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SFI WORKING PAPER: 2013-02-006

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Quantifying Morphological Computation

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January 29, 2013

Abstract

The field of embodied intelligence emphasises the importance of the morphology and environment with respect to the behaviour of a cognitive system. The contribution of the morphology to the behaviour, commonly known as morphological computation, is well-recognised in this community. We believe that the field would benefit from a formalisation of this concept as we would like to ask how much the morphology and the environment contribute to the embodied agent’s behaviour, or how an embodied agent can maximise the exploitation of its morphology within its environment. In this work we derive two concepts of measuring morphological computation, and we discuss their relation to the Information Bottleneck Method. The first concept asks how much the world contributes to the overall behaviour and the second concept asks how much the agent’s action contributes to a behaviour. Various measures are derived from the concepts and validated in two experiments which highlight their strengths and weaknesses.

Keywords. Information Bottleneck Method; Embodied Artificial Intelligence; Morphological Computation; Information Theory; Quantification; Measure; Sensori-Motor Loop

1 Introduction

Morphological computation is discussed in various contexts, such as DNA computing, and self-assembly [see 1, 2, for an overview]. This work is concerned with morphological computation in the field of embodied intelligence. In this context it is often described as the trade-off between morphology and control [3], which means that a well-chosen morphology can reduce the amount of required control substantially. Hereby, a morphology refers to the body of a system, explicitly including all its physiological and physical properties (shape, sensors, actuators, physical properties, etc.) [4]. The consensus is that morphological computation is the contribution of the morphology and environment to the behaviour, that cannot be assigned to a nervous system or a controller. The following quote very nicely describes how the shape of an insect wing in flight is not entirely determined by the muscular system, but by the interaction of the wings’ morphology with the environment:

However, active muscular forces cannot entirely control the wing shape in flight. They can only interact dynamically with the aerodynamic and inertial forces that the wings experience and with the wing’s own elasticity; the instantaneous results of these interactions are essentially determined by the architecture of the wing itself: its plan form and relief, the distribution and local mechanical properties of the veins, the local thickness and properties of the membrane, the position and form of lines of flexion. The interpretation of these characters is the core of functional wing morphology. [5, see p. 188]

The last sentence of this quote nicely summarizes that the function of the wing is determined by the interaction of the environment with the physical properties of the wing. It is the quantification of this sort of contribution of the morphology to the behaviour of a system that is in the focus of this work. The difference to previous literature by Paul [6] and Lundh [7], who investigated morphological with respect to either the actuation
(Paul) or the sensors (Lundh) only, is that we will measure morphological computation of embodied agents acting in the sensori-motor loop, including both, sensors and actuators.

To understand which aspects a measure must cover and which it should omit, this paragraph will discuss two different artificial systems which show morphological computation. To most vivid example of morphological computation is the Passive Dynamic Walker by McGeer [8]. In this example, a two-legged walking machine preforms a naturally appealing walking behaviour, without any need of control, as a result of a well-chosen morphology and environment. There is simply no computation available, and the walking behaviour is the result of the gravity, the slope of the ground and the specifics of the mechanical construction (weight & length of the body parts, deviation of the joints, etc.). If any parameter of the mechanics (morphology) or the slope (environment) are changed, the walking behaviour will not persist. Hence, we will only investigate morphological computation as an effect that emerges from the interaction of the control system, the body and the environment, also known as the sensori-motor loop (see next section). The Passive Dynamic Walker is, in this context, understood as an embodied agent without actuation.

One may argue that the Passive Dynamic Walker is a purely mechanical system, and that speaking of morphological computation is an overstatement in this case. This is a valid point of view, as purely mechanical system do not perform calculations as we intuitively understand the term. Nevertheless, we claim that it is not an overstatement in the case of the Passive Dynamic Walker, as this system was explicitly built to simulate the morphological computation that is present in the human walking behaviour. For this purpose, a mechanical system was constructed that reflects the morphological properties of the lower half of a human body as far as it is required to understand and model the principles involved in human walking. Therefore, the Passive Dynamic Walker is allegoric for morphological computation present in the locomotion of humans.

A second impressive example for morphological computation is BigDog [9]. This robot is a four-legged walking machine, which is built as a companion for humans operating in the field. The outstanding feature, with respect to this paper, is its morphological design. Instead of classical electric motors, BigDog has hydraulic actuators, which enable it to handle situations in which other walking machines would fail and even suffer severe damage. Most impressive is the video, which shows how BigDog handles a very slippery ground\(^1\). Watching the video (see footnote below), it seems impossible to program such a balancing behaviour for an arbitrarily slippery ground. That BigDog is nevertheless able to cope with it, is also the result of the hydraulic actuators and the well-chosen morphological design in general.

The next step is to analyze, what both examples have in common, and in which aspects the two applications differ. The first obvious difference is the amount of available computation. Morphological computation is most often associated with low computational power that leads to sophisticated behaviours [e.g. 6]. This intuitive understanding is well-reflected in the case of BigDog, as the on-board computation accounts for only a fraction of BigDog’s ability to cope with a slippery ground. This intuitive understanding is not well-reflected in the case of the Passive Dynamic Walker, as there is no computation available at all. Therefore, measuring morphological computation in relation to the computational complexity of the controller does not seem reasonable, as the amount of computation used to generate the action does influence the amount of computation conducted by the morphology.

There is something that both examples have in common, and which is well-suited for a measure of morphological computation. In both cases, there is an observed (coordinated) behaviour of the system that is not assigned to any control. In the case of the Passive Dynamic Walker, this is obviously the case. In the case of BigDog, it was stated above, that the behaviour of the robot on slippery ground is not fully assigned to the control, but also assigned to the design of the robot, and especially its hydraulic actuation. Therefore, a quantification should capture how much of the behaviour is assigned to the morphology and environment, and not to the controller.

This work is organised in the following way. The next section (see Sec. 2) describes the sensori-motor loop and it representation as a causal graph, and thereby, presents the conceptual foundation that is required for the remainder of this work. In the third section (see Sec. 3) we will then derive the different measures for morphological computation. The section begins with a summary of the intuitive understanding of morphological computation, which is then followed by a discussion of two concepts to measure it. The section

\(^1\)Watch [http://www.youtube.com/watch?v=WlczBcmX1Ww](http://www.youtube.com/watch?v=WlczBcmX1Ww), at about 1min 24s.
ends with relating them to the Information Bottleneck Method by Tishby et al. [10]. Two experiments to validate and analyse the derived measures are conducted in the fourth section (see Sec. 4). The fifth section (see Sec. 5) discusses the results and the final section (see Sec. 6) concludes this work. All calculations are carried out in the appendix.

2 Sensori-Motor Loop

To derive a quantification of morphological computation of an embodied system, it requires a formal model of the sensori-motor loop. This will be briefly summarised in the following paragraph (see also Fig. 1(a)). A cognitive system consists of a brain or controller, which sends signals to the system’s actuators. The actuators affect the system’s environment. We prefer the notion of the system’s Umwelt [11], which is the part of the system’s environment that can be affected by the system, and which itself affects the system. The state of the actuators and the Umwelt are not directly accessible to the cognitive system, but the loop is closed as information about both, the Umwelt and the actuators are provided to the controller by the system’s sensors. In addition to this general concept, which is widely used in the embodied artificial intelligence community [see e.g. 12] we introduce the notion of world to the sensori-motor loop, and by that we mean the system’s morphology and the system’s Umwelt. We can now distinguish between the intrinsic and extrinsic perspective in this context. The world is everything that is extrinsic from the perspective of the cognitive system, whereas the controller, sensor and actuator signals are intrinsic to the system.

