{Normal}[ (C_t - C_{t-1}) / C_{t-1} , \sigma^2 ].$$ 

(3) Then pull K random draws of $r_{kt}$ from this profit/loss rate probability distribution, and give them to the K companies in the population. In this setup, stochastic profit and loss rates are affected greatly by the overall state of the Florentine economy, but by nothing else. Parenthetically, this fact creates great pressure on the Florentine political system.

How do such profits and losses affect the companies’ capital and capacity to generate returns? And, as a consequence, the banks’ capital and capacity to generate “interest”? To answer these questions, we need an accounting framework for the tracing the capital flows of firms. For both companies and banks, the following accounting identity holds:

$$c_{kt} = c_{kt-1} + \Delta \text{investments}_{kt} + \text{profit/loss}_{kt} - \text{gifts/returns}_{kt}.$$ 

$c_{kt}$ here is the capital of company k at time $t$.

The definition of a company’s capital is

$$c_{kt} = \text{corpo}_t + \sum_{s=0}^{2-1} \sum \text{corpo}_{kt-s}.$$ 

That is, a company’s operating capital is the personal $\text{corpo}$ of the partners, plus whatever outstanding loans have been given to the firm. These same two accounting formulas will apply to banks.

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9 In fact, this model can be made complicated (that is, made more realistic) by subdividing this macro-economy into industry-specific effects: $C_{zt}$, if z is our new index for industry. Thereby wool can go down, while silk can go up, etc. This is a very serious point indeed from the empirical perspective. Great books, like Hoshino (1980), have been written about it. From a modeling point of view, however, this extension is basically trivial—so I leave it for later. In the mean time, if the reader prefers to think of the current setup as applying to a single industry (like wool) instead of to the entire economy, that is fine.

10 See Goldthwaite (1968) on household finance.

11 For large K, one need not worry about the possible difference between the sample mean of $r_{kt}$’s and the population mean of $(C_t - C_{t-1}) / C_{t-1}$. They will converge.
For simplification, assume all bank loans have fixed-terms $\tau_2$. Then the capital-flow accounting identity becomes

$$c_{kt} = c_{kt-1} + (\Sigma_l c_{kt} - \Sigma_l c_{kt-\tau_2}) + r_{kt} c_{kt-1} - \Sigma_l x_{kh}$$

$$= c_{kt-1} + r_{kt} c_{kt-1} + \Sigma_l (c_{kt} - c_{kt-\tau_2}) - \Sigma_l x_{kh} .$$

In other words, while new loans $c_{kt}$ come in, old loans $c_{kt-\tau_2}$, whose $\tau_2$ term is up, must be given back. Bankruptcy ensues if these old obligations cannot be met.

At this point, we need a specification for how firms process profit. For me, a realistic such specification is the following:

(a) If profit, then the entrepreneur keeps a fixed percentage of that profit for himself, and gives the rest to his creditors.

(b) If loss, then the entrepreneur “eats” the loss, and gives nothing to his creditors, in that time period.

The same rules expressed formally are:

(a) If $r_{kt} > 0$, then the entrepreneur keeps $(1 - G_e) r_{kt} c_{kt-1}$ for himself. Hence,

$$corpo_t = corpo_{t-1} + (1 - G_e) r_{kt} c_{kt-1} ,$$

and he gives $(G_e r_{kt} c_{kt-1})$ back to his creditors:

$$c_{kt} = \Sigma_l x_{kh} = G_e r_{kt} c_{kt-1} .$$

[$c_{kt}$ is notation for total budget flow at $t$ from $k$ back to all his creditors. $G_e$ represents the “giving” propensity of companies. Conversely, $(1 - G_e)$ is the “keeping” propensity of companies. I will investigate the aggregate behavior of the banking system as a function of this parameter.]

