# Inefficient Delays in Strategic Trades

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**Abstract**

Inefficient delays in trades can sometimes be observed after the arrival of important public news. This paper explains these phenomena with a model in which agents defer trades in the fear that they may be taken advantaged of by better informed trading partners. Under certain conditions, delay is inevitable, yet total collapse of trade can be avoided.

## 1 Introduction

The main goal of this paper is to show that the sheer fear of being taken advantage of by better informed trading partners may result in inefficient delays in trades. We observe a variety of delays in socially desirable trades mainly due to this fear. For example, after the Pound Sterling broke away from the European Exchange Rate Mechanism on September 16, 1992, most foreign exchange markets involving Pound Sterling halted for a while. When compared with the usual tick-by-tick high frequency foreign exchange markets, the halt could be regarded as an astonishing delay.[1] Another example is the temporary disappearance of consumer goods markets at the brink of the

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[1]The following is an excerpt from the *Wall Street Journal*:

> "At 11:15 a.m., a grave Mr. Farrell stands up and tells the other traders: 'The central banks are gone here' — meaning the world's central banks appear to have temporarily abandoned their efforts to support the wobbling British pound.
>
> A few moments later, Joseph Greene, a sandy-haired, bespectacled 'sterling-mark' trader, stands and announces that a customer wants to sell $100 million and buy German marks. 'There are no prices!' shouts Arnold Neimanis, who trades marks and is on the phone constantly to other brokers who are making price quotes.

first hyperinflation in Argentina in 1989. Store owners deferred trades by putting up signs which said, "Closed for lack of prices." (Heymann and Leijonhufvud (1995)) According to Haymann and Leijonhufvud (1995), a plausible explanation of why store owners declined trading was the fear of being taken advantage of by better informed customers.

This paper demonstrates how in certain circumstances the fear of being taken advantage of makes inefficient delays inevitable, whereas total collapse of trades (or the "missing-markets" results) can be avoided. The model in this paper shares the same spirit of Milgrom and Stokey's (1982) No Trade Theorem, but not their "no trade" result.

There are other studies on strategic interactions that generate inefficient delays. Jehiel and Moldovanu (1995) show how deadline effect generates inefficient delays in trades. Because the seller has to sell the good before the deadline, the deadline in effect serves as an device to make the seller's threat of selling the good to other buyers credible. Seller hence defers trades until some moments close enough to the deadline. Chamley and Gale (1994) show how informational externality generates inefficient delays in investments. Due to the lack of communication among investors,[2] useful private information about the payoff of the project cannot be pooled together. Investors hence play a mixed strategy of either wait and see or invest right away.

Delays have also been studied in the literature of investment under uncertainty, mostly in the cases of isolated learning. "Delays" may arise from specific features of the underlying uncertainties,[3] or from the irreversible nature of the investments (see, for example, Henry (1974), McDonald and Siegel (1986) and Dixit (1989)). Not surprisingly, "delays" are optimal in all these cases.

The paper is organized as follows. Section 2 will introduce the model. Section 3 presents the main results of this paper, and section 4 contains some discussions.

## 2 The Model

There is one buyer and one seller in the model. Seller can produce an indivisible good at cost $C$. The good will be of value $U$ to the buyer. The gain of trade is

$$U - C \equiv G > 0.$$

Instead of asking Chase to make a bid and take the risk of holding pounds with the market so disorderly, the customer asks Chase to execute the order bit by bit at prevailing market prices. Over the next few minutes, Chase executes part of the trade, \$5 million to \$10 million at a time; but with the pound plummeting, the customer decides to stop temporarily at \$50 million before completing the trade later in the day." (*Wall Street Journal*, September 17, 1992, **C**1)

[2]Gale (1996) provides some justifications for this lack of communication.

[3]For example, Zeira (1987) demonstrates how structural uncertainty results in investment behavior similar to that under convex adjustment costs.