The distinction between intrinsic and extrinsic is also captured in the representation of the sensori-motor loop as a causal or Bayes’ian graph (see Fig. 1(b)). The random variables $C$, $A$, $W$, and $S$ refer to the controller, actuator signals, world and sensor signals, and the directed edges reflect causal dependencies between the random variables. Everything that is extrinsic is captured in the variable $W$, whereas $S$, $C$, and $A$ are intrinsic to the system. The random variables $S$ and $A$ are not to be mistaken with the sensors and actuators. The variable $S$ is the output of the sensors, which is available to the controller or brain, the action $A$ is the input that the actuators take. Consider an artificial robotic system as an example. Then the sensor state $S_t$ could be the pixel matrix delivered by some camera sensor and the action $A_t$ could be a numerical value that is taken by some motor controller to be converted in currents to drive a motor.
Throughout this work, we use capital letter \((X, Y, \ldots)\) to denote random variables, non-capital letter \((x, y, \ldots)\) to denote a specific value that a random variable can take, and calligraphic letters \((\mathcal{X}, \mathcal{Y}, \ldots)\) to denote the alphabet for the random variables. This means that \(x_t\) is the specific value that the random variable \(X\) can take a time \(t \in \mathbb{N}\), and it is from the set \(x_t \in \mathcal{X}\). Greek letter refer to generative kernels, i.e. kernels which describe an actual underlying mechanism or a causal relation between two random variables. In the causal graphs throughout this paper, these kernels are represented by direct connections between the corresponding nodes. This notation is used to distinguish generative kernels from others, such as the conditional probability of \(s_{t+1}\) given that \(c_t\) was previously seen, denoted by \(p(s_t|c_t)\), which can be calculated or sampled, but which does not reflect a direct causal relation between the two random variables \(C_t\) and \(S_{t+1}\) (see Fig. 1(b)).

We abbreviate the random variables for better comprehension in the remainder of this work, as all measures consider random variables of consecutive time indices. Therefore, we use the following notation. Random variables without any time index refer to time index \(t\) and hyphenated variables to time index \(t+1\). The two variables \(W, W'\) refer to \(W_t\) and \(W_{t+1}\).

We have now defined the conceptual framework in which we will derive the measures for morphological computation. This is done in the next section, which starts with an overview.

### 3 Measuring Morphological Computation

We will derive the measures for morphological computation in four steps (see Fig. 2):

1. Descriptive definition of morphological computation
2. Framing of two concepts which follow from step 1
3. Formal definition of the two concepts given in step 2
4. Intrinsic adaptations of the definitions given in step 3

The fourth step is required because the two definitions given in the third step require information about the world state \(W\), which is generally not available. This will be discussed in detail later in this section.

#### 3.1 Morphological Computation

We understand morphological computation as the contribution of the morphology and environment to the overall behaviour of an embodied system. This implies that morphological computation must be studied in the sensori-motor loop, as it is the result of the interaction of the controller, morphology and environment.
Measuring morphological computation means that an observer has recorded a behaviour of interest of an embodied agent and post-hoc asks the question how much morphological computation was present in the recoded sequence.

Measuring morphological computation does not mean to capture the complexity or the quality of a behaviour. This are aspects of the behaviour of cognitive systems, which are not handled in our measures.

### 3.2 Concepts of measuring morphological computation

In the following, we assume that an embodied agent is well-captured by the sensori-motor loop as it was presented in the previous section (see Fig. 1). Explicitly, we will use the term world as it was defined above, as the system’s morphology and Umwelt.

Generally speaking, morphological computation is the effect of the current world state $W$ on the next world state $W'$, which is not assignable to the action $A$. Given the sensori-motor loop as we have defined it above (see Fig. 1), this cannot be measured in isolation, as there is a path from $W$ to $W'$ which goes through the agent. Hence, we need to measure the effect of the action $A$ on the next world state $W'$ so that we are able to deduce the effect of $W$ on $W'$ that does not include the pathway over $A$. This has two implications. First, the resulting value of such a measure is not an absolute value per se. In fact, it can only be measured as the relation of the two effects $W \rightarrow W'$ and $A \rightarrow W'$. Second, there are two ways of comparing the effects.

The first method of measuring morphological computation assumes that the next world state $W'$ is only determined by the current world state $W$, which is equivalent to assuming maximal morphological computation. Any measured effect of the action $A$ on the next world state $W'$ displays itself in a reduction of the resulting measurement. This is summarised in the following concept:

**Concept 1 (Negative effect of the action)** Given a behaviour of interest of an embodied system, the amount of morphological computation is inversely proportional to the contribution of the actions of the system to the overall behaviour.

The second method of measuring morphological computation assumes that the next world state $W'$ is only determined by the current action $A$, which is equivalent to assuming zero morphological computation. Any measured effect of the current world state $W$ on the next world state $W'$ displays itself in an increase of the resulting measurement. This is summarised in the following concept:

**Concept 2 (Positive effect of the world)** Given a behaviour of interest of an embodied system, the amount of morphological computation is proportional to the contribution of the world to the overall behaviour.

### 3.3 Formalising the concepts

The next step is to formalise the two concepts presented above. The causal diagram (see Fig. 1(b)) shows that the world kernel $\alpha(w'|w,a) = p(w'|w,a)$ captures the influence of $W$ and $A$ on $W'$. Therefore, it is the basis for our further considerations. If the action $A$ has no effect on the next world state $W'$, then the world kernel reduces to $p(w'|w,a) = p(w'|w)$ and we would state that the system shows maximal morphological computation. Analogously, if the world state $W$ has no influence on the next world state $W'$, i.e. $p(w'|w,a) = p(w|a)$, we would state that the system shows no morphological computation. Measuring the differences of the world kernel to the two conditional probability distributions $p(w'|w)$ and $p(w'|a)$ leads us to the formalisations of the two concepts discussed above. The Kullback-Leibler divergence measures the differences of two probability distributions [13] and applying it to our scenario gives us the definitions for the two concepts. Their formalisations are given below, such that the value zero refers to no morphological computation and one refers to maximal morphological computation.

**Definition 1 (Morphological Computation as negative effect of the action)** Let the random variables $A,W,W'$ denote the action, the current and the next world state of an embodied agent, which is described by the causal diagram shown in Figure 1(b). The quantification of the morphological
computation as the negative effect of the action on the behaviour is then defined as

\[ MC_A := 1 - \frac{1}{\ln |W|} D(p(w'|w, a)||p(w'|w)) \]

\[ = 1 - \frac{1}{\ln |W|} \left( \sum_{w, w' \in W, a \in A} p(w', w, a) \ln \left( \frac{p(w'|w, a)}{p(w'|w)} \right) \right). \]

**Definition 2 (Morphological Computation as positive effect of the world)**

Let the random variables \( A, W, W' \) denote the action, the current and the next world state of an embodied agent, which is described by the causal diagram shown in Figure 1(b). The quantification of the morphological computation as the positive effect of the world on the behaviour is then defined as

\[ MC_W := 1 - \frac{1}{\ln |W|} D(p(w'|w, a)||p(w'|a)) \]

\[ = 1 - \frac{1}{\ln |W|} \left( \sum_{w, w' \in W, a \in A} p(w', w, a) \ln \left( \frac{p(w'|w, a)}{p(w'|a)} \right) \right). \]

Both definitions given above require full access to the world states \( W \) and \( W' \). This is undesired for two reasons. First, it limits the applicability of the measure to systems of low complexity which live in the domain of simple grid-world environments. This contradicts our interest in presenting measures that can be used to analyse natural or non-trivial artificial cognitive systems. Second, we believe that the measures derived here, may model an intrinsic driving force in the context of guided self-organisation of embodied systems. Hence, we require that any measure must operate on intrinsically available information only. The resulting intrinsic measures are presented below, in the order of the concepts they relate too. The next sections list the four measures without discussing them in detail. They are evaluated and discussed in the subsequent sections.

