Modeling Florentine Republicanism

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The following model of political process is intended to be a moderately realistic representation of Florentine republicanism, during the period of the Renaissance (~1300-1500). The model is composed of two parts: (a) patronage—namely, the building up of partisan networks through the exchange of office-based favors, and (b) policy—namely, the collective deliberation and choice of “public goods” for Florence, in the Priorate (i.e., the city-council governing body). It was the tension and interplay between these two modes of governance, I contend, that produced Renaissance Florence’s distinctively turbulent, and creative, history of party formation and constitutional design.

This memo proceeds in three sections: the official structure of the Florentine state, the patronage networks that grew up through these offices, and the “sacred” institutions at the center1 that may or may not have transcended patronage.

Florentine constitutional structure

Although the all-important rules for eligibility, nomination, and voting varied considerably over this period (Najemy 1982, Rubinstein 1966), from the “birth of the Republic” in 1282 until the Savonarola reforms of 1494 these rule changes evolved within a fairly fixed overall constitutional framework.2 I describe graphically the basics of this republican framework in figure 1. Official Florentine republicanism was built on a crisscross of two organizing principles: guild and neighborhood. To be a citizen of Florence, eligible for voting, required both guild membership and permanent residence.3 For the most part,4 for political governance, neighborhoods5 aggregated into
FLORENTINE REPUBLICANISM:

Random draw:

NEIGHBORHOOD COUNCILS: (tax assessments)

Pool of Eligibles: ("in bag")

Scrutiny voting:

Voting:

Industries:

Families:

Priorate:

SS SC SMN SG

MEC ANZA (Commercial Court)

Guild Councils

Pool of Eligibles

Neighborhood 1 (SS) Neighborhood 2 (SC) Neighborhood 3 (SMN) Neighborhood 4 (SG)
FLORENTINE REPUBLICANISM:
NEIGHBORHOOD SIDE

PRORATE

Eligibles for Priorate:

Scrutiny voting:

NEIGHBORHOOD COUNCILS:

Citizens = Guildsmen

Eligibles for Gonfalon:
neighborhood councils and the Priorate; guilds aggregated into guild consuls and the Mercanzia (a commercial court). In addition to adjudicating cross-guild commercial disputes, the Mercanzia set international foreign-trade policy (Astorri 1998). In addition to deciding domestic policy, the Priorate controlled non-economic foreign policy (e.g., wars). In any policy conflict between the Mercanzia and the Priorate, the Priorate would trump.

The purpose of this memo is to concentrate on the political, not the economic, side of governance. Hence figure 2 highlights the neighborhood half of this crisscross structure. This half could be considered the “dominant” side, because it aggregated into the overall ruling body: the Priorate.

A simplified version of how elections to political office proceeded, within the “dominant” neighborhood half of republicanism, is as follows:

(a) Citizens voted among their neighborhood selves about who would be eligible for neighborhood office. Citizens receiving above a threshold number of votes would become eligible to hold that office. Political scientists would call this plurality voting; the Florentines themselves called it “the scrutiny.” Apart from the monks who administered the voting, the results of these scrutiny elections were kept secret.

(b) The names of the secret “eligibility pool” winners were then literally placed, by the monks, into closed bags— one bag for each office. Every three or four months, the monks would reach into the bags and randomly draw out the set of office-holders for that office term. Only at that point would winners of the scrutiny become public information. Since scrutinies were held roughly every five years, most names in the bags were drawn eventually.

(c) Citizens voted among successful-in-the-past neighborhood office holders for eligibility to the Priorate. The two-step voting procedure—first plurality voting, then

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6 Florentine electoral rules were incredibly complicated. To make any headway at all here, I have to simplify down to basics. Ultimately, Florentine political historians will have to judge whether the results of this modeling exercise are sufficiently rich and informative to justify the inevitable simplifications involved. (Being one of them myself, I can certainly appreciate their skepticism.)

7 And for that matter, within the guild half as well.

8 Sociologists might call it a popularity contest.

9 This randomization procedure was quite intentional. Florentine constitutional designers had the goal of inhibiting the ability of any one faction to gain monopoly control. The model in this memo will be able to explore the conditions under which this constitutional objective can be achieved.

10 In addition, there were all sorts of restrictions, administered by the monks, about which drawn “eligible” names were able to be seated. These restrictions often had to do with relatives of the same family and with tax delinquents. The more baroque these restrictions became, the more likely the number of names drawn but not seated (veduti) could actually exceed the number seated (seduti). I intend to deal with none of these complications here.

11 In political emergencies, however, Florentines often behaved extra-constitutionally: they would throw away (perhaps even burn) the existing bags and re-do the scrutiny—in order to get more regime supporters into the bags.

