Alternate Stable States in a Social-Ecological System

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Alternate stable states in a social-ecological system†

J. Stephen Lansing *1,2,3, Siew Ann Cheong 4, Lock Yue Chew4, Murray P. Cox5, Moon-Ho Ringo Ho6, Wayan Alit Arthawiguna7

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Abstract

Ecosystems may undergo nonlinear responses to stresses or perturbations. Hence there can be more than one stable state or regime. It is not known whether alternate regimes also occur in coupled social-ecological systems, in which there is the potential for intricate feedbacks between natural and social processes. To find out, we investigated the management of rice paddies by Balinese farmers, where ecological processes impose constraints on the timing and spatial scale of collective action. We investigated responses to environmental and social conditions by eight traditional community irrigation systems (subak) along a river in Bali, to test the intuition that older and more demographically stable subaks function differently than those with less stable populations. Results confirm the existence of two attractors, with sharply contrasting patterns of social and ecological interactions. The transition pathway between the two basins of attraction is dominated by differences in the efficacy of sanctions and

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the ability of subaks to mobilize agricultural labor.

1 Introduction

Most research on cooperation, public goods and collective action problems is organized around ideas of stability. Both micro-economics and evolutionary game theory are equilibrium theories, which examine the properties of various fixed points and analyze the conditions under which they are selected. In contrast, studies of ecosystem dynamics, especially long-term ecological research, suggest that ongoing change and variability are typical [1][2][3][5][4][6]. Nonlinear transitions between alternate stable states are characteristic of lakes and rangelands, and there is growing evidence that these occur at all scales up to the planetary [7].

Consequently, combining linearized equilibrium models of cooperation or collective action with nonlinear dynamical models of ecosystems, in what are called coupled social-ecological systems (SES), creates a mismatch. Equilibrium theories assume that any given combination of state variables will produce a unique result. But in a coupled SES, in which both social and ecological components may interact nonlinearly, it is possible that alternate steady states exist [8][9]. If we are only looking for a single equilibrium, evidence of alternate steady states will be mistaken for noise.

This point was brought home by the discovery of alternate stable states in Dutch lakes. For decades, excess fertilizer flowed into the lakes, triggering algae blooms and eutrophication. But simply reducing the amount of fertilizer entering the lakes was not enough to restore them to clarity. It turned out that alternate stable states existed, one turbid and the other clear. In ecology, such alternate stable states are known as “regimes.” The effects of nutrient flows depended on which regime a lake was in, so generalizing across all lakes obscured these differences. But once the existence of alternate regimes was recognized, a simple intervention was sufficient to restore the lakes to health. Temporarily removing the fish allowed sediment to settle and zooplankton populations to increase, whereupon water clarity could be improved by reducing the amount of fertilizer flowing into the lakes [10].

But what of more tightly coupled SES, where there is the potential for intricate feedbacks between natural and social processes? Rice paddies, for example, are shallow artificial lakes that are brought into existence by the collective action of groups of farmers. While the farmers are nominally in control, ecological processes impose constraints on the timing and spatial scale of collective action required to sustain the rice crop [11]. Paddies must be flooded and drained to deliver nutrients and promote plant growth, while also control-
ling weeds and rice pests. Synchronized harvests can reduce pest populations by removing their habitat, but for this to work, the extent of the fallow period must be large enough to prevent the pests from migrating to fields that are still in cultivation (SI Appendix 1.1). Historians have long argued that under such conditions, landscapes and institutions will co-evolve. Is this a simple linear process? Or do nonlinear transitions occur in what one historian calls “the reciprocal influences of a changing nature and a changing society”[12]? 

To find out, we investigated the interaction of social and ecological processes in an ancient and extensively studied system of wet-rice irrigation, the subaks of Bali [11][13]. Subaks are traditional, community-scale institutions that manage irrigation water and planting schedules. Balinese farmers regard water as the gift of a Goddess, and subaks are entrusted with the fair and equitable use of her gift for the purpose of growing paddy rice. Subaks have existed at least since the eleventh century [14]; today there are approximately 1000 [15]. From one year to the next, they must adjust to changing environmental and social conditions. This requires ongoing collective action, which typically includes a heavy burden of ritual obligations as well as agricultural labor. Too little investment could cause crop losses, angry neighbors or indeed the wrath of the gods, but too much might incur the wrath of one’s family. Ethnographic observations suggest that subaks vary in their ability to increase or decrease these investments, as conditions require [11]. When this capacity declines, the subak becomes vulnerable. But failures in cooperation may be temporary; crop losses may prompt a return to high investments in the subak. Thus while low investment could mean that a subak is close to collapse, it could also mean that the farmers are enjoying a period of low stress, which may or may not be temporary. This study was designed to test the intuition that as a result of their different histories of local adaptation, the older and more demographically stable subaks respond more effectively to both environmental and social challenges than subaks with less shared history.

Subaks are particularly well suited to the analysis of coupled social-ecological interactions for three reasons. First, they are functional institutions: the economic welfare of traditional Balinese rice-growing villages is largely dependent on their efficacy. Second, the time lag between failures in cooperation within the subak and perceptible consequences is short due to the small scale of subak irrigation, the fragility of terraced fields and the potential for crop losses from pests or water shortages. Hence there is a strong potential for social learning and adaptation. Third, there is large variation in relevant variables such as the age of subaks, their demographic composition and local environmental conditions: for centuries, subaks have evolved independently in neighboring catchments. Based on 30 years of ethnographic observations [13], we predicted that independent of environmental conditions, pro-social behavior would be more
common in subaks where nearly all families share a long history of managing their rice terraces, bringing with it a heightened awareness of the consequences of failure.

We tested this hypothesis in a comparative study of eight subaks located along the Sungi river in the district of Tabanan, the largest rice-growing region in Bali (Fig. 1). In the rainy season, rivers flowing down steep Balinese volcanos are hard to control with traditional engineering techniques; consequently the older subaks tend to be located upstream, where the smaller flows are easier to manage. One of the earliest dated royal inscriptions is kept in a temple near the headwaters of the Sungi, and contains references to wet-rice agriculture and nearby streams (Babahan 1, 947 A.D. in [14]). Newer and larger subaks and irrigation systems are located along the lower stretch of the river; they cannot be dated as precisely but were probably constructed beginning in the eighteenth century [16].