A piece of public information comes at the beginning of the game. The public news is something like "The Central Banks are gone!" or "Hyperinflation!" It essentially tells you that "You are in trouble!", "Everything is turned upside down!", "Chaos!", "No one knows what will happen tomorrow!"..., etc.

The "chaos" in this model is that the absolute values of $(C, U)$ can now be either $(C_H, U_H)$ or $(C_L, U_L)$, each with half chance; where $C_H > C_L$ and $U_H > U_L$. In either case, the gain of trade is still $G$, so the "chaos" is not chaotic enough to kill off the benefit of trade. Trade is still socially desirable, and delay in trade socially costly.

An ordinary seller in this model doesn't know the realized $C$ until she has concluded the trade contract with the buyer and really goes to produce the good. Symmetrically, an ordinary buyer doesn't know the realized $U$ until he really consumes the produced good. I assume the realized $C$ and $U$ are both non-verifiable to a court and hence the trade contract cannot be written in terms of them. This assumption is arguably realistic.

The key assumption in this model is that some people may be smart enough to see through the "chaos." Consider the "chaotic" situation that the CPI tomorrow will be a function of the 123456th decimal point of $\pi$. Yet some crasy professor in Cornell may use the supercomputer over there to calculate this decimal point. Is the supercomputer fast enough to finish this job before dawn? I bet most people have absolutely no clue at all. Anyway, the spirit is that you allow for some probability that your trading partner is indeed a Cornell professor accessible to a *super*-supercomputer and can calculate whether the true state is $H$ or $L$. Some readers may want to interpret this assumption as that with some probability your opponent is an "informer" — I don't mind, and I shall actually call these Cornell professors "informers" in the sequel. But I like my interpretation much more. In the end, who is an "informer" in the Sterling-Mark market?

I assume that the probabilities that the seller and the buyer are Cornell professors are independent, and both equal to $\alpha$.

The trading game is as follows: In period one, both agents quote a price. If the buyer's bidding price is higher than the seller's asking price, trade is concluded and the trading price will be the mid-point of the bid-ask spread; otherwise no trade occurs and the agents quote prices again in the next period..., and so on. Delay is costly, as both agents have the same discount factor $\delta$ which is smaller than unity.

## 3 The Main Results

I would like to make three remarks before we go into the main results of this paper.

First, let's make clear what we are going to prove in this section. We are *not* going to prove something like "under some circumstances the equilibrium of the game has no delay whereas under others it has." Actually this cannot possibly be true. Any game which has an equilibrium without delay will also has many "silly" equilibria in which both players defer making reasonable quotes until the $n$th period and then play the no-delay equilibrium strategies. So our question is *not* whether some game has a no-delay equilibrium, but whether some game doesn't have any no-delay equilibrium (and hence delay is inevitable).

The second remark is about the distinction between "delay" and "collapse." In Chamley and Gale (1994), collapse is just a special case of delay — one of infinite delay. In the model in this paper, collapse is an uninteresting phenomenon. As long as we assume $C_H > U_L$ and the agents are risk adverse enough, no trade will ever occur. But in that situation, finite delay is also impossible. So in order to make my point, I not only need to prove that the class of games I am considering have no no-delay equilibrium, but also that finite delay is possible and hence collapse can be avoided.

The third remark is about a trick we shall employ either when we are proving that certain strategy profile can be supported as an equilibrium, or when we are proving that certain class of strategy profiles cannot be supported as equilibria. In both of these cases, it is handy to employ the "harshest possible out-of-equilibrium penalties," i.e., whenever there can be multiple equilibria following any out-of-equilibrium history-of-play, I shall pick the one which gives the deviator the lowest payoff. This is the easiest way to support any particular strategy profile as an equilibrium, provided that it can really be supported as an equilibrium.

To be concrete, if the buyer takes some action out of the support of the equilibrium, an uninfomred seller or an informed $H$-seller will believe that he is an informed $H$-buyer and ask up to $U_H$ hereafter, whereas an informed $L$-seller will believe that he is an informed $L$-buyer and ask up to $U_L$ hereafter. The symmetric cases hold for a deviating seller.