12 I call this the “democratic” variant of Florentine republicanism. An “oligarchic” variant would have only successful-in-the-past neighborhood office-holders vote among themselves for eligibility to Priorate. The oligarchic variant, of course, creates a self-recruiting elite.
random draw—was identical for Priorate and for lower offices. The only procedural difference was that the number of candidates eligible for office shrunk as one moved up to more prestigious offices.

Obviously the randomization element in this republican constitution is a highly distinctive Florentine trait. One can easily make this constitution converge into a more “modern” plurality-voting setup: just lengthen terms of office, thereby restricting eligibility pools of office-holders, thereby eliminating the need for randomization. But that would be to deny the genius of the Florentine system, which was to induce widespread and active participation of citizens in their own governance.\(^{14}\)

PARTISAN VOTING: THE PATRONAGE SIDE OF FLORENTINE REPUBLICANISM

It is a clear empirical fact that, constitutional design notwithstanding, Florentine republicanism was “riddled” with patronage (Brucker 1969, 1977; Kent 1978). This is not to say that Florentine patronage did not have its own dynamic history,\(^{15}\) but the central functioning of patronage in Renaissance Florentine politics seems virtually impossible to deny.\(^{16}\) This memo does not claim that this was all there was to Florentine politics, but this section develops a model of that one side of republicanism.

Imagine offices (among other things) as founts for favors. That is, offices are assets that endow their holders with the ability to grant particularistic benefits to whomever the officeholder wishes. In Florence in fact, there were a whole raft of such offices. In this memo, however, I concentrate on the neighborhood council as my exemplar. The reason for me choosing this particular office as exemplar is that neighborhood councils were the

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\(^{13}\) Actually, here as elsewhere, reality was more complicated than the simplified version I am presenting here. In fact, there was a very elaborate, and historically variable, nomination procedure for selection of “who was eligible to be voted on for Priorate.” This nomination procedure was contested, and subject to all sorts of partisan manipulations. (For a prominent example, see the analysis of Rubinstein 1966.) I simplify greatly by just having “successful-in-the-past neighborhood office-holders” be that nomination list. This simplification, however, does capture nicely the shrinkage in nomination list as one moves up; it also captures nicely the notion of an electoral career. I am open, however, to exploring in the future other modeling simplifications of the actual (incredibly complex) Florentine nomination procedure(s).

\(^{14}\) Certainly I do not deny the P.C. point that only adult white males (with enough wealth to make the guilds) were involved here. In deference to that point, I would label the Florentine system not “democracy” but “pure republicanism.” I borrow this phrase from Thomas Jefferson, whose aristocratic populism the Renaissance Florentines would have understood well.

\(^{15}\) Curiously, Florentine historiography implicitly seems to deny this point: without necessarily intending it this way, the literature’s portrait is almost of patronage as a timeless cultural universal. So much attention has gone into the documentation of the existence (and consequences) of Florentine patronage that little analysis has been developed about its history.

\(^{16}\) Early twentieth-century American progressivism has made the word “patronage” appear odious in our ears. But Baxandall (1972) and other Renaissance art historians have drawn a direct link between Florentine patronage and the artistic innovation of the period.
sites for determining tax assessments (i.e., neighbors assessing neighbors)—probably the most potent form of “favor” imaginable, especially during periods of war.\textsuperscript{17}

A voter i (“client”) becomes a partisan of candidate j (“patron”) if voter i has promised to support candidate j with his vote, in whatever political arena that candidate chooses to be active: election to neighborhood council, election to Priorate, decision-making within Priorate. In contrast to the global commitment of partisans, “neutral” voters (in my formulation) support candidates only on an issue-by-issue basis, dependent upon those candidates’ policy stands.\textsuperscript{18}

Why do voters decide to become partisans? Shortly I will give the secondary answer of “preexisting network ties” (especially relevant at the initiation of a partisan relation), but my primary answer is “patronage” (the food that sustains the relation). Given that patronage is a flow of particularistic favors that office-holders distribute to their partisans, the first step in developing a model of partisan choice is to specify formally the meaning of “flow of favors.”

Let us say that the office “asset” grants to the office-holder at time t a “fount of favors”: \(F_t\). This, if you will, is the “favor budget” of the office-holder.\textsuperscript{19} This budget certainly could be a continuous variable, but for modeling simplicity I will reduce it to a discrete pile of equal-sized “balls”: \(N_t\). Patrons distribute favors (that is, “throw favor balls”) down upon their clients, whenever those patrons are in office. Whenever patrons are not in office, they have no balls to throw.\textsuperscript{20}

Given the Florentine constitution, being in or out of office is a random variable. Once elected to the eligibility pool, a candidate j’s probability of being randomly drawn is \(\rho_j\). This is equal to one over the number of eligibles left in the scrutiny bag—in other words, “sampling without replacement.”