2 Methods

Analysis of this model system began as a follow-up to a prior study that used neutral genetic markers to investigate the demographic histories of 21 subaks in different regions of Bali [17] [18]. The data collected for the eight Sungi river subaks were re-analyzed to discover whether there are significant differences between the demographic histories of the upstream and downstream subaks (SI Appendix 1.2). The methods used for the genetic analysis are fully described in [18]; here we summarize the key points (SI Appendix 1.3). Genetic samples were obtained from farmers in the eight subaks along the Sungi river (N=120) as well as thirteen subaks located elsewhere in Bali (N=287), and a geographically distributed sample of 180 other Balinese men to provide context for the genetic patterns observed in the subaks. Results of this prior study showed that older rice-growing isolated villages typically have less genetic diversity than elsewhere on the island, reflecting the cultural preference for endogamous marriages within these villages, along with very low rates of in-migration. Newer villages, or those with more in-migration, show greater diversity.

To discover whether upstream subaks vary systematically from downstream subaks in their responses to social and environmental challenges, two methods were used: a survey of farmers in each subak (N=83), and an experimental game (N=43). For the survey, in each of the eight subaks ten or more randomly chosen farmers were invited to answer a questionnaire administered by
Figure 1: Map of Bali showing the eight subaks in this study, on the Sungi river. All four of the upstream subaks and two of the downstream subaks are small (mean size $67 \pm 16$), the other two downstream subaks are much larger (231 and 535 members). Left histogram: mean Y-STR genetic diversity: 4 lower subaks 0.980, 4 upper subaks 0.955; mean mtDNA diversity: lower subaks 0.945, upper subaks 0.923, $P = 0.031^*$. Right histogram: responses of 10 farmers in each subak to “What is the overall condition (state) of your subak?” Mean lower subaks 2.150, upper subaks: 2.775, $P<4.993 \times 10^{-10}^{***}$
one of us who speaks Balinese (Lansing), assisted by local agricultural extension agents assigned to each subak. Farmers who participated were paid the equivalent of a laborer’s day wage (30,000 Indonesian rupiah). Survey questions were divided into two sections. The first section asked farmers about their harvest yields for the past two years and whether they experienced losses due to water shortages or pest infestations. They were also asked how much farmland they owned or sharecropped over the same time period. The second section asked for the farmer’s opinions about factors that affect the subak’s ability to respond to both social and environmental problems. These included the effectiveness of sanctions against norm violators; the ability of the subak to mobilize collective labor for maintenance of the irrigation works and the performance of temple rituals; the general condition or state of their subak and its capacity to cope with either technical or social problems. The farmers were also asked about the effects of differences in caste and class on the functioning of their subak. Because rivalry between castes is endemic in Bali, the proportion of high versus low caste members in each subak could affect its ability to respond to problems. Similarly, farmers were also asked to assess the effects of class differences (indexed by the proportion of sharecroppers versus landowners in the subak) on the efficacy of their subak (SI Appendix 1.5).

To facilitate comparisons of pro-social behavior both between subaks and with other CPRs, the same farmers who answered our survey questions also played the Dictator Game. In this simple game, each player was given another day’s wage and told that he could share as much or as little as he chose with another member of his subak who was present at the time. Players were assured that the identity of both givers and receivers would be kept confidential. The purpose of this game was to see whether the range of anonymous gifts (“offers”) to members of one’s subak varies between subaks (SI Appendix 1.5).

Because of the guarantee of confidentiality, data on offers in the game and genetic data were aggregated at the scale of subaks, while survey data was tabulated at the level of individual farmers (SI Appendix 1.6). Survey results were analyzed using multivariate ANOVA and principal component analysis, which produced two clusters of subaks. Eigen-decomposition of the correlation matrices of the clusters clarified the harmonic well structure of each regime within the 11-dimensional principal component space (SI Appendix 1.7). The transition path between the local minima was constructed using an interpolation approach, and projected into the dominant principal component space (SI Appendix 1.8).
3 Results

The genetic diversity of maternal and paternal lineages is significantly reduced ($P = 0.031$) in the four upper subaks relative to lower subaks (Fig.1). These results suggest that the upstream subaks are more demographically stable; e.g., they have a long history of continuous occupation by the same families. All subaks experienced both water shortages and pest damage to varying degrees. Water shortages were perceived to be a greater problem by upstream subaks, while pest infestations were perceived to be more problematic by downstream subaks ($P = 0.032$). This difference was also noted by the agricultural extension agents who assisted with the surveys. In the most recent harvest (2010), average rice harvests were slightly larger in the upstream subaks (5.42 tons/hectare) than downstream (4.83), though this difference was not significant. Subaks also experience problems stemming from social conflicts, which may be affected by tensions stemming from differences in either caste or social class. In our survey, subaks varied in the proportion of their members who belong to the upper castes (0 to 24%). Comparative analysis of the effects of class differences were facilitated because the mean proportion of owners versus sharecroppers (#5, “ownership”) is virtually identical for the two groups [upper subaks: mean 75.2%, S.D 40%; lower subaks: mean 78.2%, S.D. 34%; N=83].

Farmers in the upper subaks responded more positively than downstream farmers to all survey questions, including the efficacy of sanctions, the ability of the subak to mobilize to carry out irrigation maintenance, perform rituals and conduct meetings; and the overall condition and resilience of the subak [Pillai’s trace of rank scores=0.49, $P=0.000029$]. Principal components analysis of the eleven variables included in the surveys showed contrasting patterns (Figs 2 & 3). Nearly all of the variance in the correlation matrices is accounted for by the first two principal components.

The differences in correlation structure shown in Figs 2 and 3 suggest that the dynamics of interaction between social and ecological variables differs between the upper and lower subaks. To assess the significance of these differences, we examined the location of each subak in the principal component space defined by the two dominant eigenvectors. The upper and lower subaks form two clusters, each of which we associate with a well on some free energy surface, with a minimum that is locally harmonic. Figure 4 shows the ellipsoids for the two regimes, with axes whose length and orientation is defined by their orientation in the 11-dimensional principal component space. The distance between the two local minima is 0.8528, larger than the sum of the
Figure 2: Biplot of lower 4 subaks showing the correlation structure of responses to the surveys. Descriptor axes extending beyond the equilibrium circle in red are significant. For reference, here and in Fig 3, the axis of #1 (effects of caste differences) points at 270. Here “ownership” (#5, the proportion of sharecroppers vs farm owners) anti-correlates with #10, the ability of the subak to resolve social problems. PC1- 92.0%, PC2-2.4%.
Figure 3: Biplot of upper 4 subaks. Most variables have different effects than in the lower subaks. For example, there is no relationship between #5, the proportion of sharecroppers vs owners, and #10, ability to resolve social problems. PC1- 85.5%, PC2 -5.4%. 
largest projected variance from the two ellipsoids to the direction that joins the two minima (0.2178). The sum arises from the third principal eigenvector of the upper regime ellipse and the second principal eigenvector of the lower regime ellipse. Notably, we observe from Fig. 4 that the upper and lower regime ellipses do not intersect each other.