### 3.3.1 Concept 1, Associative Measure

The first measure that operates on intrinsic information only is the canonical adaptation of the first definition to the intrinsic perspective. It must be mentioned here, that the sensor states \( S \) and \( S' \) are understood as the intrinsically available information about the external world. Hence, the conditional probability \( p(s'|s) \) refers to the intrinsically available information about \( p(w'|w) \).

**Definition 3 (Associative measure of the negative effect of the action)**

Let the random variables \( A, S, S' \) denote the action, the current and the next sensor state of an embodied agent, which is described by the causal diagram shown in Figure 1(b). The quantification of the morphological computation as an associative measure of the negative effect of the action is then defined as:

\[ ASOC_A := 1 - \frac{1}{\ln |S|} D(p(s'|s, a)||p(s'|s)) \]

\[ = 1 - \frac{1}{\ln |S|} \sum_{s, s' \in S, a \in A} p(s', s, a) \ln \left( \frac{p(s'|s, a)}{p(s'|s)} \right). \]

The Kullback-Leibler divergence used in the definition of \( ASOC_A \) (see Def. 3) is also known as the transfer entropy of \( A \) on \( S \) [14] and it was investigated in the context to quantify the informational structure of sensory and motor data of embodied agents [15].

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Figure 3: Reactive system. In the context of this work, a reactive system is defined by a direct coupling of the sensors and actuators. There is no form of memory present in the system.

Figure 4: Visualisation of the causal measure $C_{A}$. The causal graph used in the Figures (a), (b), and (c) is the reduction of sensori-motor loop shown in Figure 3(b) to two consecutive time steps. Figure (a) shows that the causal information flow $CIF(S \rightarrow S')$ measures all causal information from $S$ to $S'$, including the information that flows over $A$. Figure (b) shows that the causal information flow $CIF(A \rightarrow S')$ only captures the information flowing from $A$ to $S'$. Both can be used to approximate the causal information from $S$ to $S'$, that does not pass through $A$, denoted by $CIF(S \rightarrow S' \backslash A)$, as shown Figure (c).

### 3.3.2 Concept 1, Causal Measure

The second measure which follows the first concept is based on measuring the causal information flow of the sensor and action states $S, A$ on the next sensor state $S'$. To simplify the argumentation, we first consider a reactive system (see Fig. 3(b)). The measure will then also be presented and discussed for non-reactive systems, as both cases result in different measures. The idea for this measure is captured in the Figure 4. Please keep in mind, that the sensor states $S$ and $S'$ are here understood as internally available information about the system’s Umwelt. If we talk about the causal information flow of the current sensor state $S$ to the next sensor state $S'$, we do mean the intrinsically detectable causal information flow of the current world state $W$ to the next world state $W'$, and not how the sensor acts on itself. The causal information flow from $S$ to $S'$, denoted by $CIF(S \rightarrow S')$ includes the information that flows from $S$ to $S'$ over all pathways. This explicitly includes the information flow from $S$ to $S'$ over the action $A$ (see Fig. 4(a)). Morphological computation is here understood as the causal information flow from $S$ to $S'$ excluding the causal information flowing from the action $A$ to $S'$ (see Fig. 4(c)). Hence, we need to subtract the causal information flow from $A$ to $S'$, denoted by $CIF(A \rightarrow S')$ (see Fig. 4(b)) to exclude it from $CIF(S \rightarrow S')$ such that we receive the causal information flow that goes from $S$ to $S'$ without passing $A$, denoted by $CIF(S \rightarrow S' \backslash A)$ (see Fig. 4(c)).

According to the general theory [16], we talk about identifiable causal effects if the are computable of observational data. We stated earlier, that we are interested in intrinsic measures. Hence, we require that the causal effects are identifiable from observational data that is intrinsically available to the agent. We refer to this type of identifiability as *intrinsically identifiable*. In our previous work [17], we showed that the
causal effects of $S$ on $S'$ and of $A$ on $S'$ are intrinsically identifiable by the following conditional probability distributions:

$$p(s' | \text{do}(a)) = \sum_{s \in S} p(s' | s, a) p(s)$$  \hspace{1cm} (3)

$$p(s' | \text{do}(s)) = \sum_{a \in A} \sum_{s'' \in S} p(a | s) p(s' | s'', a) p(s'')$$  \hspace{1cm} (4)

$$= \sum_{a \in A} p(a | s) p(s' | \text{do}(a))$$  \hspace{1cm} (5)

These distributions need to be explained. The notation $p(x | \text{do}(y))$ (also denoted by $p(x | \hat{y})$) refers to probability of measuring $x$ when the state of $Y$ was set by intervention to $y$ [16]. Explicitly, this means that generally intervention is required in order to determine causation. Therefore, it is important to note, that the equations above (see Eq. 3 and 4) allow us to determine the causal effects without any intervention. An agent can act in the sensori-motor loop, and from its observation determine e.g. the causal effect of its actions $A$ on its next sensor states $S'$ [for a discussion, see 17]. From the two probability distributions given in the Equations 3 and 4 we can construct the two required causal information measures for $CIF(S \rightarrow S')$ and $CIF(A \rightarrow S')$. The derivations for both measures are given in the appendix (see Sec. A.1). The resulting equations are given by

$$CIF(S \rightarrow S') = \sum_{s \in S} p(s) \sum_{s' \in S} p(s' | \text{do}(s)) \ln \left[ \frac{p(s' | \text{do}(s))}{\sum_{s'' \in S} p(s' | \text{do}(s'')) p(s'')} \right]$$  \hspace{1cm} (6)

$$CIF(A \rightarrow S') = \sum_{a \in A} p(a) \sum_{s' \in S} p(s' | \text{do}(a)) \ln \left[ \frac{p(s' | \text{do}(a))}{\sum_{a' \in A} p(s' | \text{do}(a')) p(a')} \right]$$  \hspace{1cm} (7)

The difference $CIF(S \rightarrow S') - CIF(A \rightarrow S')$ is always negative (see Eq. 57 in Sec. A.1), and hence, the resulting measure is given by the following definition.

**Definition 4 (Causal measure of the negative effect of the action for a reactive system)**

Let the random variables $A, S, S'$ denote the action, the current and the next sensor state of a reactive embodied agent, which is described by the causal diagram shown in Figure 1(b). The quantification of the morphological computation as a causal measure of the negative effect of the action is then defined as:

$$C_A := 1 + \frac{1}{\ln |S|} (CIF(S \rightarrow S') - CIF(A \rightarrow S'))$$  \hspace{1cm} (8)

$$= 1 - \frac{1}{\ln |S|} D(p(s' | \text{do}(a)) || p(s' | \text{do}(s)))$$  \hspace{1cm} (9)

$$= 1 - \frac{1}{\ln |S|} \sum_{s \in S, a \in A} p(s, a) \sum_{s' \in S} p(s' | \text{do}(a)) \ln \frac{p(s' | \text{do}(a))}{p(s' | \text{do}(s))}$$  \hspace{1cm} (10)

The causal information flow $CIF(S \rightarrow S')$ is so far not shown to be intrinsically identifiable for non-reactive systems [17]. Therefore, we consider the causal information flow from the internal controller state $C$ to the next sensor state $S'$ (see Fig. 1(b)). This is legit, because $C$ represents the entire history of the system, and therefore, also the internal representation of the entire history of the world. All further calculations are analogous to the previous case [see also 17, for a discussion] and lead to the following definition.