The real issues are to whom are these favors distributed, and how does that distribution change over time? To answer these questions, I employ following modeling strategy: first calculate how favors are distributed within a fixed partisan network, second define a learning algorithm for partisans receiving favors, and finally derive the consequences of partisan learning for elections. Feedback between partisan adaptation and candidate success leads to evolution in the partisan network itself. I call this partisan network evolution “political party formation,” even though disjoint competing clusters are only

\textsuperscript{17} The 1425-1435 “rise of the Medici” period is especially replete with examples, on both sides of the partisan divide, of tax assessments being used both to reward (Molho 1971) and to ruin (Kent 1978) wealthy people.

\textsuperscript{18} A precise mathematical definition of “policy stands” will be given in the third “neutral voting” section of this memo.

\textsuperscript{19} This “budget” of course can vary over time, depending upon circumstances. The relative value of the tax-assessment “favor”, for example, was increased dramatically during a war.

\textsuperscript{20} I could easily modify this aspect of the model by endowing some patrons with private resources that they could use to supplement their office-based favors. Certainly this would be a realistic extension: Cosimo de’ Medici, for example, gave his clients loans/gifts, out of his own Medici bank, to help them pay their taxes (Molho 1979). Such private resources would “kick start” some candidates more easily into creating a partisan network than others.
one possible outcome. [In this section, I temporarily assume that partisan voting is all there is; the other side of the story—policy voting of “neutrals”—is postponed to the next section.]

Let me start with a simple fixed-partisan-network example, to make subsequent exposition clearer:

One-way partisan promises of support, called clientage, are denoted by arrows (\(\rightarrow\)); mutual partisan promises of support, called alliances, are denoted by two-headed arrows (\(\leftrightarrow\)). The matrix version of this particular partisan network (“direct ties”) is:

\[
P = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}.
\]

The two-step version of this particular partisan network (“indirect ties”) is:

\[
P^2 = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}.
\]
and so forth for n-step paths.

Votes flow up these partisan networks, and favors flow down. In plurality voting, voters can vote for more than one candidate. Call the maximum number of possible votes per voter K, a parameter that institutions can manipulate. Without worrying about K limits right now, I will posit

(a) that partisan voters keep their promises with probability v, and
(b) that partisan voting promises are transitive: namely, in addition to voting for their own patron, clients vote for their patrons’ patrons.

Under these specifications, for all j,

\[
\{\text{Expected vote}_j\} = \text{(vote of self)} + \text{(votes of direct partisans)} + \text{(votes of 2-step partisans)} + \text{(votes of 3-step partisans)} + \cdots
\]

\[
= 1 + \sum_{\text{one-step partisans}} v + \sum_{\text{2-step partisans}} v^2 + \sum_{\text{3-step partisans}} v^3 + \cdots
\]

\[
= \sum_i \sum_{n=0}^{\infty} v^n P^n.
\]

To illustrate for the above example, summing over all n-step paths, the expected voting of each voter i for each candidate j is:

<table>
<thead>
<tr>
<th>Candidates:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voters: 1</td>
<td>1</td>
<td>0</td>
<td>0</td>
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<td>0</td>
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<td>1</td>
<td>v</td>
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<td>0</td>
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<tr>
<td>3</td>
<td>0</td>
<td>v</td>
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<td>v</td>
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<td>0</td>
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<tr>
<td>7</td>
<td>0</td>
<td>0</td>
<td>v</td>
<td>0</td>
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<td>1</td>
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<td>8</td>
<td>v</td>
<td>2</td>
<td>v</td>
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<td>9</td>
<td>v</td>
<td>2</td>
<td>v</td>
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<td>0</td>
<td>v</td>
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<td>1</td>
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<tr>
<td>10</td>
<td>v</td>
<td>2</td>
<td>v</td>
<td>0</td>
<td>0</td>
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<td>0</td>
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<td>0</td>
<td>0</td>
<td>v</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>v</td>
<td>3</td>
<td>v</td>
<td>0</td>
<td>0</td>
<td>v</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

The expected overall vote total for each candidate j is derived simply by summing each column, down the rows.

Successfully elected candidates are those whose vote total equals or exceeds the institutionally defined threshold, V. That is,

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21 In other words, assume K larger than the expected number of patrons a partisan will vote for. “Left over votes” the partisan can allocate on purely “neutral” policy grounds.
\[ j \text{ wins if } \{\text{Expected vote}_j\} = \sum_i \sum_n v^n P^n \geq V. \]

Calculating expected favors that voters receive is pretty much the same procedure, except for summing favors over candidate columns, rather than summing votes over voter rows. With one exception: favors can only be received from successful candidates.