Locally, the two regimes are differentiated by the difference in the correlation structure among the variables as shown in Figs 2 and 3. The transition between two free energy minima separated by an energy barrier will proceed along a path that crosses the barrier at the lowest point. Such a path will leave the first minimum along the direction where the free energy increases the slowest, and approach the second minimum along the direction where the free energy also decreases the slowest. These directions can be approximated by the eigenvectors with the largest eigenvalues of the covariance matrices of the respective minima. Globally, the variables that dominate the transition pathway between the local minima are: condition of the subak (30%), efficacy of sanctions (14%), and the ability of the subak to mobilize labor (10%).

Finally, offers (gifts) in the Dictator Game were more generous in the upstream subaks (mean 33.75%) than the downstream subaks (mean 25.5%, P=0.047, Welch t-test). The size of the gifts varied with the farmer’s estimate of the overall state of their subak, but the nature of this relationship varied between the two groups. Among the downstream subaks, the worse the condition of one’s subak, the smaller the offer. The opposite pattern emerged in the upstream subaks: the worse the state of one’s subak, the more generous the offer. These correlations have opposite effects, so at the level of the global system of eight subaks there was no correlation.

4 Discussion

Responses of these eight Balinese communities to social and environmental challenges fall into two contrasting patterns. The upstream and downstream groups experience similar social and environmental conditions, but they respond to them in different ways. These differences only become apparent through a cross-scale comparison; they would be invisible to a study of either individual subaks or the whole system. For example, the average offer in the Dictator Game for the entire sample of farmers was 30% with a large variance, at the low end of the scale of offers in cross-cultural studies [21]. Buried within this variance, however, are contrasting patterns: farmers in downstream subaks are less generous overall, while the most generous farmers of all are those...
Figure 4: Ellipsoids depict the potential wells within the 11-dimensional principal component space. A 2D representation of the minimum energy pathway between the two equilibria is indicated by the green line, indicating the most probable transition pathway for a hypothetical subak undergoing a regime change.
in the two upstream subaks experiencing water shortages.\footnote{This is consistent with the results of a recent study by Lamda and Mace: "That behavioral variation is at least partly contingent on environmental differences between populations questions the existence of stable norms of cooperation"\cite{22}.}

To explain these differences, size does not appear to matter: the two small downstream subaks resemble the large downstream subaks more closely than the small upstream subaks. Differences in genetic relatedness between the two groups are too small to affect cooperation directly (e.g. kin selection), nor do they suggest the exclusion of outsiders. Instead they are consistent with the view expressed by several upstream farmers, that their subaks benefit from generations of shared history. But what makes the difference? The transition pathway between the two regimes is dominated by the efficacy of sanctions and the ability of the subak to mobilize its members for agricultural labor. The functional significance of these variables is obvious; they are plausible as drivers pushing subaks towards one or the other regime, and the depth of the respective free energy wells provides information about how difficult this journey may be.

In ecology, the discovery of alternate stable states led to a shift from the investigation of equilibrium or near-equilibrium states, to the study of stability boundaries for different regimes \cite{8}. Here we have extended this approach to a fully coupled social-ecological system, revealing a clear separation between two regimes, significant at four sigma. The more successful upstream subaks flourish in a small but deep basin of attraction. Confident in their collective ability to meet any challenge, they are exceptionally public-spirited. Their neighbors downstream cluster around their own attractor, revealing that muddling through can also be a steady state, with different dynamical relationships among state variables than in the upstream group. Failure is also an option: subaks and their rice terraces are presently vanishing at a rate of about a thousand hectares each year. Thus there are at least three attractors for the subaks, each with its own dynamics.

That the existence of alternate attractors in SES like the subaks has gone unnoticed until now may reflect differences in the observable behavior of SES as compared to natural ecosystems. Regime shifts in ecosystems are hard to miss, since they produce visible changes in the biotic community. Moreover, the dynamic behavior of ecosystems- their inner workings- are relatively clearcut, compared to SES like Balinese subaks. Intuitively, it is not surprising that the coupling of social and ecological dynamical systems does not converge on a simple linear equilibrium. If- as in this case- multiple attractors may be hidden in the data, cross-scale comparisons are necessary. As Paul Samuelson observed in his early work on stability analysis, “...in order for the comparative-statics
analysis to yield fruitful results, we must first develop a theory of dynamics” [23].

Bibliography


SI Text: Alternate stable states in a social-ecological system

1.1 Balinese subaks: effects of ecological processes on the spatial scale of cooperation

Balinese Subaks are associations of farmers who shared water from a common source, such as a spring or irrigation canal. Their role in water management is attested in royal inscriptions beginning in the eleventh century: the first appearance of the term subak is as the root of the word kasuwakan in the Pandak Bandung inscription of 1071 AD (No. 436: Setiawan 1995: 104–5). Because of Bali’s steep volcanic topography, the spatial distribution of Balinese irrigation canals, which by their nature cross community boundaries, made it impossible for irrigation to be handled at a purely community level. (Christie 1992; Scarborough et al 1999, 2000). Typical ancient irrigation systems included multiple subaks. For example, an inscription dated AD 1072 refers to a single subak comprising fields located in 27 named hamlets (Er Rara inscription, Sukarto Atmodjo 1986: 34-5). In 1993 the number of subaks in Bali was estimated to be 1,611 farming 108,000 hectares of rice paddies. By 2010 the extent of irrigated paddies had been reduced to approximately 82,000 hectares, according to the Ministry of Agriculture. The number of subaks remaining has not been systematically counted since 1993 (unpublished statistics, Badan Penelitian dan Penilaian Teknologi Pertanian, Ministry of Agriculture, Bali).

Because irrigation depends on seasonal rainfall, each subak’s choice of an irrigation schedule affects the availability of water for their neighbors downstream. The timing of irrigation can also be used to control rice pests like rats, insects and insect-borne diseases. This is accomplished by synchronizing rice harvests and then briefly flooding the fields, depriving the pests of their habitat. The larger the area that is encompassed by the post-harvest flooding, the fewer the pests. But if too many subaks try to flood their fields at the same time, there will not be enough water. This creates a feedback relationship between the choice of irrigation schedules and the occurrence of water shortages or pest infestations. Consequently, there is a continuing need to sustain tightly synchronized irrigation schedules, in order to balance these constraints.
To test the ability of the subaks to discover effective solutions to this trade-off between pest control and water shortages, a forward-in-time simulation model was constructed (Lansing and Kremer 1993; Lansing 2006). Through a process of trial and error, each subak seeks to discover an irrigation schedule that minimizes losses both to pests and water shortages. In less than a decade, a patchwork of synchronized irrigation schedules comes into existence, which closely resembles the schedules observed on the ground. As this occurs, rice harvests improve because water shortages and pest damage are reduced for the entire watershed. When the key environmental parameters are stabilized, variation in harvests declines because these benefits spread across the entire system. This model captures an evolving feedback relationship between the decisions of the subaks and the responses of the environment. As shown in Figures S1 & S2, simple trial-and-error at the local level produces a patchwork of synchronized irrigation schedules which over time improves harvests and also reduces variance in harvests. The reduction in variance is potentially significant, because large differences in harvests could discourage cooperation by farmers with suboptimal harvests. These simulated results are supported by responses from farmers to another question in the survey: 97% stated that their own harvest is about the same as that of the other farmers in their subak (Lansing and Miller 2005).