**Definition 5 (Causal measure of the negative effect of the action for a non-reactive system)**

Let the random variables $A, C, S'$ denote the action, the controller and the next sensor state of a non-reactive or deliberative embodied agent, which is described by the causal diagram shown in Figure 1(b). The quantification of the morphological computation as a causal measure of the negative effect of the action is
then defined as:

\[ C_A^d := 1 + \frac{1}{\ln |S|} (CIF(C \rightarrow S') - CIF(A \rightarrow S')) \] (11)

\[ = 1 - \frac{1}{\ln |S|} D(p(s'|do(a))||p(s'|do(c))) \] (12)

\[ = 1 - \frac{1}{\ln |S|} \sum_{c \in C, a \in A} p(c, a) \sum_{s' \in S} p(s'|do(a)) \ln \left[ \frac{p(s'|do(a))}{p(s'|do(c))} \right] \] (13)

The last definition (see Def. 5) is given for the reason of completeness only. Morphological computation is mainly discussed in the context of behaviours which are well-modelled as reactive behaviours (e.g. locomotion). To the best of our knowledge, it has so far not been discussed in the context of non-reactive or deliberative behaviours. Therefore, the first definition of the causal measure (see Def. 4) suffices for all currently discussed cases of morphological computation in the field of embodied artificial intelligence.

This concludes the measures of the first concept, in which morphological computation is calculated inversely proportional to the influence the action \( A \) has on the next world state \( W' \). The next section discusses the measures for morphological computation in which it is calculated proportionally to the effect the world has on itself.

3.3.3 Concept 2, Associative Measure

Analogous to the associative measure of the first concept, the associative measure of the second concept adapts the Definition 2 to the intrinsic perspective by replacing \( W \) by \( S \) and \( W' \) by \( S' \). This leads to the following definition.

**Definition 6** (Associative measure of the positive effect of the world)

Let the random variables \( A, S, S' \) denote the action, the current and the next sensor state of an embodied agent, which is described by the causal diagram shown in Figure 1(b). The quantification of the morphological computation as an associative measure of the positive effect of the world is then defined as:

\[ ASOC_W := \frac{1}{\ln |S|} D(p(s'|s, a)||p(s'|a)) \] (14)

\[ = \frac{1}{\ln |S|} \sum_{s, s' \in S, a \in A} p(s', s, a) \ln \left[ \frac{p(s'|s, a)}{p(s'|a)} \right]. \] (15)

The next measure approximates the dependence of the next world state \( W' \) on the current world state \( W \) solely based on the internal world model \( p(s'|s, a) \).

3.3.4 Concept 2, Conditional Independence

The idea of this measure is to calculate how much the conditional probability distribution \( p(s'|s) \) estimated from the recoded data differs from the assumption that the world did not have any effect on itself, denoted by the conditional probability distribution \( \tilde{p}(s'|s) \). The Figure 5 shows the difference between the two conditional probability distributions graphically. The following equations show how they can be calculated from the observed data.

\[ p(s'|s) = \sum_{a \in A} p(s'|s, a)p(a|s) \] (16)

\[ \tilde{p}(s'|s) = \sum_{a \in A} p(s'|a)p(a|s) = \sum_{a \in A} p(a|s) \sum_{s'' \in S} p(s'|s'', a)p(a|s)p(s'')/p(a) \] (17)

The measure is then defined as the Kullback-Leibler divergence of \( p(s'|s) \) and \( \tilde{p}(s'|s) \).
Figure 5: Visualisation of the conditional independence measure $C_W$. The left-hand side shows how the conditional probability distributions $p(s'|s)$ can be calculated from the world model $p(s'|s,a)$ and the policy $p(a|s)$. The right-hand side shows how $p(s'|s)$ changes, if one assumes that the world does not influence itself (gray arrow between $W$ and $W'$), and if this is reflected in the internal world model (gray arrow between $S$ and $S'$). The difference of both measures the morphological computation.

**Definition 7 (Conditional dependence of the world on itself)**

Let the random variables $A, S, S'$ denote the action, the current and the next sensor state of an embodied agent, which is described by the causal diagram shown in Figure 1(b). The quantification of the morphological computation as the error of the assumption, that the next world state is conditionally independent of the previous world state is then defined as:

$$C_W := \frac{1}{\ln |S|} D(p(s'|s)||\hat{p}(s'|s)) \quad (18)$$

$$= \frac{1}{\ln |S|} \sum_{s \in S, s' \in S} p(s'|s)p(s) \ln \left[ \frac{p(s'|s)}{\hat{p}(s'|s)} \right] \quad (19)$$

This concludes the presentation of definitions of morphological computation measures. The next section discusses their relation to the Information Bottleneck Method [10].

### 3.4 Relation to the Information Bottleneck Method

Figure 6: Visualisation of the relation of the concepts to the Information Bottleneck Method. Figure (a) shows the general concept of morphological computation. The world states are highly correlated (red arrow), whereas the world and action are only weakly correlated (blue arrow). The Figure (b) show the concept of the Information Bottleneck Method. The difference is that a strong correlation between the action $A$ and $W'$ and a weak correlation between $W$ and $A$ are required. For a discussion, please read the text below.

We will discuss the relation of morphological computation to the Information Bottleneck Method by Tishby et al. [10] along with the two graphs shown in Figure 6. The graph on the left-hand side (see Fig. 6(a)) shows how we understand morphological computation. It is present, if the current world state $W$ has a high influence on the next world state $W'$ (denoted by a red arrow) and the current action $A$ has a
low influence on the next world state $W'$ (denoted by a blue arrow). Hereby, it is not relevant how much the world has influenced the action (denoted by a black arrow).

The Information Bottleneck Method is visualised in the Figure 6(b). Here, the influence of the world on the action is minimised and the influence of the action on the next world state is maximised. The action $A$ is the bottleneck variable $\tilde{X}$ (see Fig. 6(b)) for the relevant information that is passed through the world. This appears connected for the following reason. Conceptionally speaking, reducing the required alphabet in $A$ to still maintain the information the world carries about itself is related to the notion of morphological computation as it means that only a few actions should be chosen by the agent to maintain the behaviour. Formally, the two concepts (Information Bottleneck and morphological computation) are not easily aligned. The Information Bottleneck Method requires to minimise the Kullback-Leibler divergence $D(p(w'|w)||p(w'|a))$ [see e.g. Eq. (27) in 10], whereas morphological computation is defined to minimise the Kullback-Leibler divergence $D(p(w'|w,a)||p(w'|w))$ (see Def. 1) or to maximise the Kullback-Leibler divergence $D(p(w'|w,a)||p(w'|a))$ (see Def. 2). We could currently not solve this contradiction, but we believe that it should be followed, as it could unify the presented concepts $MC_A$ and $MC_W$.

This concludes the presentation and discussion of the two concepts, their formalisations, and the resulting four measures that rely on internal variables only. The next step in this work is to evaluate how these different measure behave, when they are applied to experiments. This is done in the next section.

4 Experiments

This section applies the morphological computation measures to two different experiments. The first experiment is a simplified parameterisable model of the one-step sensori-motor loop for reactive systems (see Fig. 3(b)). The model is defined by the four transition maps $\alpha_{\phi,\psi}(w'|w,a)$, $\beta_{\zeta}(s|w)$, $\pi_{\mu}(a|s)$, and $p_{\tau}(w)$. For each map, the indices refer to its parameters. Hence, the entire model is parameterised by five parameters, of which two are kept constant in the experiments below.