To maintain symmetry, but also to model the concept of patronage chains, I posit

\begin{enumerate}
  \item discrete needs or requests from randomly located clients are passed up all partisan paths, searching for termination at an office-holder, who can satisfy the request;
  \item requests successfully are passed up each link with probability \( f \);\(^{22}\)
  \item if the request makes it to an office-holder, it is granted, as long as the office-holder’s budget \( N_t \) has not been exceeded.
\end{enumerate}

This bottom-up request process is equivalent to office-holders throwing “favor balls” randomly down partisan networks, until their \( N_t \) supply is finally absorbed.

Quite similarly to the expected vote case,

\[ \{\text{Expected favors}_i\} = N_t \rho \sum_{\text{winning } j} \sum_n f^n P^n. \]

\( \rho \), to remind the reader, is the probability of random draw into office, given successful election to the eligibility pool. \( N_t \) is like the \( N_t \) budget, except that \( N_t \) is the number of balls that need to be thrown in order that \( N_t \) of them are received.

The number of favors any partisan \( i \) will expect to receive from any particular successful candidate \( j \) at time \( t \) is

\[ (\text{Expected favors}_{ijt}) = N_t \rho \sum_n f^n P^n. \]

Again, this can be illustrated by the same example:

<table>
<thead>
<tr>
<th>Candidates: 1 2 3 4 5 6 7 8 9 10 11</th>
<th>( \sum_{j=1,2,3} \sum_n f^n P^n ):</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voters: 1</td>
<td>1 0 0 0 0 0 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>2 0 1 f 0 0 0 0 0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>3 0 f 1 0 0 0 0 0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>4 f 0 0 1 0 0 0 0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>5 f f f^2 0 1 0 0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>6 0 f^3 f 0 0 1 0 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>7 0 0 f 0 0 0 1 0 0 0 0</td>
<td></td>
</tr>
<tr>
<td>8 f^3 0 0 f 0 0 0 1 0 0 0</td>
<td></td>
</tr>
<tr>
<td>9 f^3 0 0 f 0 0 0 0 0 1 0</td>
<td></td>
</tr>
<tr>
<td>10 f^3 f^2 f^3 0 f 0 0 0 0 1</td>
<td></td>
</tr>
<tr>
<td>11 0 f^3 f^2 0 0 f 0 0 0 1</td>
<td></td>
</tr>
</tbody>
</table>

\(^{22}\) With probability \((1-f)\), in other words, requests are “forgotten.”
So much for the partisan network from the patron’s point of view.

The issue for the client, of course, is whether or not to buy into this partisan network in the first place. Are the favors he receives from it worth it?

Instead of building up to the partisan’s adjustment or learning equation, let me just state it and then explain it:

\[
p(p_{ijt}) = \frac{\sum_j \sum_n \rho \sum Nr \cdot fn \cdot Pt^n}{\sum_j \sum_n \rho \sum Nr \cdot fn \cdot P^n},
\]

as long as \[\sum_j \sum_n \rho \sum Nr \cdot fn \cdot Pt^n \geq cp; \text{ otherwise } p(p_{ijt}) = 0.\]

Here, \[\epsilon_{ij} = (\epsilon^{pm}/pd^{m}_{ij}) + (\epsilon^{e}/pd^{e}_{ij}) + (\epsilon^{p}/pd^{p}_{ij-1}) + \epsilon.\]

First, notation: \(p_{ijt}\) is the binary partisanship tie, from \(i\) to \(j\) at \(t\). The set of such ties, \(\{p_{ijt}\}\), is assembled into the now dynamic matrix \(P_t\). Hence, \(p(p_{ijt})\) is the probability of a partisanship tie from \(i\) to \(j\) at \(t\). Up to a maximum of \(K\) partisan ties (“promises to vote”) can be allocated through this \(p(p_{ijt})\) mechanism.

This is a form of learning equation that, in my earlier memos,\(^{23}\) I labeled “the rule of reciprocity.” Put simply, “the more favors the partisan receives, the more likely he is to become or to stay a partisan.”

I just derived that \((\rho \sum Nr \cdot fn \cdot P^n)\) is the expected number of favors \(j\) gives to \(i\) at time period \(t\). Thus, \(\sum_t (\rho \sum Nr \cdot fn \cdot P^n)\) is the number of favors that \(j\) gave to \(i\) over the past \(\tau\) time periods. Again as discussed in my earlier memos, \(\tau\) is a “memory” parameter: how much of the past is considered relevant in evaluating giving?\(^{24}\) The comparison of numerator to \(\sum_j\) denominator means that \(j\)’s giving is evaluated relative to that of other \(j\)’s. In other words, there is favor-giving competition here.