**Figure S1:** Effects of increasing pest virulence on the synchronization of irrigation schedules in a simulation model of subaks along the Oos and Petanu rivers in Bali. Subaks are depicted as small squares; their color identifies their irrigation schedule for the year. The outcomes reflect three levels of pest damage ($p$): low (left), current (middle), and high (right). As synchronized irrigation schedules expand, pest damage is reduced by effective fallowing, but water shortages increase. Lansing & Fox 2011: 931.
The formation of different-sized clusters of subaks practicing synchronized cropping creates an ecological inheritance of modified selection pressures for descendant populations, and is thus consistent with a process of niche construction (Lansing and Fox 2011). A real-world test of the functional significance of the subak clusters began in the 1970’s as an unintended consequence of changes in agricultural policy. At that time, the Asian Development Bank became involved in an effort to boost rice production in Indonesia. The Bank’s consultants saw two ways to improve harvests in Bali. The first was to encourage the farmers to grow higher-yielding “Green Revolution” rice varieties, which produce more grain than native Balinese rice. The second recommendation took advantage of another feature of the new rice: it grows faster than native rice. Consequently, the farmers could plant more frequently. The Ministry of Agriculture adopted both recommendations, and competitions were created to reward the farmers who produced the best harvests. By 1977, 70% of the southern Balinese ricebowl was planted with Green Revolution rice, and subaks stopped coordinating their irrigation schedules.

At first, rice harvests improved. But a year or two later, Balinese agricultural and irrigation workers began to report “chaos in water scheduling” and “explosions of pest populations.” At the time, planners dismissed these
occurrences as coincidence, and recommended higher doses of pesticides. They urged the farmers to apply higher doses of pesticides, while still competing to grow as much rice per year as possible. This actually intensified both the pest problem (Machbub et al. 1988) and water shortages (Horst 1998). It was only when farmers spontaneously returned to synchronized planting schemes that harvests began to recover, a point subsequently acknowledged by the final evaluation team from the Asian Development Bank: “Substitution of the “high technology and bureaucratic” solution in the event proved counter-productive, and was the major factor behind the yield and cropped area declines experienced between 1982 and 1985...The cost of the lack of appreciation of the merits of the traditional regime has been high” (Lansing 2007: 124-5). Similar results can be modeled by running the simulation model in reverse: beginning the simulation with the evolved cluster patterns of subaks, and instructing the subaks to plant as often as possible. This quickly leads to the fragmentation of subak clusters, triggering increases in pests and water shortages. Thus both analytical and empirical results indicate that breakdowns in cooperation leading to unsynchronized irrigation schedules rapidly cause crop losses.

References:


1.2 Study Populations

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<th>Longitude</th>
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<th>Game offer</th>
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Table S1: Study Populations
1.3 Genetic Data and Analysis

We compared population genetic structure for subaks located in two regions of Bali (Figure S3). The first group consisted of 287 farmers who belong to 13 subaks associated with a water temple network in the vicinity of the village of Sebatu on the upper reaches of the Petanu river. Archaeological evidence suggests that these are among the oldest rice terraces and water temples in Bali. The second group consists of 120 farmers belonging to eight subaks located along the Sungi river in the district of Tabanan (Tables S1 and S2). All samples were collected from volunteer donors with written informed consent and appropriate permits from the Indonesian Government via the Eijkman Institute for Molecular Biology. The University of Arizona Human Subject Committee approved sampling protocols.

Polymorphic markers on the non-recombinating portion of the human Y chromosome were screened, included a set of 74 previously published binary markers (Karafet et al., 2005) as well as four additional polymorphisms: M208, M210, M347, M356 (Underhill et al., 2000; Kayser et al., 2003, 2006; Hudjashov et al., 2007). Binary markers were analyzed using a hierarchical typing strategy (Hammer et al., 2001) where increasingly detailed sample genotyping was restricted to relevant downstream mutations as defined by the known Y chromosome haplogroup tree. For microsatellite analysis, ten short tandem repeats (STRs: DYS19, DYS388, DYS389I, DYS389II, DYS390, DYS391, DYS392, DYS393, DYS426 and DYS439) were typed as described by Redd et al. (2002). Some individuals were additionally scored for DYS438 and DYS457.

A 519 base pair sequence of the first hypervariable segment 1 (HVSI) of the human mtDNA was sequenced as described in Lansing et al. (2008) and analyzed with Sequencher® v. 5.0 (Gene Codes Corporation, Ann Arbor, MI, USA; http://www.genecodes.com).

The genetic structure of these two subak groups was compared with background levels of genetic diversity determined from an additional sample of 100 men selected randomly from each of the nine geographical regions of Bali. Within population diversity was determined using Nei’s measure of haplotype diversity (h) (Nei 1987). The results of genetic analyses on these populations are fully explored in Lansing et al. (2009). Here we summarize only the relevant findings. Subaks located at the furthest positions upstream on their respective irrigation systems on both rivers demonstrate reduced levels of genetic differentiation and diversity, suggesting that they came into existence before their younger downstream neighbors. There is a strong correlation between Y-STR and mtDNA diversity in the Sebatu subaks (Pearson’s r = 0.629). Strong correlations were also found between Y-STR variation and geography (Mantel test r = 0.541), and mtDNA variation and geography (Mantel test r = 0.361), possibly reflecting shared demographic events in the history of these populations. No such correlations were identified for the Sungi river subaks or the all-Bali sample.
Differences in genetic diversity between the upper and lower subaks were explored using a bootstrap method. The null hypothesis ($H0$) states that the upper and lower subaks are drawn from a single undifferentiated group. Subak diversities were selected without replacement, and four were assigned randomly as upper subaks, while four were assigned randomly as lower subaks. The test summary considered how often i) the upper subaks had less diversity than the lower subaks and ii) the difference between the means of the upper and lower subaks was equal to or greater than that observed for the real data. The Y chromosome and mtDNA were analyzed simultaneously for $10^4$ iterations of the bootstrap permutation test. The level of population differentiation observed between the upper and lower subaks is significantly unlikely ($P = 0.0312$). On this basis, the null hypothesis is rejected; genetic diversity is statistically smaller in the upper subaks compared to the lower subaks, and this pattern holds across both maternal and paternal lineages.

