Incomplete Markets, Borrowing Constraints, and the Foreign Exchange Risk Premium

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SFI WORKING PAPER: 1998-06-050

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January 15, 1998

Abstract

A large body of literature documents that returns from currency speculation are highly volatile and possess a predictable component, which is itself highly volatile and serially correlated. Explaining the returns from currency speculation through the presence of a risk premium has proven difficult, however. In particular, models with complete markets and time-separable preferences generate risk premia that are nearly constant. This paper solves a model consisting of two monetary economies with incomplete markets, in which agents are subject to borrowing constraints. The paper investigates if such a framework is able to account for the volatility and the size of the foreign exchange risk premium. Under very restrictive borrowing constraints, the model succeeds in increasing substantially the volatility of the risk premium and generates predictable excess returns, although not sufficiently large to match the data. It thus appears unlikely that excess returns from currency speculation can be uniquely explained by a time-varying risk premium in an incomplete-markets economy.

* I thank Anthony Smith, David Bowman, Luca Dedola, Wouter den Haan, John Knowles, Per Krusell, Kevin Moran, and Alan Stockman for helpful discussions. John Miller kindly provided his Sun workstation at Carnegie-Mellon University. This paper was written in part at the Board of Governors of the Federal Reserve System. Support from the Social Sciences and Humanities Research Council of Canada is also gratefully acknowledged.

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A well-known fact in international economics is that forward exchange rates are biased predictors of expected future spot rates and that there are consequently predictable excess returns from currency speculation. A large body of literature documents that these predictable expected returns, although small, are highly volatile and serially correlated. Two main approaches have been offered to explain this feature of the data, but, to date, no consensus has emerged. The first approach assumes that agents are risk-neutral and explains the bias by systematic forecast errors on the part of the traders (Lewis (1989); Frankel and Froot (1987); Tornell and Gourinchas (1996)).[1] The second avenue retains the assumption of agents' rationality and explains the expected excess returns by the presence of a time-varying risk premium. This approach has had only limited success. Specifically, Arrow-Debreu economies composed of a moderately risk-averse representative agent with time-separable preferences generate risk premiums that have nearly no variance (see Macklem (1991); Engle (1992); Bekaert (1994)).[2] Their failure stems mainly from their inability to generate enough variability in an agent's intertemporal marginal rate of substitution (IMRS). In general, this lack of variability leads the models to fail the test proposed by Hansen and Jagannathan (1991), in which the ratio of the standard deviation of the IMRS to its expected value has to be greater than the estimated Sharpe ratio[3] of any zero net investment portfolio. For instance, Bekaert (1994) shows that a coefficient of relative risk aversion of at least fifty is necessary for his complete-markets framework to pass the Hansen-Jagannathan test.

This paper investigates whether the presence of undiversifiable risks, in a general equilibrium two-country monetary model in which markets are incomplete and agents face borrowing constraints, can explain the forward discount puzzle. The inability to insure fully against idiosyncratic risk implies that the agent's

[1]Some authors have also proposed bulding models with artificially adaptive agents in order to make sense of the behavior of financial markets (e.g., Sargent (1993) and Marimon (1996)). See Arthur et al. (1996) for an application to the stock market and Dittmar et al. (1997) for an application to the foreign exchange market.

[2]Empirical tests of general equilibrium models with complete markets have also been unsuccessful in uncovering a time-varying risk premium (see Hansen and Hodrick (1983); Hodrick and Srivastava (1984); Mark (1985); Domowitz and Hakkio (1985); Cumby (1988); Kaminsky and Peruga (1990)). Hodrick (1987) and Engle (1995) survey the empirical literature.

[3]The Sharpe ratio is the ratio of the expected return to the standard deviation of the return, $\frac{E(r)}{\sigma(r)}$. The ratio of the standard deviation of the IMRS to its expected value, $\frac{\sigma(IMRS)}{E(IMRS)}$, is also called the market price of risk.