The \(\epsilon\) terms hearken back to, and operationalize, the secondary reason for partisan ties: previous network connections. Numerous reasons could be adduced for folding in this realistic secondary factor: propinquity, trust, sanctions. I consider here three such “previous network connections”: marriage, economic partnerships, and partisanship. \(pd^{m}_{ij}, pd^{e}_{ij},\) and \(pd^{p}_{ijt-1}\) are the path distances between \(i\) and \(j\) in these three networks, respectively.\(^{25}\) The greater the network distances between \(i\) and \(j\), the less likely a partisanship tie will form.

\(^{23}\) In particular, in my earlier memo “Gift-Economy Model: Base Version,” dated 10/12/00.

\(^{24}\) Low \(\tau\) means, essentially, “What have you done for me lately?” High \(\tau\) individuals, in contrast, remember favors from long ago. At the system level, \(\tau\) controls speed of adjustment: high \(\tau\) leads to sluggish or inertial adjustment of partisan networks to current gift-giving conditions; low \(\tau\) leads to fast, perhaps even volatile and noisy, adjustment.

\(^{25}\) Note that in a full and grand “general equilibrium” version of this model, marriage, economic partnership, and partisanship networks would all co-evolve: hence \(t\) subscripts on all three path distances. Such ambition exceeds my current grasp, however, so I treat marriage and partnership as exogenous.
The various $\epsilon$ coefficients weight the relative importance of such previous network connections, both relative to favor-giving and relative to each other. A constant $\epsilon$ term is also thrown in for good measure, to preserve the small random chance of a partisanship tie being formed between two persons “for no good reason.”

Note that, as experience with favor-giving accumulates over $\tau$, the relative importance of “previous network connections” diminishes (although it never completely goes away). In this model, therefore, previous network connections are especially important at the onset of a partisan tie. A trial partisanship may be attempted because of previous network connections, but if that isn’t backed up by favors (especially if others are giving favors) that trial partisan tie will be dropped.

Finally, I impose a “cost of partisanship” truncation threshold: $c_p$. If expected benefits (previous network connections, plus favors) are below this threshold, then partisanship “just isn’t worth it,” and $p(p_{ijt})$ becomes zero. This threshold cost could be rationalized numerous ways: partisanship is work; opportunity cost of promises; Progressive cultural values. For now, I just treat it as a parameter. In social-network parlance, this parameter has the effect of making partisanship a “strong tie” (where “strong” is $\geq c_p$).

With this partisan learning mechanism, all the pieces are now in place to study partisan-network evolution (viz., “political party formation”). We “only” have to solve the model (analytically or numerically) for $P_t$ and to observe its dynamic behavior under various $N_t$ and parametric (institutional, behavioral, and prior-network) circumstances.

“NEUTRAL” VOTING: THE POLICY SIDE OF FLORENTINE REPUBLICANISM

But not everyone in Florence was a partisan, addicted to patronage and to “friends.” There developed in Florence a civic humanist tradition, which celebrated, and was even committed to, the Greek political ideals of philosophy and of serious collective discourse. Arguably, the patronage and the humanist sides of Florence were not as disjoint as they appear. Both were aspects of the emergence and consolidation of an elite. But at the level of public decision-making procedure, patronage and civic humanism certainly implied two distinct ways of doing political business, a fact that induced tensions and occasionally contradictions into the polity.

In this memo, I certainly am going to do no intellectual history. But I do want to develop a “conversational model” of Florentine politics, based not on particularistic self-interests, but rather on principled compromise among represented group interests. This model will not be so jejune as to assume politicians with no interests, but it will assume a procedural commitment on their part to aggregate their group interests into a jointly constructed perception of the “public good.” When the two models (patronage and conversation) are assembled together, politicians will become simultaneously both patrons and sages—a situation that I believe accurately captures the Florentine reality.

26 See Martines (1963) for the social context of these ideals.
27 Avoiding that history, however, does not imply that I regard it as irrelevant.
Before operationalizing “conversation,” the first step is to operationalize “group interests,” the positions from which politicians converse. Next I will develop my model of conversation-based decision-making in the “sacred” council of the Priorate. This model will be developed in two variants: without partisans on the Priorate and with partisans on the Priorate. After that, I will specify a model of “neutral” (that is, non-partisan) voting, based on the spatial proximity of politicians and voters in k-dimensional group-interest space. The payoff of the whole exercise will be to analyze dynamic feedback between Priorate decision-making and office elections. Again, this will be operationalized in two variants: without partisans in the electorate and with partisans in the electorate.