**Figure S3**: Map of Bali showing locations of Sebatu and Tabanan subaks, and boundaries of the nine districts from which genetic samples were taken. While the older Sebatu subaks are tightly clustered, the subaks along the Sungi river are spread along the full length of the river. The average geographical distance between the Sebatu subaks is only 3.2 km; it is 13.9 km for those along the Sungi river.
Alternate stable states in a socio-ecological system

References


**Alternate stable states in a socio-ecological system**

### Y-SNP

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### mtDNA

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**Table S2: Genetic Data.**

246 N_S, sample size; N_h, number of haplotypes; h, Nei’s haplotype diversity (plus standard error).

249 **1.4 Dictator Game**

250 After completing the survey, farmers in each subak were invited to play the Dictator Game with one another. As the game script indicates, the identity of both givers and receivers was not recorded or revealed to participants.
Script:

You are being invited to take part in a research study being conducted by Professor Lansing from the University of Arizona in America. The purpose of this research study is to understand the problems facing Balinese subaks. You are being asked to participate in this study because you are a member of a subak.

The aim of this research is to seek data concerning the causes that influence the welfare of Balinese subaks. These causes include environmental factors (such as adequacy of irrigation water and agricultural pest infestations); labor; economic factors such as farmer’s incomes; and social factors such as the readiness of farmers to participate in cooperative labor; subak institutions, and the ability of the subak to stay on top of problems like environmental conditions. You are asked to answer some questions about those matters in a questionnaire. You may choose not to answer some or all of the questions. During the interview(s) or survey(s), written notes will be made in order to help the investigator review what is said. Your name will not appear on these notes. Any questions you have will be answered and you may withdraw from the study at any time.

We also ask if you would be willing to participate in a game involving a small amount of money called "Dictator". The aim of this game is to clarify the basis for people’s ways of thinking about justice. You will be given 30,000 rupiah and asked to decide if you want to share some of the money with another member of the subak, who will be chosen randomly. How much you share is up to you, from none of it to all of it! You will not know which subak member will receive the money, and they will not know who gives the money. We guarantee that no one will ever know. There are no known risks from your participation.

Results

Offers in the Dictator Game were more generous in the upstream subaks (mean 33.75%) than the downstream subaks (mean 25.5%), P= 0.047, Welch t-test), Table S3. Because the identity of players was kept confidential, it was not possible to correlated each individual’s offer with their answers to survey questions. However, it was possible to correlate mean offers in each subak with mean answers to the question, “What is the overall condition of your subak?” (Table S4).

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<tr>
<th>Subak</th>
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### Table S3: Offers in the Dictator Game

“Offers” in Indonesian rupiah (column 1) and as a percentage of the money provided to the donors (30,000 Indonesian rupiah, a laborer’s day’s wage)

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SI: Alternate stable states in a socio-ecological system

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<td>Gadon</td>
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Table S4: Relationship of average game offers to average of responses to survey question, “what is the overall condition of your subak?” Upstream subaks: \( r=-0.955, \ P=0.044 \). Downstream subaks: \( r=0.504, \text{n.s.} \)

1.5 Survey Questionnaire

1) Identity
1. Name
2. Age
3. Number of family members sharing one kitchen
4. Address (village, district)
5. Names of subaks where you farm
6. Location of your largest farm
7. Membership of your subak: total
   a. Brahmana caste
   b. Ksatriya caste
   c. Sudra caste
   d. Persons without caste

2) Harvests
1. Harvets for previous 2 years (kg/hectare, rainy and dry seasons). How satisfied were you with each harvest: (Very satisfied to very unsatisfied)
2. Your wet rice farms (sawah): ______ owned ______ sharecropped
3. If you sharecrop, where does the owner live?
4. Fraction of your total income from farming compared to other sources
5. What is your opinion of participation in the subak?
   a. Attendance of subak members at subak meetings is:
SI: Alternate stable states in a socio-ecological system

i. More than 80%
ii. Between 60% - 80%
iii. Less than 60%

b. Participation of subak members at communal labor is:
i. More than 80%
ii. Between 60% - 80%
iii. Less than 60%

c. Participation of subak members in social activities connected with the
subak, such as subak rituals:
i. More than 80%
ii. Between 60% - 80%
iii. Less than 60%

6. Approximately what percent of your labor, time and resources are
invested in subak-related activities per year, compared with other
activities?

3) Participation of subak members in solving problems
1. What is the average magnitude of problems like these during each
growing cycle?
a. Technical problems such as:
i. shortages of irrigation water
ii. rice pests
iii. shortages of fertilizer
b. Social problems such as:
i. Conflicts between subak members
ii. Theft of irrigation water
iii. Subak rituals or other problems

2. Does the subak impose sanctions against wrong-doers? If so what are
they?
a. Fines of money
b. Social sanctions
c. Other sanctions

3. How effective are the sanctions?
a. More than 80% of the members accept and follow them
b. About 60-80% accept and follow them
c. Less than 60% accept and follow them

4. In your opinion, is there a relationship between the effectiveness of the
subak and caste differences among subak members?
a. There is a strong connection
b. There is some connection
c. There is no connection at all

5. In your opinion, is there a relationship between the effectiveness of the
subak and differences in the level of prosperity or poverty among subak
members?
a. There is a strong connection
b. There is some connection
4) Farmer’s Perceptions

1. The condition of the subak now is:
   a. Still strong, with no deterioration
   b. Some deterioration
   c. Deteriorated

2. Causes of deterioration of the subak include:
   a. Water shortages
   b. (Outside) investors buying land
   c. Conflicts within the subak
   d. Alternative employment or income
   e. Effects from the external environment
   f. Pest damage

3. The condition of the irrigation works now, compared to the usual:
   a. Excellent
   b. Fair
   c. Poor

4. During the past two years (wet and dry growing seasons of 2009-10), did the subak members all follow the same irrigation schedule (yes/no)?