(b) If $r_{kt} \leq 0$, then the entrepreneur “eats” the entire loss. Hence,

$$corpo_t = corpo_{t-1} + r_{kt} c_{kt-1} ,$$

and nothing is given to creditors:

$$\Sigma_l x_{kh} = 0 .$$

Substituting these behavioral rules about splitting profits into the accounting equations produces the overall “law of motion” for company capital, both versions of which reduce properly:
If \( r_{kt} > 0 \), then

\[
c_{kt} = c_{kt-1} + r_{kt} c_{kt-1} + \sum_l (c_{kt} - c_{kt-2}) - \sum_l x_{kt}
\]

\[
= c_{kt-1} + \left\{ G_c r_{kt} c_{kt-1} + (1 - G_c) r_{kt} c_{kt-1} \right\} + \sum_l (c_{kt} - c_{kt-2}) - G_c r_{kt} c_{kt-1}
\]

\[
= \{ \text{corporate}_t + \sum_l \sum c_{kt-s} \} + (1 - G_c) r_{kt} c_{kt-1} + \sum_l (c_{kt} - c_{kt-2})
\]

\[
= \text{corporate}_t + \sum_l \sum c_{kt-s} + \sum_l (c_{kt} - c_{kt-2})
\]

\[
= \text{corporate}_t + \sum_l \sum c_{kt-s} + \sum_l (c_{kt} - c_{kt-2})
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\[
= \text{corporate}_t + \sum_l \sum c_{kt-s} + \sum_l (c_{kt} - c_{kt-2})
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= \text{corporate}_t + \sum_l \sum c_{kt-s} + \sum_l (c_{kt} - c_{kt-2})
\]

\[
= \text{corporate}_t + \sum_l \sum c_{kt-s} + \sum_l (c_{kt} - c_{kt-2})
\]

\[
= c_{kt}.
\]

And if \( r_{kt} \leq 0 \), then

\[
c_{kt} = c_{kt-1} + r_{kt} c_{kt-1} + \sum_l (c_{kt} - c_{kt-2}) - \sum_l x_{kt}
\]

\[
= c_{kt-1} + r_{kt} c_{kt-1} + \sum_l (c_{kt} - c_{kt-2}) - 0
\]

\[
= \{ \text{corporate}_t + \sum_l \sum c_{kt-s} \} + r_{kt} c_{kt-1} + \sum_l (c_{kt} - c_{kt-2})
\]

\[
= \text{corporate}_t + \sum_l \sum c_{kt-s} + \sum_l (c_{kt} - c_{kt-2})
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\[
= \text{corporate}_t + \sum_l \sum c_{kt-s} + \sum_l (c_{kt} - c_{kt-2})
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\]

\[
= \text{corporate}_t + \sum_l \sum c_{kt-s} + \sum_l (c_{kt} - c_{kt-2})
\]

\[
= c_{kt}.
\]

For bankers, exactly this same machinery holds, with appropriate changes in notation:

\[
c_h = c_{lt-1} + \Delta \text{investments}_{lt} + \text{revenue}_{lt} - \text{gifts/"interest"}_h.
\]

\[
= c_{lt-1} + \left( \sum_i w_{ih} - \sum_i w_{il+h-2} \right) + \left( \sum_k c_{lt-r} - \sum_k c_{lt} \right) + (\sum_k x_{kh} - \sum_l x_{lh}).
\]

Here, there are two sets of net investment flows to keep track of: deposits from investors, and loans to companies. Bank profit is not (directly) stochastic draws from the “environment” of the macro-economy. Rather, it is “revenue minus costs.” In this context, that means the inflow from returns on loans to companies minus the outflow in “interest” to depositors.

In this equation, the terms referring to forward flow (i.e., the solid arrows in the diagram) have essentially already been derived in the first section of this memo, on investment
behavior. All that remains for those terms is to translate from probabilities into expectations:

\[ \Sigma_i w_{it} = \Sigma_i E(w_{it}) = \Sigma_i w_{it} p(w_{it}) \]

\[ = \Sigma_i w_{it} \left\{ \xi_1 \delta_{it} + (1 - \xi_1) \left[ \sum_{s=1}^{\tau_1} \frac{x_{l+s}}{\sum_{s=1}^{\tau_1} x_{l+s}} \right] \right\}, \text{ and} \]

\[ \Sigma_k c_{kt} = \Sigma_k E(c_{kt}) = \Sigma_k c_{kt} p(c_{kt}) \]

\[ = \Sigma_k c_{kt} \left\{ \xi_2 \delta_{kt} + (1 - \xi_2) \left[ \sum_{s=1}^{\tau_2} \frac{x_{k+s}}{\sum_{s=1}^{\tau_2} x_{k+s}} \right] \right\} . \]

Similarly, the deposits and loans due for repayment, \( \Sigma_i w_{it+1} \) and \( \Sigma_k c_{kt+2} \), are just these same equations, back \( \tau_1 \) and \( \tau_2 \) periods.