Now, let's start with the definition of *no-delay* equilibrium.

**Definition 1** *An equilibrium is said to have the no-delay property if trade occurs in period one with probability one.*

The following theorems make clear why we shall restrict our attention to cases where $0 < \alpha < 1$ and $K \equiv C_H - U_L > 0$.

**Theorem 1** *If $\alpha = 0$ or 1, there exists a no-delay equilibrium.*

PROOF When $\alpha = 0$, both players quoting $\frac{C_H+U_L}{2}$ every period is a perfect Bayesian equilibrium. When $\alpha = 1$, both players are informed, and hence the game reduces to one with certain $C$ and $U$. Both players quoting, for example, $\frac{C+U}{2}$ every period is a perfect Bayesian equilibrium. In both cases, trade occurs in the first period with probability one. *Q.E.D.*

**Theorem 2** *If $K \equiv C_H - U_L \leq 0$, there exists a no-delay equilibrium.*

PROOF I shall show that it is a perfect Bayesian equilibrium for everyone to quote the price

$$B_H = B_L = B_U = S_H = S_L = S_U = \frac{C_U + U_L}{2} \tag{1}$$

in every period. Since

$$C_L < C_H \leq \frac{C_H + U_L}{2} \leq U_L < U_H,$$

(1) satisfies the individual rationality constraints for all agents of all types. Given the seller of any type is playing strategy (1), a buyer bidding higher will have lower payoff, and bidding lower will cause delay which is costly to him as $\delta < 1$. For the same reason, the seller also will not deviate for one single stage. To complete the proof, it suffices to specify the out-of-equilibrium beliefs such that at any out-of-equilibrium node everyone's belief on the opponent's type is the same as that at the beginning of the game. *Q.E.D.*

Hereafter we shall restrict our attention to cases where $0 < \alpha < 1$ and $K > 0$.

The following theorem says that when $0 < \alpha < 1$, whether the game has a no-delay equilibrium depends on how "different" are the two possible states. Note that since the agents are risk neutral, large difference between the two possible states kills the no-delay equilibrium *not* because the environment is "too risky."

**Theorem 3** *For any fixed $0 < \alpha < 1$, there exists a threshold $\bar{K}$, which is uniformly bounded from above by $2G$, such that no-delay equilibria exist if and only if*

$$K \equiv C_H - U_L \leq \bar{K}.$$

I shall first prove that there is no no-delay equilibrium with mixed first-period behavioral strategies.

**Lemma 1** *There is no no-delay equilibrium in which players play strictly mixed price-quoting strategies in period one.*

PROOF Let $B_H$, $B_L$, $B_U$, $S_H$, $S_L$ and $S_U$ be arbitrary elements in the supports[4] of the first-period price-quoting strategies of an infomred $H$-buyer, an informed $L$-buyer, an uninformed buyer, an informed $H$-seller, and informed $L$-seller, and an uninformed seller, respectively.

If trade is going to occur in the first period with probability one (together with the individual rationality constraints that $S_H \geq C_H > U_L \geq B_L$), we must have

$$B_H, B_U \geq S_H \geq C_H > U_L \geq B_L \geq S_U, S_L.$$

Take infimums of the first-period bidding prices and supremums of the first-period asking prices

[4] I shall use the definition that the support of a probability distribution is the smallest closed set with probability one.

over their respective supports, we have

$$\inf B_H, \inf B_U \geq \sup S_H \geq C_H > U_L \geq \inf B_L \geq \sup S_U, \sup S_L.$$

By lowering the first-period bid to $\sup S_H$, an informed $H$-buyer and an uninformed buyer can pay less while maintaining the same probability of trade; hence the first-period price-quoting strategies for them must both collapse at $\sup S_H$. By similar reasoning, the behavioral strategies of both players of all types collapse as

$$B_H = B_U = S_H \equiv h \geq C_H > U_L \geq l \equiv B_L = S_U = S_L, \tag{2}$$

which is a pure strategy profile. *Q.E.D.*

Note that the above proof also proves that in any no-delay equilibrium, the first-period behavior strategy profile is in the form of (2). We shall now prove Theorem 3.