5. How many subak members planted at the same time?
   a. All (100%)
   b. Most (about 75%)
   c. Half (about 50%)
   d. Less than half

6. How frequent are pest infestations?
   a. Very frequent
   b. Frequent
   c. Rare
   d. Very rare

7. How frequent are shortages of irrigation water?
   a. Very frequent
   b. Frequent
   c. Rare
   d. Very rare

8. How frequent are subak meetings?
   a. Very frequent
   b. Frequent
   c. Rare
   d. Very rare

9. Are decisions made at subak meetings usually effective?
   a. Very frequently
   b. Frequently
   c. Seldom
1.6 Survey Results

Survey results are reported below in Table S5 and Figure S4.
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<th>Effects of social condition of subak</th>
<th>Overall success of ownership Sharecropper meetings</th>
<th>Ratio of owners to rituals</th>
<th>Ability to perform sanctions</th>
<th>Effectiveness of mobilize work</th>
<th>Ability to solve technical problems</th>
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## SI: Alternate stable states in a socio-ecological system

Table S5: Survey results

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Figure S4: Responses to survey questions and results of genetic analysis. A: What is the overall state of your subak? Mean upper subaks: 2.775, lower subaks 2.150, P > 4.993 e-11. B: How well can your subak cope with problems? Mean upper subaks 2.50; lower subaks 2.325, P > 0.114. C: How effective is your subak at mobilizing collective labor for tasks like irrigation maintenance? Upper subaks: mean 2.82, S.D. 0.384. Lower
SI: Alternate stable states in a socio-ecological system

**subaks:** mean 2.575, S.D. 0.500, \( P > 0.0143^* \). **D:** How effective are the sanctions imposed by your subak? Upper subaks: 2.90, lower subaks 2.55, \( P > 0.010^* \).

**E:** Genetic diversity.

**Y-STR diversity:** upper subaks 0.955; lower subaks 0.980. **Mean mtDNA diversity:** upper subaks 0.923, lower subaks 0.945, \( P = 0.031^* \). **F:** Proportion of farm owners (vs. sharecroppers). Upper subaks: Mean 75.2%, S.D 40%. Lower subaks: mean 78.2%, S.D 34%.

1.7 Principal Components Analysis

Let us first define the data matrix \( X \) of size \( M \times N \) for the analysis. The number of rows \( M \) relates to the number of descriptors, which are socio-ecological factors that affect the farmers in the subaks. There are eleven descriptors and they are: Caste, Class, Condition, Meetings, Ownership, Ritual, Sanctions, Work, Technical Problems, Social Problems and Resilience. These eleven descriptors are arranged in the same order in the data matrix. On the other hand, \( N \) gives the number of farmers participated in the survey, which amounts to 80 for farmers from the eight subaks. Thus, \( M=11 \) and \( N=80 \) in our studies. The data entries in \( X \) give the response of the farmers to the survey and in particular, the data in element \( X_{ij} \) represents the response of farmer \( j \) to questions that relate to the descriptor \( i \).

For the purpose of principal component analysis, we first need to take the mean of each descriptor by averaging over all the farmers’ response, leading to a \( M \times 1 \) vector with the following element:

\[
\bar{X}_i = \frac{1}{N} \sum_{j=1}^{N} X_{ij}.
\]

This allows us to form a zero-mean data matrix \( Y \) whose element is given by:

\[
Y_{ij} = X_{ij} - \bar{X}_i.
\]

We then obtain the \( M \times M \) covariance matrix of the descriptors as follow:

\[
C = \frac{1}{N} YY^T
\]

where the superscript \( T \) represents the matrix transpose. Since the descriptors are of different nature and dimension, it is necessary to normalize them. Hence, the correlation matrix \( R \) is more appropriate. Its elements \( R_{ij} \) is given as follow:

\[
R_{ij} = \frac{C_{ij}}{s_is_j}
\]
where

\[ s_i = \sqrt{c_{ii}} \]

Alternatively, we can also express \( R = DCD \) with \( D \) being a diagonal matrix whose element \( D_{ii} = 1/s_i \).

Next, we perform an eigen-decomposition of \( R \) by means of the singular value decomposition:

\[ R = E\Lambda E^T \]

where \( \Lambda \) is a diagonal matrix of positive singular values such that \( \Lambda_{ii} \geq \Lambda_{jj} \) for \( i < j \); and \( E \) is the eigenvector matrix with the \( ith \) column being the \( ith \) eigenvector (or principal component) that corresponds to the \( ith \) singular values. We then extract the two most dominant principal component of a particular descriptor \( i \) and plot their values, which are \( E_{i1}\sqrt{\Lambda_{11}} \) and \( E_{i2}\sqrt{\Lambda_{22}} \). We then draw an arrow to this point which represents the axis of the descriptor as shown in the biplot in Figure S5. By repeating this process, a total of \( M \) descriptor axes are obtained. Note that we have rotated the axes such that the axis of Caste always points at 270° for reference purposes. The biplot is useful in providing a pictorial view of the correlation between the descriptors through the angle between the descriptor axes. However, since our biplot is basically a projection from an eleven-dimensional space to a two-dimensional space, the angle between descriptor axes in a biplot is a projected angle. The true angle between descriptor axes may be larger in the eleven-dimensional space. Nonetheless, by drawing an equilibrium circle of descriptors in the biplot, whose radius is \( \sqrt{d/n} = \sqrt{2/11} = 0.43 \) where \( d \) and \( n \) are the dimensions of the projected and full space respectively, descriptors axes with length larger than this radius can be judged to be significant. The angle between such significant descriptor axes would give a close approximation to the true angle in the full space.

With this understanding, let us now discuss on the descriptor relations for the eight subaks. We observe that Caste and Class are positively correlated with each other. On the other hand, Meetings, Work and Social Problems are also positively correlated with each other. However, Meetings and Work are observed to be uncorrelated with Caste and Class. Finally, Ownership is anti-correlated with Sanctions. In our multi-scale analysis, this is the result we obtain at the highest level. We next look one level down in scale, by splitting the eight subaks into two groups: one group of four upstream subaks, and a second group of four downstream subaks. We call the first group the upper subak, and the second group the lower subak. By performing a similar analysis as above, except that the data matrix is now \( X_{Upper} \) and \( X_{Lower} \) for the upper and lower subaks respectively with \( N=40 \) each, we obtain the biplot of descriptor relations for these two groups of subaks. These are illustrated in Figs. 2 and 3 in the main paper. For the upper subaks, we again observe that Caste and Class are positively correlated with each
The descriptors Work and Sanctions are positively correlated but they are uncorrelated with Caste and Class. In addition, Meetings and Social Problems are positively correlated, while Ownership is anti-correlated with Caste and Class. For the case of lower subaks, all descriptors are significant except Resilence. While Caste and Class are less correlated now, we observe that the group of descriptors: Caste, Class, Sanctions and Technical Problems tend to conglomerate together. There is positive correlation between Meetings and Work. Lastly, Condition is anti-correlated with Ritual, while Ownership is anti-correlated with Social Problems.