The total budgets available for investment by banks and by investors are denoted in these formulas by \( q_{lt} \) and by \( w_{it} \), respectively. What are these? Let me deal with \( q_{lt} \) in this section and with \( w_{it} \) in the next.

The total budget a bank has available for new loans depends upon how much of their revenue stream that bankers want to keep for themselves, and also on how they choose to allocate the remainder between new loans to companies and “interest” payments to their investors.

A bank’s revenue stream at \( t \) is \( \Sigma_k x_{kt} \). Following the notation above, define \( (1 - G_b) \) as the percent of that revenue stream that bankers keep for themselves. \( G_b \), thus, is the percent that bankers give back, either to investors or to companies. Likewise, define \( \xi_3 \) as our third “exploration” propensity, this time of banks toward new loans. High \( \xi_3 \) means that bankers aggressively pursue new loan opportunities, at the cost of “interest” payments to their investors. Low \( \xi_3 \) means that bankers look out primarily for the short-term benefit of their investors. Like \( G_c, \xi_1, \) and \( \xi_2 \), \( G_b \) and \( \xi_3 \) are behavioral parameters, whose effect on the aggregate dynamics of the market I will explore.\(^{12}\)

With this notation,

\[ \Sigma_k x_{kt} = \Sigma_k x_{kt} \left\{ (1 - G_b) + \xi_3 G_b + (1 - \xi_3) G_b \right\}, \]

such that, a bank’s total budget for new loans becomes

\(^{12}\) As I said in footnote 5, all of these parameters could be made heterogeneous, but I do not pursue that option here. I do have one conjecture, however, about the consequences of doing so: Competition among firms will drive all selfish profits for entrepreneurs and for bankers, the \((1 - G_{ak})'s\) and the \((1 - G_{bk})'s\), to zero! Such is the counterfactual prediction of neo-classical models of perfect competition; I expect that result to carry over here. Were this conjecture to prove true, then homogeneous non-zero \((1 - G_c)\) and \((1 - G_b)\) could be interpreted as the collusive regulation by guilds of “fair profits.” This interpretation would be consistent with the not-deeply-competitive flavor of actual Florentine markets, so eloquently described by Goldthwaite (1987), who builds on Melis (1962).
While I am at it, the bank’s corresponding total budget for “interest” payments to investors is

\[ \tilde{j}_n = (1 - \xi_3) G_b \sum k x_{k/t} . \]

**REPAYMENT (“GIFT”) BEHAVIOR**

Finally, we are ready to model reverse “gift” flows\(^ {13} \) (i.e., the dotted arrows in the diagram): namely, returns from companies and “interest” from banks. We have already derived the total budget volume of these flows, \( c_{kt} \) and \( \tilde{j}_n \), now how are they allocated?

Staying true to my adaptive-learning framework, I posit once again our old friend “the rule of reciprocity.” With one difference: For returns and for “interest,” firms engage in no exploration. They give back only to those who have invested in them. Since investments always precede counter-gifts, there is no time period in which this rule is undefined.

With no exploration, the learning rule of reciprocity is very straightforward: “give in proportion to what you have received.”

For companies, this rule implies the following allocation:

\[
p(x_{kh}) = \left[ \sum_{s=1}^{r_2} c_{kt-s} / \sum_{s=1}^{r_2} c_{kt-s} \right].
\]

Hence, the flow of returns from companies \( k \) to banks \( l \) is

\[
E(x_{kh}) = c_{kt} \cdot p(x_{kh})
= \sum_i x_{kh} \left\{ \sum_{s=1}^{r_2} c_{kt-s} / \sum_{s=1}^{r_2} c_{kt-s} \right\}
= G_c \cdot r_{kt} \cdot c_{kt-1} \left\{ \sum_{s=1}^{r_2} c_{kt-s} / \sum_{s=1}^{r_2} c_{kt-s} \right\}.
\]

For banks, this rule implies the following allocation:

\[
p(x_{li}) = \left[ \sum_{s=1}^{r_1} w_{lt-s} / \sum_{s=1}^{r_1} w_{lt-s} \right].
\]

Hence, the flow of “interest” from banks \( l \) to investors \( i \) is

\(^{13} \) An anthropologist would call these “counter-prestations.”
$$E(x_{it}) = \sum_{j} p(x_{it})$$

$$= \left[ (1 - \xi_3) G_{b} \sum_{k} x_{kt} \right] \{ \sum_{s=1}^{t-1} w_{ist} / \sum_{s=1}^{t-1} w_{ist} \} .$$

The only thread left hanging in the model is investor budgets, $w_{ist}$. We know how investors will allocate their investment money once they have it, but how much do they have to allocate?