IMRS becomes more volatile. The model incorporates two endowment economies composed of a continuum of (types) of infinitely-lived agents relying on their savings in their home currency and in the two countries' bond to insure themselves against bad times. The agents face both aggregate uncertainty, in the form of aggregate income and money growth rate shocks, and idiosyncratic income shocks. Although the agents can trade assets among themselves to smooth their consumption, the presence of borrowing constraints limits their ability to do so.

Comparing frameworks with and without idiosyncratic risk, the paper shows that although introducing idiosyncratic uncertainty significantly increases the standard deviation of the risk premium, the model still fails to produce volatile enough risk premium to match the data. As a result, excess returns are predictable, but not to the extent one sees empirically. The results are best understood by noticing that, notwithstanding the model's ability to increase the market price of risk in the economy to 20% of that estimated in the data, substantially more risk would need to be introduced to account for the statistical properties of the risk premium.

Recently, other papers have followed different approaches to resolve the puzzle through the presence of a risk premium. Compared to the standard framework, these papers generally have more volatile risk premiums, but they are unable to replicate the volatility that the risk premium shows in the data. This paper reaches a similar conclusion. In particular, Canova and Marrinan (1993) generate more volatile risk premiums by introducing exogenous shocks that follow GARCH processes in a Lucas (1982) model. However, the larger variation in the risk premium in their paper is due to an increase in the variance of the convexity term,[4] which is believed to be empirically small. In an attempt to increase the variability of the IMRS, some studies have introduced time-nonseparable preferences. Backus et al. (1993) show that habit persistence raises the standard deviation of the risk premium. However, their result comes at the cost of generating negatively autocorrelated forward premiums, which are highly positively autocorrelated in the data. By introducing both habit persistence and consumption durability, Bekaert (1996) succeeds in increasing the variance of the risk premium without generating an unrealistic autocorrelation of the forward premium. Bekaert et al. (1994) show that although allowing for preferences that exhibit first-order

[4]The convexity term is due to Jensen's inequality. Since expected profits can be measured in terms of both currencies, expected profits must exist, at least in terms of one of the two currencies, even if the agents are risk neutral.

risk aversion increases the standard deviation of the risk premium, their model still fails to produce empirically plausible risk premium volatility. Finally, Sibert (1997), using an overlapping-generation model, demonstrates that contrary to the these previous studies, habit persistence has nearly no impact on the variance of the risk premium in her framework.

This paper differs from the above frameworks by departing from the complete-markets framework and by assuming the presence of borrowing constraints. The main idea is that incomplete markets and borrowing constraints increase the variability of the IMRS and may lead to a more volatile risk premium. Moreover, contrary to Backus et al. (1993) and Macklem (1991), the paper does not take prices to be a given random variable. In particular, prices are determined by the interaction of the agent's decisions in the home and the foreign country. The difficulty associated with an incomplete-markets framework is that the distribution of wealth matters for the determination of these prices. Typically, the cross-sectional distribution of agents' characteristics is a high dimensional object that is part of the set of state variables. In this regard, the paper provides an algorithm to solve international monetary models with incomplete markets. The algorithm adapts the work of Krusell and Smith (1995; 1997), in which the distribution of wealth is approximated by a function of the state variables, to an international monetary environment.