**Figure S5**  *Biplot of the 8 subaks showing the correlation structures of responses to survey.*

The relation between descriptors through the first two principal components can be better visualized by means of a network diagram. This is done
by representing each descriptor as a node in the network. The connection
between two nodes is given by an edge whose weight is the correlation between
the descriptors of the respective nodes. Note that descriptors with axis length less
than 0.43 are to be regarded as isolated nodes. Next, we remove edges with
weights less than 0.79 such that relations between descriptors are preserved only
for those with good correlation. The resulting network diagrams for the case of:
all subaks, the upper subaks and lower subaks, are shown in Figures S6 and S7.
For the case of all subaks, we observe that a fully connected subgraph of three
nodes: OWN, SAN and SOC is linked to an almost fully connected subgraph of five
nodes: MEE, RIT, WOR, SOC and RES, through the node SOC. Note that we have
used the first three letters of the socio-ecological variables to denote the nodes of
the graph. In addition, node CAS only connects to node CLA; nodes CON and TEC
are isolated nodes. On the other hand, for the upper subaks, we observe that the
fully connected subgraph of three nodes: CAS, CLA and OWN is connected to the
fully connected subgraph of four nodes: MEE, SAN, WOR and SOC, through the
node RES. There are three isolated nodes in this case: CON, RIT and TEC. Finally,
for the lower subaks, we observe that a fully connected subgraph of four nodes:
MEE, OWN, WOR and SOC is connected to another fully connected subgraph of
four nodes: CAS, CLA, SAN and TEC, through node SOC. While node CON connects
only to node RIT, node RES is an isolated node.

Figure S6  Network diagram showing the correlations between socio-
ecological variables for all subaks. Abbreviations: OWN= ownership, the
ratio of owners to sharecroppers. SAN= efficacy of sanctions. SOC=
subak’s ability to solve social problems. MEE= success of subak meetings.
RIT= subak’s ability to perform rituals. WOR= success at mobilizing work
(collective labor). RES= resilience. CLA= effects of class. CAS= effects of
caste. TEC= subak’s ability to solve technical problems. CON= overall
condition of the subak.
These results indicate a marked difference in the socio-ecological circumstances for the farmers in the upper and lower subaks. The first observation is that more descriptors are closely correlated for the lower subaks than the upper subaks. Furthermore, node SOC, which corresponds to Social Problems, plays a more dominant role for the lower subaks. This results from it having the largest number of edges (i.e. the highest degree in complex network nomenclature) and also the highest centrality betweenness (i.e. the greatest number of shortest path that passes through it from all vertices). On the other hand, node RES, which is Resilience, seems more dominant for the upper subaks. Although it has one less degree than nodes SAN and WOR, it possesses the largest centrality betweenness. The clusters of closely correlated descriptors between the upper and lower subaks are also different. While the clusters are CAS, CLA, OWN, and MEE, SAN, WOR, SOC for the upper subaks, it is CAS, CLA, SAN, TEC and MEE, OWN, TEC, SOC for the lower subaks. By combining the descriptors in a cluster into one common axis, this axis is observed to be orthogonal (or uncorrelated) to the common axis formed by the second cluster. More specifically, the axis formed by CAS, CLA, OWN is orthogonal to that formed by MEE, SAN, WOR, SOC for the upper subaks; the axis for CAS, CLA, SAN, TEC is orthogonal to that for MEE, OWN, WOR, SOC for the lower subaks. These results imply that the upper and lower subaks operate at different regimes. Nevertheless, there exist descriptor relations that are common for both groups of subaks. They are the Caste (node CAS) and Class (node CLA) correlation, and the Meetings (node MEE) and Work (node WOR) correlation. Interestingly, we observe the additional feature that the Caste-Class descriptors are not correlated with the Meetings-Work descriptors for each case. This trend continues to hold as we examine the network diagram of all the subaks. At the same time, we find the more dominant role of node SOC (i.e. Social Problems), followed by node RES (Resilience), node MEE (Meetings) and node WOR (Work) in this case (see Figure S6).
1.8 Regime Shifts and Transition Path between the Upper and Lower Subaks

The differences in relations between the socio-ecological descriptors between the upper and lower subaks discussed in Supplementary Section 1.7 suggest that the two groups of subaks lie in different regimes. The goal of this section is to demonstrate this convincingly. In principle, if we could know the free energy function in this problem, we would find the local minima in this landscape, and show that the upper and lower subaks fall into different minima. We would then proceed to work out the transition path between the two regimes, and the bifurcation properties at the saddle point. Unfortunately, it is very difficult to infer the free energy landscape in such problems. We therefore take an indirect approach, and ask: if the upper and lower subaks do indeed occupy different minima in a high-dimensional free energy landscape, what kind of statistical differences do we expect to observe?

Figure S7  Network diagram showing the correlations between socio-ecological variables for upper subaks (top figure) and lower subaks (bottom figure).
SI: Alternate stable states in a socio-ecological system

Figure S8. Two clusters of points within two potential wells, and their projections to a plane orthogonal to the line joining the two wells (clusters not separated) and a plane parallel to the line joining the two wells (clusters well separated).

To begin, we would expect the upper subaks to cluster around one minima, and the lower subaks to cluster around a different minima. This is indeed observed, based on the genetic, survey, and dictator game results reported in Supplementary Sections 1.3 through 1.7. To visualize this separation in the high-dimensional space through projections down to lower dimensions, we must be careful. As shown in Supplementary Figure S8, if we project the data onto subspaces orthogonal to the line joining the two minima, we will find little or no separation between the two groups of subaks. On the other hand, if we project the data onto subspaces containing the line joining the two minima, we expect to find maximum separation between the two groups of subaks. In other words, we would like to find coordinate axes along which the two groups of subaks are most different. This is achieved once again by principal component analysis of the full covariance matrix $C$, which we write as

$$C = V^A \Sigma (V^A)^\top,$$

where $V_i^A$ are the eigenvectors of $C$.

Visualization will be done in the two-dimensional subspace formed by the first two principal components $V_{i1}^A$ and $V_{i2}^A$ (or $V_1^A$ and $V_2^A$ in vector form), since these capture most of the variance between the two groups of subaks. By averaging over the farmers in subak $k$,

$$\bar{X}_{sk} = \frac{1}{N_k} \sum_{j=1}^{N_k} X_{sk}^j$$

with $k$ ranging from 1 to 8 and $N_k$ being the number of farmers in the $k$th subak, and $X_{sk}^j$ is the $11 \times 1$ descriptor vector of the $j$th farmer in the $k$th subak, we
obtain eight \(11 \times 1\) mean descriptor vectors \(\bar{X}^{1}, \bar{X}^{2}, \bar{X}^{3}, \bar{X}^{4}, \bar{X}^{5}, \bar{X}^{6}, \bar{X}^{7}\) and \(\bar{X}^{8}\), which corresponds to the subaks Apityeh, UmaPoh, Padang, Bena, Tinjak, Sungi, Jaka and Gadon respectively. We then find the projections of these mean descriptor vectors onto the two principal components,

Principal Component \(i\) of the \(k\)th subak = \(\left(\bar{X}^{\text{sk}}\right)^{T} V_{i}^{A}, i = 1, 2, \)

and use these to plot each subak as a point in the two-dimensional subspace. As we can see from Fig. 4 in the main text, there are two clusters of subaks with four members each. Remarkably, members of the first cluster belong to the upper subaks and members of the second cluster belong to the lower subaks. This indicates that the upper and lower subaks indeed reside in two different regimes.