The second experiment is designed as a minimal physical system, that allows a transition between the Dynamic Walker [18] and a classically controlled humanoid.

4.1 Binary Model Experiment

In the introduction to this paper, it was stated that the measures presented in this work should be applicable to biological systems. In the previous section (see Sec. 3), it was stated that the measures should not operate on the world state $W$, also because it is not accessible for any example other then simple toy worlds. This section now evaluates the two concepts and four measures based on such a simple toy world example. This might appear as a contradiction to the previous statements, and therefore, the application of the measures to this model needs a further explanation.

In order to understand how the measures differ and in which aspects they are alike, it is best to analyse them under a fully controllable setting. Using real robots only allows to fully control a very limited set of parameters of the entire system, namely the parameters of the policy and partially those of the embodiment. This improves only slightly in simulated robotics, depending on the chosen implementation. These uncontrollable or hidden parameters require robustness of the applied methods, and therefore, are the main argument favouring virtual or real robots over grid world environments. The goal of this section is different. Here, we want to validate the measures by controlling the influence of the world on itself in addition to controlling the systems parameters. Therefore, it is necessary to choose an experiment which allows us to fully control every aspect of the sensori-motor loop, explicitly including the world transition kernel $\alpha(w'|a,w)$. The next section will then validate the measures in more realistic experiment.

The minimalistic model discussed in this section is shown in Figure 7 and define by the following set of equations:
Figure 7: Visualisation of the binary model. This figure shows the causal graph that is used in the binary model experiment (see text below). It is the graph representing the single step sensori-motor loop of a reactive system, where the indices of the transition maps refer to their parametrisation.

\[
\alpha_{w',w} = e^{\phi_{w'}w + \psi_{w'}a} \quad (20)
\]

\[
\beta_{\zeta}(s|w) = e^{\zeta_{sw}} \quad (21)
\]

\[
\pi_\mu(a|s) = \sum_{a' \in \Omega} e^{\mu a' s} \quad (22)
\]

\[
p_\tau(w) = \sum_{w' \in \Omega} e^{\tau w'} \quad (23)
\]

where all random variables are from the same binary alphabet, i.e. \(w', w, a, s, s' \in \Omega = \{-1, 1\}\). The model is parametrisable by the variables \(\phi, \psi, \zeta, \mu, \tau \in \mathbb{R}^+\), which control how deterministic the kernels are. If we consider the policy \(\pi_\mu(a|s)\) which is controlled by the parameter \(\mu\), then we see from Equation 22 that \(\mu = 0\) results in \(\pi_\mu(a|s) = 1/2\) for all \(a, s \in \Omega\). The policy is completely random, because both actions \(a = 1\) and \(a = -1\) occur with equal probability, independent of the current sensor value \(s\). On the other hand, if we set \(\mu \gg 0\), then the policy changes to a Dirac measure on the sensor and actuator states, i.e. \(\pi_\mu(a|s) = \delta_{as}\), where \(\delta_{xy} = 1\) if \(x = y\) and zero otherwise. The parameter \(\mu\) allows a linear transition between these two cases.

For simplicity, but without loss of generality, the following two assumptions are made. First, it is assumed that all world states \(w \in \Omega\) occur with equal probability, i.e. \(p(w = 1) = p(w = -1) = 1/2\). Furthermore, we assume a deterministic sensor, i.e. \(\zeta \gg 1 \Rightarrow p(s|w) = \delta_{sw}\), which means that the sensor is a copy of the world state. The first assumption does not violate the generality, because it only assures that the world state itself does not already encode some structure, which is propagated through the sensori-motor loop. Second, in a reactive system, as it is shown in Figure 7, the sensor state \(S\) and \(A\) could be reduced to a common state, with a new generative kernel \(\gamma(a|w) = \pi(a|s) \circ \beta(s|w)\). Hence, keeping one of the two kernels deterministic and varying the other in the experiments below, does not affect the generalisation properties of this model. This leaves three open parameters \(\psi, \phi, \text{ and } \mu\), against which the morphological computation measures are validated (see Figures 9, 10, and 11).

To be able to understand if the plots validate the measures, it is necessary to understand how the parameters affect the morphological computation in this model. To simplify the argumentation, it is first assumed that the policy is completely random (\(\mu = 0\)), and hence, the action \(A\) is independent of the previous world state \(W\) (this assumption is dropped below). What follows is that the world transition kernel \(\alpha(w'|a, w)\) (see Eq. 35) can be divided into four cases (see Fig. 8), which are classified by the effect of \(\phi\) and \(\psi\) on the morphological computation. To understand how the parameters affect the measure, they are first discussed with respect to the world kernel \(\alpha(w'|a, w)\). This is then related to how the Kullback-Leibler divergences used in the two definitions (see Def. 1 and Def. 2) are affected. The analysis is conducted with respect to the two concepts \(MC_A\) and \(MC_W\) will it starts with the two unambiguous cases. The intrinsic measures are discussed later in this section.

1st Case: \(\phi \gg 0\) high, \(\psi \approx 0\). This case refers to a world, which is only influenced by the last world state, as the world kernel \(\alpha(w'|a, w)\) reduces to a Dirac measure on the current and next world state (see
Figure 8: Visualisation of the four discussed cases of the world model. High values are indicated by an arrow pointing upwards (↑), and low values with an arrow pointing downwards (↓), i.e. φ↑, φ↓ refers to the case, where φ ≫ 0 and φ ≈ 0, resulting in a high influence of W → W′ and low influence of A → W′. The figures are discussed in the text below.

Fig. 8): \[ \alpha(w'|w,a) = \delta_{w'w}. \] (24)
The result is a high measured morphological computation for both measures MC_A and MC_W, as \( p(w'|w,a) = p(w'|w) \), and hence, \(-D(p(w'|w,a)||p(w'|w))\) and \(D(p(w'|w,a)||p(w'|a))\) are both maximal.

2nd Case: φ ≈ 0, ψ ≫ 0. This case refers to a world, which is only influenced by the last action, as the world kernel \( \alpha(w'|w,a) \) reduces to a Dirac measure on the action and next world state (see Fig. 8):

\[ \alpha(w'|w,a) = \delta_{w'a}. \] (25)
The result is a low measured morphological computation for both measures MC_A and MC_W, as \( p(w'|w,a) = p(w'|a) \), and hence, \(-D(p(w'|w,a)||p(w'|w))\) and \(D(p(w'|w,a)||p(w'|a))\) are both minimal.

Figure 9: Numerical results for the measures MC_A and MC_W and the binary model. The three figures in the first row shows the results for MC_A, which increasingly deterministic policy \( \pi \) (from left to right). The second row shows the results for MC_W, also with increasingly deterministic policy. For all plots, the base axes are given by world kernel parameters \( \psi \) and \( \phi \). The plots confirm the considerations discussed in the text below.

3rd Case: φ ≈ 0, ψ ≈ 0. This case refers to a world, in which the next world state is independent of the previous world state and the previous action, which is described by

\[ \alpha(w'|w,a) = 1/2. \] (26)
The two measures $MC_A$ and $MC_W$ give different results, because the two Kullback-Leibler $- D(p(w'|w, a)||p(w'|w))$ and $D(p(w'|w, a)||p(w'|a))$ are both zero. Consequently, the two measures lead to $MC_A = 1$ and $MC_W = 0$.