In Renaissance Florence, wealth came in many forms: land, real estate, moveable objects, government bonds, church benefices, personal credit, and private investments in firms.\(^{14}\) Hence private investment, the subject of this memo, was only one element in the Florentine’s overall wealth portfolio.\(^{15}\) The “money supply” side of the Florentine banking market, therefore, has to do with how Florentines moved their wealth among these components of their overall portfolios.\(^{16}\)

Rather than get into this “general equilibrium” issue explicitly, let me do the usual economist’s trick:

(a) Assume that, from the perspective of investment in banks, overall wealth supply essentially is inexhaustible.\(^ {17}\)

(b) Assume, however, that there is an “opportunity cost” rate of “interest,” below which investors will withdraw their money from banks.

In Florence, the most logical opportunity-cost peg for evaluating the relative value of private-bank “interest” was the fluctuating government bond market. Investments in the Florentine private economy and investments in the Florentine government-bond market were of approximately equal liquidity, and hence very comparable.\(^ {18}\)

With this, the trick becomes easy to specify: Define the opportunity-cost peg as $(w_{gb,t})$, which is one plus the fluctuating interest rate in the government-bond market. Each investor adjusts his overall investment budget in accordance with how his bank investments have done, on average, relative to this opportunity-cost peg.

The money that came back at $t$ to investor $i$, from his past investments $\tau_1$ periods before, is

$$w'_{it} = \sum_{s=1}^{\tau_1} \left( w_{ist} + \sum_{s=1}^{\tau_1} x_{ist} \right).$$

\(^{14}\) Arguably also political offices.

\(^{15}\) Goldthwaite’s first book (1968) describes this wealth portfolio well.

\(^{16}\) There were huge historiographical debates about alleged “return to the land” impulses among Florentine aristocrats. I will not touch these with a ten-foot pole.

\(^{17}\) A rather lousy assumption for 1430. Not terrible under normal economic circumstances, however.

\(^{18}\) See Conti (1984) and Molho (1971, 1994) for more information about the complex world of Florentine public finance. It seems clear to me, although not established in the literature, that increased reliance on government bonds during the Medici period (e.g., the ingenious Monte delle doti dowry fund) caused the collapse of private domestic banking, which I observe in this same period. (The latter trend was also noted by DeRoover (1966: p. 374), who probably over-generalized his correct observation.)
Hence, the average annual rate of “interest” that he observes from all of his banks together is

\[
\dot{w}_{it} = \left\{ \frac{\sum_j (w_{iht-1} + \sum_{s=1}^{\tau} x_{it-s})}{\sum_j w_{iht-1}} \right\} / \tau_t
\]

The adaptive-adjustment rule for total-investment budgets that I hypothesize is

\[
\ddot{w}_{it} = \left[ \frac{\dot{w}_{it}}{w_{gb,t}} \right] \dot{w}_{it}.
\]

If investor i experiences an average rate of bank “interest” that is higher than the government-bond interest rate, then he will increase his total investments. If lower, then he will decrease his total investments. All in an adaptively smooth way.

Given this investment budget, the investor allocates his money to banks in accordance with the rules derived in the first section of this memo.

What do these behavioral rules imply about aggregate money supply, and hence about the trajectory of the banking market as a whole?

If \( \sum_i w_{it} > \sum_i w_{it}' \), then the domestic banking market is growing.

If \( \sum_i w_{it} < \sum_i w_{it}' \), then the domestic banking market is shrinking.

Absent other sources of funding, those manufacturing industries in Florence dependent on bank finance will be stimulated or depressed accordingly.19

With this, model specification is complete. “All” that remains is to investigate the model’s dynamic behavior.

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19 Any evaluation of Lopez’s (1962) controversial Renaissance-depression thesis, which DeRoover (1966) embraced prematurely, depends in part on estimating these “other sources of financing.” Goldthwaite (1993) relied in part on “functional banking” (see footnote 1) to argue forcefully against that thesis. Certainly models alone cannot adjudicate this important matter.
Bibliography