Mathematically, I could just posit some abstract (i.e., arbitrary) k-dimensional issue space, in order to represent “group interests.” But that is not very helpful for Florence. Instead, I see eight dimensions as capturing the gist of what crisscrossing “group interests” meant concretely in Renaissance Florence. These are:

- **fixed (“attributes”):** fluid (“resources”):
  - kinship: Lineage x Family Size x
  - economics: Guild/Occupation x Wealth x
  - politics: Social Status x Office-Holdings x
  - urban: Neighborhood x Parish.

Households, and for that matter individuals, can be conceived of as points in this 8-dimensional attribute space. Households can more easily move around in the dimensions on the right than they can in those on the left.

Public policies can also be conceived as points in this 8-dimensional attribute space—as long as particular such policies are measured in terms of who benefits. Dimensions can be organized into or out of politics. This means that opposed coalition blocs can line up on any of these dimensions: for example, *popolani* vs. new men, major vs. minor guilds, rich vs. poor, large vs. small families, neighborhood vs. neighborhood, Franciscan vs. Dominican, politically included vs. excluded. “Realignment” is when the dominant discourse and cleavage lines in politics shift from one dimension to another. In Florence, while some of these dimensions were empirically correlated, they most definitely were not equivalent.

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28 By “Priorate” I basically mean the specific 9-person city council of that name, with rotating 2-month terms. For purposes of reality check, however, it is well to bear in mind that there actually were three legislative bodies in the traditional Florentine constitution (the so-called *Tre Maggiore*), not to mention the numerous other councils alluded to in footnote 2. Thus “Priorate” also could be read as that entire set of interlocking legislative councils.
Let me begin analysis with the 9-person Priorate itself, without saying anything (yet) about how office-holders got there. By “conversation” I mean the interaction of two persons, each with their own position in attribute space. By “public spirited conversation” I mean the convergence of the two parties on at least temporary agreement on the mean position between them. “Public discourse” then means many such conversations.

Within this basic dyadic-conversation setup, without partisans, there are many procedural variants:

(a) Full consensus: Everyone talks to everyone. Here the Priorate converges with certainty to the office-holders’ group mean.
(b) Majority first-past-the-post voting among random conversation partners: The first connected subgroup of majority size (here, five) votes its subgroup position through. Here, on average, the office-holders’ group mean will be attained, but with some sample variation.
(c) Majority first-past-the-post voting among spatially contiguous (“similar”) conversation partners: Here subgroup means could differ substantially from global means; hence the variance in outcomes will be large. Assuming no bias in who speaks with whom first, however, sample means will still on average equal the global office-holders’ mean.
(d) Majority competition-among-subgroups voting among random conversation partners: Here two or more subgroup positions emerge, as always. However, instead of the first one to reach majority winning, dyads continue talking, with individuals permitted to switch allegiances (“change their minds”) if another subgroup’s mean is closer to them than is their current subgroup’s mean. It is conceivable to me (although I need to check further) that, under this procedural regime, some spatial configurations of office-holders could lead to perpetual cycling. That is, there will always be someone who is interested in switching.
(e) Majority competition-among-subgroups voting among spatially contiguous (“similar”) conversation partners: Again I am not sure of the outcome in this case, but perpetual cycling, if it exists, would in this case appear like a rotating fan. In all such cycling cases, the Priorate’s final decision is entirely determined by when exactly the vote is called.

A little example of the last procedural method might aid intuition. For the uniformly distributed 2-dimensional case, a 9-person Priorate might look like this:

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While I cannot get my graphics perfect, the x’s define the mean of the subgroups.
At t=6, there is an attempt to get the middle office-holder to defect, but it fails. However, the second try for defection at t=10 succeeds, and the lower bloc’s position carries, 6 to 3. [When cross-group defection attempts are made, the potential defectee calculates subgroup means, both ways, with his own participation included.] In this particular example, there are no cycles. What the example does illustrate, however, is that majority voting can easily lead even consensual Priorates to outcomes different from the global Priorate mean (at least temporarily).29

To add partisanship to this conversational framework is simple. Partisanship means a promise to vote for the other, in all circumstances, both elections and councils. A one-way partisanship tie (\(\rightarrow\)), i.e., clientage, between two members of the Priorate in effect gives the patron two votes. A two-way alliance (\(\leftrightarrow\)) between two members of the Priorate leads to compromise within their dyad, like any conversation. The difference is that alliance compromises are defection-proof; allies will stick together in a subgroup, no matter what. When two partisans are both on the council, operationalizing the effect of partisanship within the Priorate thus seems very straightforward.