PROOF OF THEOREM 3 For any $\alpha > 0$, we shall set $\bar{K} = \frac{2\alpha G}{2-\alpha}$. It is clear that $\bar{K}$ is uniformly bounded from above by $2G$.

When $K \equiv C_H - U_L \leq \bar{K}$, I shall show that

$$h = C_H > U_L = l \tag{3}$$

can be supported as an equilibrium, where $h$ and $l$ are defined in (2).

We first check that (3) satisfies all individual rationality constraints. The individual rationality constraints for informers are clearly satisfied. The individual rationality constraint for an uninformed buyer is that his expected payoff must be non-negative, which will be

$$\frac{1}{2}\alpha\frac{h+h}{2} + \frac{1}{2}\alpha\frac{h+l}{2} + (1-\alpha)\frac{h+l}{2} \leq \frac{U_H + U_L}{2}, \tag{4}$$

conditioned on the no-delay property. The three terms on the left are the expected payments to an informed $H$-seller, an informed $L$-seller and an uninformed seller, respectively. Plug in (3) and simplify, we shall find that (4) is satisfied when $K \leq \frac{2G}{\alpha}$. But this is the case, because

$$K \leq \bar{K} = \frac{2\alpha G}{2-\alpha} < \frac{2G}{\alpha}$$

when $\alpha < 1$. Since both the model and the proposed equilibrium is symmetric, the individual rationality constraint for an uninformed seller is also satisfied.

It remains to check the incentive compatibility constraints. By employing the harshest possible out-of-equilibrium penalties" (see the third remark at the beginning of this section), we guarantee that the only possibly profitable deviation is to bid $l$ when one should bid $h$, or to ask $h$ when one should ask $l$. It can be shown that the incentive compatibility constraints for the informed players are redundent, and it suffices for us to check those for the uninformed players.

For an uninformed buyer, lowering the bid to $l$ may entail in no trade in the first period. This occurs only when the seller is indeed informed and the state is $H$, in which case the seller will know that the buyer is cheating. Following the spirit of harshest possible out-of-equilibrium penalty, we can specify the continuation equilibrium to be such that both the informed $H$-seller and the cheating uninformed buyer quote the price $U_H$ from the second period onward. This specification of the continuation equilibrium minimizes the payoff of deviation to

$$\frac{1}{2}\alpha(U_L - l) + (1-\alpha)(\frac{U_H + U_L}{2} - l).$$

This term cannot be bigger than the payoff from not deviating, which equals to the right side of (4) minus the left side. Plug in (3) and simplify terms, we shall find that this incentive compatibility constraint is satisfied when $K \leq \frac{2\alpha G}{2-\alpha}$, which is the same condition that $K \leq \bar{K}$. By symmetry, the incentive compatibility constraint for an uninformed seller is also satisfied.

To see that there is no no-delay equilibrium when $K > \bar{K}$, we can add the incentive compatibility constraints

$$\begin{aligned}
\frac{1}{2}\alpha(U_L - l) + (1-\alpha)(\frac{U_H + U_L}{2} - l) &\leq \frac{U_H + U_L}{2} - \frac{1}{2}\alpha\frac{h+h}{2} - \frac{1}{2}\alpha\frac{h+l}{2} - (1-\alpha)\frac{h+l}{2} \\
\frac{1}{2}\alpha(h - C_H) + (1-\alpha)(h - \frac{C_H + C_L}{2}) &\leq \frac{1}{2}\alpha\frac{l+l}{2} + \frac{1}{2}\alpha\frac{h+l}{2} + (1-\alpha)\frac{h+l}{2} - \frac{C_H + C_L}{2}
\end{aligned}$$

of an uninformed buyer and an uninformed seller, respectively, together and simplify. Again, we obtain the inequality

$$K \leq \frac{2\alpha G}{2-\alpha},$$

which is now violated. Note that the incentive compatibility constraints we are considering are already the most "favorable" ones for any $(h, l)$ pair, thanks to the harshest possible out-of-equilibrium penalties. Hence any strategy profile with (2) as the first-period behavioral strategy profile cannot be supported as an equilibrium. But, as I remarked immediately after the proof of Lemma 1, this in effect also rules out any no-delay equilibrium. *Q.E.D.*