Now that we have verified the existence of two regimes, let us go further, to characterize their statistical properties. In both regimes, the subaks are scattered about some average descriptors

\[
\bar{X}^{U} = \frac{1}{4} \sum_{k=1}^{4} \bar{X}^{sk}
\]

and

\[
\bar{X}^{T} = \frac{1}{4} \sum_{k=5}^{8} \bar{X}^{sk}
\]

which represent the free energy minima of the two regimes. If we are not too far away from the minimum, the free energy surface will be locally flat. The level sets of the free energy well centered at \(\bar{X}\) are thus ellipsoids given by

\[x^{T}Ax = K,\]

where \(x = X - \bar{X}\) is the displacement from the free energy minimum, \(A\) is the Hessian matrix associated with the free energy well, and \(K\) is an arbitrary constant that sets the scale (size) of the ellipsoid. In particular, the shape of the standard ellipsoid \(x^{T}Ax = 1\) is determined by the reciprocal of the square roots of the eigenvalues of \(A\) (the semi-axes of the ellipsoid), while its orientation is determined by the eigenvectors of \(A\).

If we assume that the subaks belonging to one regime move in the free-energy well in the presence of noise, then like water filling up a pond the distribution of subaks about the free energy minimum will have the same ellipsoidal shape as the free energy well itself. This means that \(A = C^{-1}\), where \(C\) is the covariance matrix of the subaks. With this statistical insight we can characterize the free-energy wells associated with the upper and lower subaks in the two-dimensional space formed by the first two principal components \(V_{1}^{A}\) and \(V_{2}^{A}\), by diagonalizing their \(2 \times 2\) covariance matrices \(C^{U} = V^{U} \Sigma^{U} \left(V^{U}\right)^{T}\) and \(C^{L} = V^{L} \Sigma^{L} \left(V^{L}\right)^{T}\). As shown in Figure 4, the ellipse for the upper regime, with
semi-axes $\sqrt{\Sigma_{ii}^U}$ and orientation defined by the two eigenvectors $V_i^U$, and the
ellipse for the lower regime, with semi-axes $\sqrt{\Sigma_{ii}^L}$ and orientation defined by the
two eigenvectors $V_i^L$, are statistically well separated.

![Contour plot showing the free energy landscape around the minima A and B, the saddle point separating them, and the transition path going from one to the other. Also shown are the first principal components of the two free energy wells, and how they relate to the transition path.](Image)

**Figure S9.** Contour plot showing the free energy landscape around the minima A and B, the saddle point separating them, and the transition path going from one to the other. Also shown are the first principal components of the two free energy wells, and how they relate to the transition path.

Finally, let us work out the transition path, along which the socio-ecological regime shift will proceed. In general, for a system with minima A and B as shown in Figure S9, the saddle point which presents the lowest free energy barrier to the transition between A and B does not lie along the straight line connecting the two. Instead, it lies somewhere between the direct midpoint between A and B, and the intersection between the semi-major axes of the two minima. Along the transition path that goes from A to B (or vice versa), the free energy increases the slowest after leaving A, reaches a maximum at the saddle point, before decreasing the slowest to reach B. Instead of finding this transition path from a given free energy function, we interpolate it using the fact that it leaves A along $V_1^A$, and arrives at B along $V_1^B$. In this interpolation, we want to use as much information on the structure of the upper and lower free energy wells as possible. Beyond the first principal components $V_1^U$ and $V_1^L$, which dictate the initial and final directions of the transition path, the ratios $\Sigma_{22}^U / \Sigma_{11}^U$ and $\Sigma_{22}^L / \Sigma_{11}^L$ determine how quickly the transition path deviates from its initial and final directions. For example, if $\Sigma_{22}^U / \Sigma_{11}^U \approx 1$, the upper free energy well is nearly circular, and thus there is little extra cost to pay in free energy for the transition path to leave the upper free energy well along a direction different from $V_1^U$. On the other hand, if $\Sigma_{22}^U / \Sigma_{11}^U << 1$, the upper free energy well is highly eccentric, and the free energy cost of deviating from $V_1^U$ becomes forbidding.

We can incorporate all these features into our approximate transition interpolation path by using the cubic Bezier interpolation formula

\[
X_i(t) = (1-t)^3 \bar{X}_i^U + 3t(1-t)^2 X_a + 3t^2(1-t)X_b + t^3 \bar{X}_i^L,
\]
SI: Alternate stable states in a socio-ecological system

where \( 0 \leq t \leq 1 \) is the interpolation parameter, \( \bar{X}^U \) and \( \bar{X}^L \) are the free energy minima of the upper and lower subaks on the plane formed by the first two principal components,

\[
X_a = \left( 1 - \frac{\Sigma_2^U}{\Sigma_{11}^U} \right) X_1 + \frac{\Sigma_2^U}{\Sigma_{11}^U} \bar{X}^U
\]

is the control point for the leg of the transition path leaving the upper free energy well, and

\[
X_b = \left( 1 - \frac{\Sigma_2^L}{\Sigma_{11}^L} \right) X_1 + \frac{\Sigma_2^L}{\Sigma_{11}^L} \bar{X}^L
\]

is the control point for the leg of the transition path arriving at the lower well. Here, \( X_1 \) is the point of intersection between the straight lines

\[
X = \bar{X}^U + r_U V_1^U
\]

and

\[
X = \bar{X}^L + r_L V_1^L
\]

defined by the first principal components of the upper and lower free energy wells. Based on the shapes and orientations of the free energy wells in Figure 4 of the main text, we extract the approximate free energy profile along the transition path shown in Supplementary Figure S10. In Supplementary Figure S10 we also show a hypothetical subak that sits at the top of the free energy barrier separating the two regimes.

Figure S10. The approximate free energy profile along the transition path between the upper and lower free energy wells. Along the transition path, the upper free energy well has a broad basin of attraction while the lower one has a narrow basin of attraction. Also shown is a hypothetical subak at the top of the free energy barrier.

References