Trickier to specify is the effect of indirect partisan ties—one or more clients of an offstage patron, for example. For one-step ties, where a direct promise has been made, it also seems straightforward to follow the same rules as above: namely, the one-way client votes the off-stage patron’s position; two-way allies consult, and the office-holder votes the joint position. The less obvious aspect of indirect ties is how to handle transitivity. Should partisan office-holders care about fellow partisans two, three, four steps up the chain? Since I think that the answer to this question is unclear, it seems safest as a modeling strategy simply to develop two variants: one variant with no transitivity, a second variant with discounted Priorate-voting transitivity, along the partisan-electoral lines (e.g., \(v^0, f^0\)) treated above.

The above diagram shows that majority voting, with no partisans, can lead to Priorate outcomes at variance with the Priorate mean. On average, however, over many votes, non-partisan Priorates will produce unbiased results, with mean of sample means equaling the global mean. When partisans are present on council, however, unbiased voting no longer holds. This is obvious in the extreme case: if five partisans are on a nine-person Priorate, they will dominate with certainty. The seriousness of the bias in settings less extreme than this requires more precise quantitative analysis.

29 I strongly suspect (but am not sure), however, that winning coalitions, in non-partisan conversational Priorates, will always include the median voter.
How do voters respond to all of this? Partisan voters are already locked in, through patronage, to a number of electoral vote commitments. That still leaves, however, many non-partisan “neutrals,” who cast their electoral votes on the basis of public policy—that is, on the basis of how well politicians represent voter interests in council.

This neutral-voting idea can be expressed in the same “rule of reciprocity” learning framework that I have used throughout these memos. As was done in the previous section, I will first just state this learning rule, and then explain it.

\[
p(v_{ij}) = \frac{[\xi/d_{ij} + \sum_k \chi_{ijkt}]}{\sum_j [\xi/d_{ij} + \sum_k \chi_{ijkt}]},
\]

as long as \([\xi/d_{ij} + \sum_k \chi_{ijkt}] \geq c_v\). (Otherwise, \(p(v_{ij}) = 0\).)

Here \(d_{ij}\) is the Euclidean\(^{30}\) spatial distance from \(i\) to \(j\), in \(k\)-dimensional attribute space.

\(\chi_{ijkt}\) are “policy favors,” to be explained below.

\(j'\) are candidates who have actually been on the Priorate in the past (and hence “have a record” to run on).

For neutral voters, up to a maximum of \(K\) votes can be allocated through this mechanism. For partisan voters, up to a maximum of \((K - \text{number of partisan commitments})\) can be allocated through this mechanism.

\(d_{ij}\) is the underlying spatial distance between voter \(i\) and candidate \(j\) in “group interests,” as defined concretely by the six attributes above (\(popolani\) vs. new men, etc.). “Policy favors” will modify this baseline, but I start off with the \(k\)-dimensional spatial-voting foundation familiar to political scientists. Neutral voters are thus anchored in group interests; partisan voters are anchored in personalistic networks. Since voters in the conjoined model can participate in both modes of behavior, it will be the interaction between networks and group-attributes that channels (and perhaps locks in) the dynamic movement of the aggregate voting “cloud” through families in Florence’s group-attribute space. Producing tippings or realignments, under various environmental perturbations, is one of the major goals of this modeling exercise.

The \(\chi_{ijkt}\)’s are “policy favors” that office-holders can produce, through Priorate decision making. I use the word “favors” to maintain formal symmetry with the partisan model; but really what these are are public policy decisions that voters approve or disapprove of, from their own \(k\)-dimensional group perspective. Through “policy favors,” current and past office-holders can build up voting strength even among groups far distant from themselves in attribute space.

\(^{30}\) If there are empirical problems with comparing distances across dimensions, then \(\sum_k (\xi_k/d_{ijk})\) could be substituted here. But this is a practical, not a theoretical, issue.
Within the conversational model of the Priorate, there are two ways to operationalize “policy favors.” If public policy is conceived of as a public good, then all majority coalition members of the Priorate get equal credit (or opprobrium) for the Priorate policy decision. If on the other hand public policy is conceived of as a summation, which mathematically it is, then majority coalition members of the Priorate get credit (or opprobrium) to the degree of their marginal contribution\(^{31}\) to the majority coalition outcome. Since the latter evaluative procedure demands a high degree of sophistication in voters, I choose the first, simpler specification—that is, winning coalition members are all seen as the same in the eyes of voters (at least for that one bill). This choice is also more consistent with the humanistic ideal of “collective discourse,” in which all are held responsible.