As I argued at the beginning of this section, merely proving that there does not exist any no-delay equilibrium is not enough, for it is possible that the only equilibrium is the total collapse of trade, in which case this model is nothing more than another model of missing markets. However, the following theorem asserts that no matter how different are the two possible states, there always exits an equilibrium with finite delay.

**Theorem 4** *For any $K > \bar{K}$, there exists an equilibrium such that there exists an $N < \infty$ such that trade occurs before period $N$ with probability one.*

PROOF For fixed $\alpha$, $\delta$ and $K$, pick $\beta \in (0, G)$ such that

$$\beta > \frac{G}{2}, \tag{5}$$

and pick $m, n \in \{1, 2, 3, \cdots\}$ such that

$$\delta^n(\frac{G+K}{2}+\beta) \quad < \quad \frac{G}{2}, \text{ and} \tag{6}$$

$$\max\{\delta^m(G+\frac{K}{2}), \delta^{m+n}(G+K+\beta)\} \quad < \quad G-\beta. \tag{7}$$

I shall show that it is a perfect Bayesian equilibrium for:

- informed $H$-buyer to bid $H$ in the first period, and then $C_H$ in all subsequent periods;
- informed $L$-seller to ask $L$ in the first period, and then $U_L$ in all subsequent periods;
- informed $L$-buyer to bid $L$ in the first period, $C_L$ from the 2nd to the $(m+n)$th periods, and then $L$ again in all subsequent periods;
- informed $H$-seller to ask $H$ in the first period, $U_H$ from the 2nd to the $(m+n)$th periods, and then $H$ again in all subsequent periods;
- uninformed buyer to bid $L$ in the first period, $C_L$ from the 2nd to the $m$th periods, $\frac{C_H+U_L}{2}$ from the $(m+1)$st to the $(m+n)$th periods, and then $H$ in all subsequent periods;
- uninformed seller to ask $H$ in the first period, $U_H$ from the 2nd to the $m$th periods, $\frac{C_H+U_L}{2}$ from the $(m+1)$st to the $(m+n)$th periods, and then $L$ in all subsequent periods;

where $H \equiv C_H + \beta$ and $L \equiv U_L - \beta$. Apparently, if this is really an equilibrium, it has finite delay.

Since the proposed strategy profile is symmetric, it suffices to show that a seller of any type has no incentive to deviate unilaterally. We first look at the individual rationality constraints. The only place where the individual rationality constraints may be violated is when an uninformed seller asks for $L \equiv U_L - \beta$ from the $(m+n+1)$st period onward. But if the buyer of any type does not deviate, the only chance that trade still has not concluded is when the buyer is informed and the state is indeed $L$. So asking for $L$ is still profitable for the seller.

It is apparent that an informed $H$-seller has no incentive to deviate in the first $m$ periods. She will not deviate in the $(m+1)$st period and mimic an uninformed seller because $\frac{C_H+U_L}{2} - C_H < 0$. It is also apparent that she has no incentive to deviate from the $(m+2)$nd period onward.

For an informed $L$-seller, she knows that her trading partner is either an informed $L$-buyer, in which case she has no chance of taking advantage of him, or an uninformed buyer, in which case the only two ways to take advantage of him are to mimic either an uninformed seller or an informed $H$-seller and trade in the $(m+1)$st and $(m+n+1)$st periods, respectively. So a very generous upper bound for the payoff of deviation is

$$\begin{aligned} & \max\{\delta^m(\frac{C_H+U_L}{2}-C_L), \delta^{m+n}(H-C_L)\} \\ = \ & \max\{\delta^m(G+\frac{K}{2}), \delta^{m+n}(G+K+\beta)\}, \end{aligned}$$

which by (7) is smaller than the payoff of not deviating, i.e., $L - C_L = G - \beta$.