This line of research has similarities with the literature on the equity premium puzzle and the risk-free rate puzzle. The low variability of the IMRS is a central difficulty in explaining the behavior of both the stock and foreign exchange markets. However, the puzzle in international finance is not so much the low mean of the expected excess return from currency speculation but rather its high variance. To explain the equity premium puzzle, Mehra and Prescott (1985) demonstrate that the representative-agent framework with time-separable preferences, complete markets, and frictionless trade needs an unrealistically high level of risk aversion. A long list of papers relaxed the assumption of complete markets (see Aiyagari (1994); Aiyagari and Gertler (1991); Constantinides and Duffie (1996); den Haan (1994); Heaton and Lucas (1996); Huggett (1993); Lucas (1994); Krusell and Smith (1995, 1997); Telmer (1993); Storesletten et al. (1997)) and show that when idiosyncratic shocks are transitory, agents are able to smooth their consumption well enough with a small set of assets that the hardships from lack of insurance are small and, consequently that the effects on asset prices are only minor. However, this literature also demonstrates that severe restrictions on borrowing[5] and the persistence of idiosyncratic shocks have important effects on equilibrium asset prices. When these factors are taken into account, Lucas (1994), Heaton and Lucas (1996), Krusell and Smith (1997), and Storesletten et al. (1997) show that the models can generate significantly more suffering from idiosyncratic risks. In particular, this leads to a large increase in the market price of risk in the model, which is a key determinant of the equity premium. As previously mentioned, the present model shows that when borrowing constraints are very restrictive, the behavior of the risk premium differs significantly from that of a complete-markets framework, although its variance still remains lower than that in the data. The model also demonstrates that the introduction of idiosyncratic risk has a large impact on the market price of risk. In that sense, the result parallels and increments those of the equity premium literature. How severe the borrowing constraints are and what fraction of the population is credit constrained remain open empirical questions, however.[6]

The reminder of the paper is organized as follows. Section 1 presents some statistical properties of the data and computes an estimate of the risk premium. The model is described in Section 2, while sections 3 and 4 describe the numerical method and the parameterization of the model, respectively. Section 5 reports the results on the risk premium, the forward premium, and the exchange rate. Section 6 concludes.

1. Data and Definitions

This section computes a measure of expected returns from currency speculation and examines the statistical properties of exchange rates for Canada, France, Italy, Japan, the UK, and the US.

Define e_t as the log of the spot exchange rate at time t and f_t as the log of the one-period ahead forward exchange rate at time t, both expressed as units of domestic currency (US dollars) per unit of foreign currency. Suppose an investor takes a long position in a foreign currency at time t. He buys the foreign currency forward at time t at a price f_t, for delivery at time $t+1$. At time $t+1$, he sells the foreign currency on the spot market at a price e_{t+1}. His realized profit from trading in the foreign exchange market is therefore $e_{t+1} - f_t$. The investor's

[5] For instance, Heaton and Lucas (1996) show that the equity premium reaches empirically plausible values when there are transaction costs in the stock markets and no bond borrowing.

[6] See for instance Zeldes (1989), Flavin (1991), Eun Young et al. (1991) and Shea (1995).

expected profit from currency speculation is then $E_t(e_{t+1}) - f_t$, where E_t is the expectation conditional on all information available at time t. Define the risk premium at time t, rp_t, as the expected return from currency speculation:

$$rp_t \equiv E_t(e_{t+1}) - f_t. \tag{1.1}$$

If agents are risk-neutral and have rational expectations, the expected return from currency speculation should be driven to zero: the forward rate, f_t, should be an unbiased predictor of the expected future spot rate, $E_t(e_{t+1})$.[7]

Another way to see this is through the concepts of uncovered and covered interest parity. The uncovered interest parity says that the return, i_t, on a unit of domestic currency invested in a domestic deposit should equal the expected return from converting this unit of currency into foreign units at the price, e_t, investing it in a foreign deposit earning i_t^*, and converting the proceeds back into domestic currency at time $t+1$ at price e_{t+1} :

$$E_t(e_{t+1}) - e_t = i_t - i_t^*. \tag{1.2}$$

Covered interest parity, on the other hand, states that the return, i_t, on a unit of domestic currency invested in a domestic deposit should equal the return from converting this unit of currency into foreign units at the price, e_t, investing it in a foreign deposit earning, i_t^*, and converting the proceeds forward back into domestic currency at time t at price f_t :

$$f_t - e_t = i_t - i_t^*. \tag{1.3}$$

Substituting 1.2 into 1.3 one gets that the forward rate is an unbiased predictor of expected future spot rates:

$$E_t(e_{t+1}) - f_t = 0.$$

However, if agents are risk-averse and some risks in the economy is undiversifiable, they will demand a premium in order to hold a riskier currency. In particular, $E_t(e_{t+1}) - f_t < 0$ implies that the agent, by buying the foreign currency forward at time t, pays a premium relative to the foreign currency's expected future value.