**4th Case:** $\phi \gg 0$, $\psi \gg 0$. This case is a mixture of all previous cases, as the world behaves according to

$$\alpha(w'|w, a) = \begin{cases} \delta w'w & \text{if } w = a \\ 1/2 & \text{otherwise} \end{cases}.$$ (27)

The fourth case is similar to the third case if the world state $W$ is not equivalent to the action $A$ as then the next world state $W'$ is independent of both. We saw from the third case, that this leads to $MC_A = 1$ and $MC_W = 0$. If the action $A$ and the world state $W$ are equal, the first measure reduces the amount of morphological computation, whereas the second measure increases the amount of morphological, leading to $MC_A < 1$ and $MC_W > 0$.

To complete the analysis of the model before the numerical results are discussed, the assumption of a random policy is dropped. We now assume a fully deterministic policy ($\mu \gg 0$). In this case, the policy reduces to a Dirac measure on the world and action state, as the sensor state is a copy of the world ($\beta(s|w) = \delta(sw)$), and therefore, $\pi_\mu(a|s) = \delta_{as} = \delta_{as}\delta_{sw} = \delta_{aw}$. Is also follows that $p(w'|a) = p(w'|w)$. For the two measures, it follows that:

$$MC_A = 1 - \frac{1}{\ln |W|} D(p(w'|w, a)||p(w'|w))$$

$$MC_W = \frac{1}{\ln |W|} D(p(w'|w, a)||p(w'|a))$$

$$= 1 - \frac{1}{\ln |W|} D(p(w'|w)||p(w'|w)) = 0$$

The effect, that a deterministic policy leads to different results for both concepts will occur in the following experiments. It will be discussed in detail in the next section (see Sec. 5).

The four cases are identical with the four corners of the $\phi$--$\psi$ plane in Figure 9, and the four cases are well-reflected in the plots. The plots also visualise the difference of the two concepts for the last two cases ($\phi \approx 5$, $\psi \approx 5$ and $\phi \approx 0$, $\psi \approx 0$ in Fig. 9).

Figure 10: Comparison of the two measure $ASOC_A$ and $ASOC_W$. The results confirm the measures are intrinsic adoptions of the measures $MC_A$ and $MC_W$ (see Figure 9).

Next, this section now applies the four intrinsic measures to the binary model. The results are shown for the variations of the three parameters $\mu, \psi, \phi$ in Figure 10 and Figure 11.
The intrinsic measures require the probability distributions \( p(s) \) and \( p(s'|s,a) \), which can be calculated from the Equations 20 to 23 in the following way:

\[
p(s) = \sum_{w \in \Omega} \beta(s|w)p(w) \tag{31}
\]

\[
p(s'|s,a) = \sum_{w,w' \in \Omega} p(s',w',w|s,a) = \sum_{w',w \in \Omega} \frac{p(s',w',w,s,a)}{p(s,a)} \tag{32}
\]

\[
= \sum_{w',w \in \Omega} \frac{p(s',w',w,s,a)}{p(a|s)p(s)} \tag{33}
\]

\[
= \sum_{w',w \in \Omega} \beta(s'|w')\alpha(w'|w,a)\beta(s|w)p(w)\frac{p(a|s)}{p(s)} \tag{34}
\]

\[
= \sum_{w',w \in \Omega} \beta(s'|w')\alpha(w'|w,a)\beta(s|w)p(w)\frac{1}{\sum_{w'' \in \Omega} \beta(s|w'')p(w'')} \tag{35}
\]

Figure 11: Comparison of the two measure \( C_A \) and \( C_W \). The results confirm the measures are intrinsic adoptions of the measures \( MC_A \) and \( MC_W \) (see Figure 9).

The results show that both measures in the first concept and both measures in the second concept show very similar results compared to the formalisation of the concepts (compare Fig. 9 with Fig. 10 and Fig. 9 with Fig. 11). This can be expected because the sensor distribution in equivalent to the world distribution due to the Equation 31 and because the sensor kernel was set to be a Dirac measure on the world \( (\beta(s|w) = \delta_{sw}) \). Nevertheless, the plots show that the intrinsic adaptations of the formalisation of the concepts capture what the concepts specify.

4.2 Rotator

The previous section verified the measures in a very simple, yet illustrative binary model of the sensori-motor loop. It was shown that all measures produce the desired output for variations on the transition probabilities of the world and the policy. This section applies the intrinsic measures to an experiment that is inspired by common examples of previously discussed systems that show high and low morphological computation.

The Dynamic Walker [18] is designed to emulate the natural walking of humans. One characteristic is that only half of the leg movement involved in the walking behaviour is actively controlled by the brain [19]. It is the stance phase, i.e. the time during which the foot touches the ground and moves the rest of the body
forwards, which is fully controlled. The swing phase, i.e. the time in which the leg swings forward before another stance is initiated, is only partially controlled. One may say that the body lets loose and gravity takes over. This is exactly what the Passive Dynamic Walker highlights, as it only exploits its body and the environment, i.e. the slope and gravity to produce a natural walking behaviour.

The world’s most advanced humanoid robot\textsuperscript{2} Asimo is an example of a system, that does not show any morphological computation as it is discussed in this work. The trajectory of each part of the morphology is carefully controlled during the stance and swing phase at all times. This is also the reason, why the motion of Asimo does not appear natural although it is very smooth.

We are now presenting a simple experiment, that allows us to vary the amount of exposed morphological computation between the two examples discussed above. For this purpose we chose a pendulum, which can rotate freely around its anchor point. The task of our controller is to consistently rotate the pendulum clockwise. The Asimo case is approximately given, if the angular velocity of the pendulum is controlled at every instance in time, whereas the Dynamic Walker case is approximated if the angular velocity of the pendulum is only altered for large deviations of the current angular velocity from the target velocity. This will be more clear, after the equations have been presented. The pendulum is modelled by the following equation:

\[ 0 = ml\ddot{\theta}(t) + \gamma l\dot{\theta}(t) + mg \sin(\theta(t)) - f(t), \]

where \( f(t) \) is the force imposed by the controller (see below), \( \gamma \) is the friction coefficient, \( \theta(t) \) is the current angle of the pendulum, \( m \) is its point mass which is located from the center by the length \( l \). The Equation 36 was numerically solved for \( t = 0.0s \ldots 0.01s \) using NDsolve method of Mathematica 8.0 \textsuperscript{[20]}. The actions \( f(t) \) were modified only for \( t = 0.01, 0.02, \ldots, T \), and kept constant while the Equation 36 was solved. This refers to a behaviour update frequency of 100Hz. The controller is defined by the following set of equations:

\[
\begin{align*}
    s(t) & = \dot{\theta}(t) + u(-\eta, \eta) \\
    g(t) & = \frac{\dot{\Theta} - s(t)}{\eta} - \frac{\text{sign}(s(t)) \beta}{\eta} + \frac{\text{sign}(s(t)) F_{\text{min}}}{\eta} \\
    f(t) & = \begin{cases} 
        g(t) \cdot F_{\text{max}} & \text{if } |\dot{\Theta} - s(t)| \geq \beta \\
        0 & \text{if } |\dot{\Theta} - s(t)| < \beta.
    \end{cases}
\end{align*}
\]

The function of the controller is explained along the three equations Eq. 37 to Eq. 39. The first Equation (see Eq. 37) adds uniformly distributed noise \( u(-\eta, \eta) \) to the sensed angular velocity \( \dot{\theta}(t) \), which is then presented as sensor value \( s(t) \) to the controller. It was discussed in the previous section that a deterministic reactive system prevents the possibility to distinguish between the information flow of \( A \rightarrow W' \) and \( W \rightarrow W' \) as \( A \) becomes a determinism function of \( W \), and hence, \( A \) and \( W \) are equivalent with respect to our measures. Therefore, noise is added to the sensors, such that \( S_t \) is not deterministically dependent on \( W_t \). The Equation 38 determines the strength of the response of the controller as a function of the difference of the sensor value \( s(t) \) and the target angular velocity \( \dot{\Theta} \) (see first term in Eq. 38). This function is only executed if the difference of the sensor value to the goal is larger than a threshold value \( \beta \). Hence, to ensure sensitivity, the threshold value is subtracted from the absolute value of the response (see second term in Eq. 38). The third term in the Equation 38 ensures a minimal response strength, and hence, an minimal effect of the action \( A_t \) on the next sensor values \( S_{t+1} \).