An uninformed seller can deviate in the first period and asks for $L$. Trade will then occur immediately, but the expected payoff to the seller is only

$$\begin{aligned}
& \frac{1}{2}\alpha(\frac{H+L}{2} - C_H) + \frac{1}{2}\alpha(\frac{L+L}{2} - C_L) + (1-\alpha)(\frac{L+L}{2} - \frac{C_H + C_L}{2}) \\
= & -\frac{1}{2}\alpha\frac{K}{2} + \frac{1}{2}\alpha(G-\beta) + (1-\alpha)(\frac{G}{2} - \beta - \frac{K}{2}) \\
= & \frac{G}{2} - \frac{2-\alpha}{2}\beta - \frac{2-\alpha}{4}K \\
< & \frac{G}{2} - \frac{2-\alpha}{2}\frac{G}{2} - \frac{2-\alpha}{4}K \qquad \text{by (5)} \\
\leq & \frac{G}{2} - \frac{2-\alpha}{2}\frac{G}{2} - \frac{2-\alpha}{4}\frac{2\alpha}{2-\alpha}G \qquad \text{by } K \geq \bar{K} \\
= & -\frac{\alpha G}{4} \\
< & 0.
\end{aligned}$$

It is also apparent that the uninformed seller has no incentive to deviate from the 2nd to the $m$th periods. She also will not deviate in the $(m+1)$st period, because deviating by that time can at most bring her

$$\begin{aligned}
& \delta^n(H - \frac{C_H + C_L}{2}) \\
= & \delta^n(\frac{G+K}{2} + \beta) \\
< & \frac{G}{2} \qquad \text{by (6)},
\end{aligned}$$

where $\frac{G}{2}$ is the expected payoff she can immediately get by following the proposed strategy. *Q.E.D.*

# 4 Discussion

## 4.1 "No Price!"

I would like to draw the reader's attention to the description of the proposed finite delay equilibrium once again. When a seller (buyer) asks (bids) for a price as high (low) as $U_H$ ($C_L$), in effect she (he) is telling the buyer (seller) to "Get out of here!" This maps naturally to "There are no prices!" in the Sterling-Mark market and "Closed for lack of prices!" in Argentina. The insight of this paper is that "No price!" actually serves as a mechanism which keeps the (impatient) informed players honest.

### 4.2 Period Length

Also note that $\delta$ plays virtually no role in the proposed finite delay equilibrium. When $\delta$ is closer to one, we simply pick larger $m$ and $n$ to satisfy (6) and (7). The magnitude of $\delta$ can be interpreted as the speed with which players can react to the moves made by their trading partners. Given a fixed common time preference for the players, a higher speed corresponds to a larger $\delta$. But then the corresponding larger $m$ and $n$ required by (6) and (7) imply more or less the same time delay in equilibrium. This independency of reaction speed is not shared by Chamley and Gale's (1994) model. In their model, whether there will be delay or collapse depends crucially on the reaction speed.

### 4.3 Equilibrium Concept

Throughout the whole paper I have been using the perfect Bayesian equilibrium concept, which places no restriction at all to the out-of-equilibrium beliefs. This is innocuous for Theorem 3, as our main goal is to show that for a class of models delay is inevitable, and more stringent restrictions on the out-of-equilibrium beliefs can only make it easier to establish this point. However, in showing that "collapse" is avoidable, the perfect Bayesian equilibrium concept makes it easier to demonstrate the existence of a finite delay equilibrium. So far I haven't investigated whether "collapse" is still avoidable if we place restrictions on the out-of-equilibrium beliefs.

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