Given this, for each of the \(k\) dimensions of a successfully passed bill, and for all the \(j\)' members of the winning office-holder coalition\(^{32}\) at \(t\),

\[
x_{ijkt} = \left(\left|x_{j|k} - x_{ik}\right|\right)^{-1}.
\]

\(x_{j|k}\) is the Priorate policy decision at \(t\), on the \(k\)-th dimension; \(x_{ik}\) is voter \(i\)'s own group interest, on the \(k\)-th dimension. Since voters prefer low, not high, spatial distance between themselves and public policy, I crank spatial distance through the inversion function.\(^{33}\)

Three obvious points are worth underlining about the aggregation of policy favors in the neutral voters’ learning rule: (a) Only office-holders can generate policy favors. To get there in the first place requires some mixture of patronage and group-interest voting support. But once there, office-holders can build an “incumbency advantage” among not-so-obvious supporters, through their coalitional performance in office. [Of course, they could alienate supporters as well.] (b) The effect on voters of the policy favors of any individual \(j'\) is relative to that of the other \(j'\)'s. In other words, there is candidate competition at the level of policy, as well as at the level of patronage. However, to the extent that Priorate coalitions are stable over time, \(j'\) effects could become intercorrelated. A high enough level of intercorrelation we might be willing to call a “political party.” (c) And also just like the partisan-voting case, favors are only remembered for \(t\) time periods. Voter memories could be long or short, but whatever they are, they are finite. At the very least, \(t\) will affect speed of system adaptation.

\(c_v\) is the cost of voting, analogous to \(c_p\) the cost of partisanship. Unlike the cost of partisanship, I assume the cost of voting is quite low: \(c_v << c_p\). I don’t assume \(c_v = 0\), however, because I presume there is some spatial distance beyond which voters simply don’t care.

\(^{31}\) That is, the difference between the winning coalition means with and without their membership in it.

\(^{32}\) \(\{j'\}\) is just my notation for the set of all members of the winning coalition in the Priorate.

\(^{33}\) Obviously other monotonically declining functional forms could be substituted here. To avoid dividing by zero, probably all of the inversion formulas in this memo should be rescaled to an origin of +1. That is, redefine all “\(d_{ij}\)” in the formulas as \(d_{ij} + 1\).
So we have office-holders forming coalitions in council, through means of their conversations, and we have policy-oriented voters responding to those office-holders actions. Hence, a feedback loop shaping the future population of office-holders. I explicitly do not posit office-holders strategically voting in council in order to maximize votes. I regard that modernist assumption as inconsistent with the civic humanist ideal of collective discourse. Moreover, normal Florentine politics was more elitist than modern democratic theory presumes. The assumption of “no behavioral maximization” notwithstanding, however, it is altogether possible that this adaptive-cum-evolutionary system might grope its way towards “as if” vote maximization. Only further analysis will tell whether or not this is true.

FITTING THE TWO SIDES INTO THE FLORENTINE CONSTITUTION

With that, sketching the formal models of partisan networks and of the Priorate is complete. The main ambiguity remaining is how exactly to fit these dynamics into the constitutional framework outlined in the first section.

The real Florentine republican state was a bewildering array of participatory councils, built on the electoral foundation of guild x neighborhood described at the outset. To make any headway at all, I drastically simplified all these councils into neighborhood councils and the Priorate, both built on neighborhood electoral foundations. This background complexity is, however, worth bearing in mind in the model’s specification of patronage flows. Voters both in the model and in Florentine reality chose from among themselves, on a neighborhood by neighborhood basis, office-holders for gonfaloni and for the Priorate. For example, only San Giovanni voters voted for the San Giovanni neighborhood council and for the two San Giovanni slots on the Priorate. Patronage, however, in reality flowed across neighborhood boundaries, because of the dizzying array of actual councils. Patronage networks functioned, indeed, to give voters access to favors from councils far removed from themselves.

How should this reality be reflected in the simplified confines of the model? Even though it was not true for real gonfaloni, I will let neighborhood councils in my model distribute patronage favors to any voter in the city, through neighborhood-based office-holders. As such, “neighborhood councils” in the model stand in for a much more complicated array of real councils. In such cross-neighborhood patronage flows, it is the broker, not the ultimate giver, who benefits electorally. Thus, such cross-neighborhood patronage flows will tend to emerge through office-holders only if they are reciprocated. I expect this model to behave first by building up neighborhood-based patronage networks, and only then gradually to link across neighborhoods. A potentially fascinating interaction between partisan and neutral voting would be if Priorate coalitions, based on humanist conversation, turned out to be the catalyst for developing cross-neighborhood alliances for patronage reciprocation.

In all this, we shall see.

34 The ninth “chairman of the board” slot was elected by a more complicated city-wide and elite procedure, which I will not go into here.
Bibliography