All parameters shown in the Equations 36–39 were evaluated systematically. The most distinctive results were found for the following values, which is why they were chosen for presentation here:

\[
\begin{align*}
    F_{\text{max}} & = 10.0 & \beta & \in [0.0, 2.0] & \dot{\Theta} & = 2\pi & m & = 1.0 \\
    F_{\text{min}} & = 0.25 & \eta & \in [0.0, 0.5] & \gamma & = 0.0 & g & = 9.81 & T & = 5000
\end{align*}
\]

\textsuperscript{2}Cited from http://asimo.honda.com
The second concept (ASOC_A, C_A, ASOC_W, and C_W). The plane in each plot is defined by the noise $\eta$ and the threshold parameter $\beta$. The values are averaged over 10 runs, for $\eta \in \{0, 0.025, 0.05, \ldots, 0.5\}$ and $\beta \in \{0, 0.01, 0.02, \ldots, 2.0\}$. The results are discussed in the text below.

**Results**  The results are presented in two figures (see Fig. 12 and Fig. 13). The Figure 12 shows three-dimensional plots in which the plane is defined by the noise $\eta$ and the threshold $\beta$, and for which the height is given by the measurements of the denoted measure. The Figure 13 shows the transients of the sensor values $s(t)$ (see Eq. 37) and the normalised action $q(t)|_{t=1}$ (see Eq. 38 and Eq. 39) for the four extremal points of the plots in Figure 12. Therefore, the five plots in Figure 13 show the transients for the configurations $(\eta, \beta) \in \{(0,0), (0.0, 2.0), (0.5, 0.0), (0.5, 2.0)\}$. The discussion of the results in the following paragraphs refers to these two figures and follows the ordering of the transient plots in Figure 13. All results were obtained by sampling the world model $p(s'|s, a)$, policy $p(a|s)$ and sensor distribution $p(s)$ from the data stream of 5000 behaviour updates (see Eq. 36). The sampling was performed according to [21] (it is additionally also briefly described in section A.2 of the appendix). The sensor values were binned in the interval $S_t \in [0, 8]$ and the actions were binned in the interval $A_t \in [-1, 1]$ with 30 bins each.

The first discussed configuration is $(\eta, \beta) = (0.2, 0)$. The transients show a clear picture, as no action of the controller is performed other than $f(t) = 0$ after about 2 seconds. Consequently, for this configuration any measure should result in maximal morphological computation leading to a value close or equal to one, as the inertia, and hence, the world $W$ is the only cause for the observed behaviour. We see that both measure of the first concept (ASOC_A and C_A) deliver a value close or equal to 1 (all values in the Figure 13 are rounded to the second decimal place and are averaged over 100 runs). The two measures of the second concept (ASOC_W and C_W) show their maximal values for this configuration at approximately 0.54 (maximal refers to all configurations shown in Figure 12). This again highlights the differences of the two concepts. The first concept reduces the amount of morphological computation by the measured effect of the action $A$ on the next world state $W'$, measured through $S'$, whereas the second concept increases the morphological computation by the measured effect of $W$ on $W'$, captured by $S$ and $S'$. This explains why both concepts show different maximal values for this configuration.

The second discussed configuration is given by $(\eta, \beta) = (0, 0)$. We expect no morphological computation, because a threshold of $\beta = 0$ means that the pendulum is controlled at every time step $t$, and no noise on the sensor values $\eta = 0$ means that the action is only dependent on the actual angular velocity $\dot{\theta}(t)$. We see that in this case, both concepts lead to very different results. The second concept matches our intuition better, as both measures (ASOC_W and C_W) deliver values close to zero. The measures in the first concept result in values close to one, as the action $A$ is deterministically dependent on the world state $W$, captured by $S$, and hence, the effect of $A \rightarrow W'$ is fully determined by the effect of $W \rightarrow A$. Consequently, in the first concept, the current world state $W$ almost fully determines the next world state $W'$, which leads to the high values. We point out, that there is no preference to any one of the two results. Both seem equally valid at the current stage of the discussion. This is strong evidence that both concept capture important
aspects, but that a clear picture cannot be presented yet. The will be discussed in detail in the next section.

\[ h = 0 \]
\[ b = 0 \]
\[ t = 0s 1s 2s 3s 4s 5s \]
\[ H_tL \]
\[ H_tL \]
\[ H_tL \]
\[ H_tL \]

Figure 13: Transients. Please see the legend to each plot and read the text below for a discussion. The values were averaged over 100 runs.

The other three configurations show that a noisy controller output is filtered by the inertia of the pendulum, resulting in an angular velocity approximating the target velocity. The amount of noise \( \eta \) increases the measured morphological computation, as well as the higher threshold value \( \beta \). Both results correspond with the expectation.

5 Discussion

Pfeifer and Scheier [3] state that one problem with the concept of morphological computation is that while intuitively plausible, it has defied serious quantification efforts. From the experiments presented in the previous section we can now understand where this problem rises from. In the context of deterministic world, a determinism embodiment, and a deterministic reactive policy one cannot distinguish between the effect of the world \( W \) and the effect of the action \( A \) on the next world state \( W' \) as the action \( A \) is given by a deterministic function of \( W \). Hence, the two states can be subsumed to a single state, which results in the obtained difference of the measures in the two concepts (see Eq. 30). We will study this with respect to a Gedankenexperiment by Braitenberg [22]. In his book, Braitenberg discusses different vehicles, which show different behaviours due to a very simple sensor motor coupling. The vehicle 3 is of such kind. It has two sensors that can detect light sources and two actuators with which the system can move and steer.

The sensors and motors are cross-coupled (see Fig. 14). Depending on the polarity of the couplings, the

Figure 14: Braitenberg Vehicle 3. The Figure shows two Braitenberg vehicles, each of them is equipped with two light sensors and two actuators. The polarity of the sensor-motor couplings determines the behaviour. The Braitenberg vehicle on the left-hand side is repelled by light, whereas the Braitenberg vehicle on the right-hand side is attracted by light.

ASOC_A: 0.99 C_A: 1.00 ASOC_W: 0.54 C_W: 0.54

ASOC_A: 0.96 C_A: 0.99 ASOC_W: 0.01 C_W: 0.01

ASOC_A: 0.92 C_A: 0.99 ASOC_W: 0.46 C_W: 0.55

ASOC_A: 0.96 C_A: 0.97 ASOC_W: 0.45 C_W: 0.51
vehicle is either attracted or repelled from a light source. For this experiment, the two measures result in \( MC_A = 1 \) and \( MC_W = 0 \). Which one is considered to be correct depends on how the policy is separated from the embodiment. One may argue, that the coupling and the coupling strength defines the policy. In this case, one has to conclude that there is low morphological computation, as the next world state is mostly determined by the policy. This favors the measure \( MC_W \). One may also argue that the couplings between the sensors and actuators are of such simplicity, that they can be considered as part of the embodiment, as the sensors signals directly feed to the actuators. In this case, the state of the system is fully determined by the current sensor state, and hence, the world state. This is in favor of the first measure \( MC_A \).

It seems that both measures capture important aspects of morphological computation, but that a final answer cannot yet be given here, as the discussion about the Braitenberg vehicle 3 shows.

6 Conclusions

This work began with an introduction of morphological computation in the context of embodied artificial intelligence. It was concluded, that the complexity of the controller or brain should not be taken into account for a measure of morphological computation. Instead, the influence of the action of an agent and the influence of the last world state need to be compared. Two concepts to measure morphological computation were then discussed and formalised, from which several intrinsic adaptations were derived. The different measures were evaluated in two experiments, which showed the conceptual difficulty in measuring morphological computation. For any fully deterministic reactive system, the influence of the action and the influence of the world are not easily separated, which leads to contradicting results for the two concepts. As this only occurs in this very special, but common case, we propose to use the contradicting results as an indication that the observed behaviour may be due to such a deterministic reactive system. In this case, noise should be introduced to break the deterministic dependency of the action on the world.

Acknowledgements

The authors want to thank Georg Martius and Ralf Der for the fruitful discussions which contributed to the conclusions of this work. This works was partly funded by the DFG Priority Program 1527, Autonomous Learning.

A Appendix

A.1 Derivation of the Causal Measure 1

The causal information flows used in the Section 3 (see Fig. 4(a) and Fig. 4(b)), are derived by the means of a mutual information measure. The mutual information of two random variables \( X \) and \( Y \) is calculated in the following way [13]:

\[
I(X;Y) = \sum_{x \in X} \sum_{y \in Y} p(x,y) \ln \left( \frac{p(x,y)}{p(x)p(y)} \right) = D(p(x,y)||p(x)p(y))
\]

This is the Kullback-Leibler divergence of the joint distribution of \( X \) and \( Y \) with respect to the marginal distribution of \( X \) and \( Y \) (see Eq. 43). In the case of the causal information flow of \( S \) on \( S' \), the measure compares the joint distribution of \( S' \) and the intervened \( S \), given by \( p(s',do(s)) := p(s'|do(s))p(s) \), with the marginal distribution of \( \hat{p}(s')p(s) \), where \( \hat{p}(s') \) is the post-interventional distribution of \( S' \) (compare Eq. 45).
and 46, below). The resulting measure is then given by:

\[
CIF(S \to S') = \sum_{s',s \in S} p(s', do(s)) \ln \left[ \frac{p(s', do(s))}{\hat{p}(s')p(s)} \right]
\]

(44)

\[
= \sum_{s \in S} p(s) \sum_{s' \in S} p(s'|do(s)) \ln \left[ \frac{p(s'|do(s))p(s)}{\hat{p}(s')p(s)} \right]
\]

(45)

\[
= \sum_{s \in S} p(s) \sum_{s' \in S} p(s'|do(s)) \ln \left[ \frac{p(s'|do(s))}{\sum_{s'' \in S} p(s''|do(s')) p(s'')} \right]
\]

(46)

The causal information flow of \( A \) on \( S' \), denoted by \( CIF(A \to S') \) is derived analogously with \( p(s', do(a)) := p(s'|do(a))p(a) \):

\[
CIF(A \to S') = \sum_{s',a \in A} p(s', do(a)) \ln \left[ \frac{p(s', do(a))}{\hat{p}(s')p(a)} \right]
\]

(47)

\[
= \sum_{a \in A} p(a) \sum_{s' \in S} p(s'|do(a)) \ln \left[ \frac{p(s'|do(a))}{\sum_{a' \in A} p(s'|do(a')) p(a')} \right]
\]

(48)

The Equation (5) implies

\[
CIF(S \to S') = \sum_{s' \in S} p(s'|do(s)) \ln \left[ \frac{p(s'|do(s))}{\sum_{s'' \in S} p(s'')p(s'|do(s'))} \right]
\]

(49)

\[
= \sum_{s' \in S} \left( \sum_{a \in A} p(a|s)p(s'|do(s)) \right) \ln \left[ \frac{\sum_{a \in A} p(a|s)p(s'|do(a))}{\sum_{a \in A} p(a|s)p(s'|do(s))} \right]
\]

(50)

\[
= \sum_{s' \in S} \left( \sum_{a \in A} p(a|s)p(s'|do(a)) \right) \ln \left[ \frac{\sum_{a \in A} p(a|s)p(s'|do(a))}{\sum_{a \in A} p(a|s)p(s'|do(a))} \right]
\]

(51)

\[
\leq \sum_{a \in A} p(a|s) \sum_{s' \in S} p(s'|do(a)) \ln \left[ \frac{p(s'|do(a))}{\sum_{a \in A} p(a|s)p(s'|do(a))} \right]
\]

(52)

\[
\Rightarrow CIF(S \to S') \leq \sum_{a \in A} p(a|s)CIF(A \to S').
\]

(53)

Summation with respect to \( s \) finally yields

\[
CIF(S \to S') = \sum_{s \in S} p(s)CIF(s \to S')
\]

(54)

\[
\leq \sum_{s \in S} p(s) \sum_{a \in A} p(a|s)CIF(a \to S')
\]

(55)

\[
= \sum_{a \in A} p(a)CIF(a \to S')
\]

(56)

\[
\Rightarrow CIF(S \to S') \leq CIF(A \to S')
\]

(57)

It follows, that the measure for morphological computation can be represented as KL-divergence in the following form:

\[
CIF(S \to S') - CIF(A \to S') = - \sum_{s \in S, a \in A} p(s, a) \sum_{s' \in S} p(s'|do(a)) \ln \left[ \frac{p(s'|do(a))}{p(s'|do(s))} \right]
\]

(58)

\[
= -D(p(s'|do(a))||p(s'|do(s)))
\]

(59)

Any Kullback-Leibler divergence is non-negative [13], i.e. \( D(p||q) \geq 0 \), which means that the measure given above (see Eq. 58) is always negative.
The derivation of the morphological computation measure for non-reactive control (see Fig. 1(b)) is analogous to the reactive control [17]. Every occurrence of $s, S, S'$ is replaced by $c, C, C'$, which leads to

$$CIF(C \rightarrow S') = \sum_{c \in C} p(c) \sum_{s' \in S} p(s' \mid do(c)) \ln \left( \frac{p(s' \mid do(c))}{\sum_{c' \in C} p(s' \mid do(c')) p(c')} \right)$$

(60)

A.2 Sampling world model, policy and input distribution in pendulum experiments

The probability distributions that were required in the rotator experiment (see Sec. 4.2) were obtained from the data series by the same sampling, that we have used in a previous publication [21]. It is briefly discussed for the world model below. Its application to the policy and input distribution is straight forward [17]. The sampling starts with an equal distribution and the updates the rows for which a sample was detected in the following way:

$$p^{(0)}(s' \mid s, a) := \frac{1}{|S|}$$

$$p^{(n)}(s' \mid s, a) := \begin{cases} 
\frac{n^a_s}{n^a+1} p^{(n-1)}(s' \mid s, a) + \frac{1}{n^a+1} & \text{if } S_{n+1} = s', S_n = s, A_{n+1} = a \\
\frac{n^a_s}{n^a+1} p^{(n-1)}(s' \mid s, a) & \text{if } S_{n+1} \neq s', S_n = s, A_{n+1} = a \\
p^{(n-1)}(s' \mid s, a) & \text{if } S_{n+1} \neq s \text{ or } A_{n+1} \neq a
\end{cases}$$

(61)

References