[7]There will still be nonzero expected nominal returns if the exchange rate covaries with the price level, even though investors are risk neutral (see Engle (1984)). Cumby (1988) provides evidence rejecting this explanation for the failure of the unbiasedness hypothesis.

A risk premium can therefore account, in principle, for the departures from the unbiasedness hypothesis.

Following Cumby (1988), an empirical measure of the risk premium is constructed by regressing the realized returns from currency speculation, $e_{t+1} - f_t$, on a constant and the forward premium at time t, $f_t - e_t$:

$$e_{t+1} - f_t = \alpha_1 + \beta_1(f_t - e_t) + u_{t+1} \tag{1.4}$$

As long as expectations are rational, this is a valid procedure. The idea is that in large samples realized profits from currency speculation should be unforecastable. Table 1 reports the results for five countries for the period covering 1974:12 to 1996:10. The data is taken from the International Finance data base of the Board of Governors of the Federal Reserve System and the exchange rates are US dollars per unit of foreign currency.

Table 1. OLS Regression:$e_{t+1} - f_t = \alpha_1 + \beta_1(f_t - e_t) + u_{t+1}$

	Canada	France	Italy	Japan	UK
$\widehat{\alpha_1}$	-0.0028	-0.0007	-0.0020	0.0096	-0.0047
	(0.0009)	(0.0020)	(0.0028)	(0.0024)	(0.0022)
$\widehat{\beta_1}$	-1.4593	-1.0907	-0.9470	-1.7757	-1.5572
	(0.1609)	(0.1984)	(0.1445)	(0.2264)	(0.2212)
R^2	0.2403	0.1042	0.1420	0.1913	0.1601

As most empirical studies found, estimates of β_1 are for the most part smaller than -1, except for the Italian Lira which has an estimate of -0.9470. Using yen, mark, and pound exchange rates against the US dollar, McCallum (1994) reports an average value for $\widehat{\beta_1}$ of -5, significantly lower than the estimates found here.[8] Nonzero estimates of β_1 imply that returns from currency speculation have a predictable component. Table 2 reports the fitted values of the regression which serves as an estimate of the risk premium.

[8] Mayfield and Murphy (1992), however, find opposite results. Using a similar test as the one above while allowing for common movements in risk premia across currencies, they do not reject the null hypothesis that $\beta_1 = 1$, for the French franc, the Swiss franc, and the german mark (relative to the US dollar) estimated simultaneously.

Table 2: Statistical Properties of the Risk Premium.

Statistical Properties of the Risk Premium					
	Canada	France	Italy	Japan	UK
Mean	0.0023	0.0059	0.0146	-0.0029	0.0052
Standard Deviation	0.0057	0.0089	0.0105	0.0136	0.0116
Autocorrelation	0.917	0.864	0.879	0.952	0.942

Table 2 shows that, for all currencies, the mean of the risk premium is close to zero. On the other hand, the risk premium is highly volatile: from 0.57 percent per month for the Canadian dollar to about 1 percent per month for the yen. The risk premium is also highly persistent, being as high as 0.952 for the Japanese yen.

The results in Table 1 also have implications for (i) the variance of the risk premium relative to the variance of the expected rate of depreciation and (ii) the covariance between the risk premium and the expected rate of depreciation. Consider the following equation:

$$e_{t+1} - e_t = \alpha_2 + \beta_2(f_t - e_t) + v_{t+1} \tag{1.5}$$

Equation (1.5) is equivalent to equation (1.4) where $\beta_2 = 1 + \beta_1$ and $\alpha_2 = \alpha_1$. The results from Table 1 therefore implies that $\widehat{\beta_2}$ is less than zero. Using the algebra of least squares, Fama (1984) shows that: (i) the variance of the risk premium must be greater than the variance of the expected rate of depreciation, and (ii) the expected rate of depreciation and the risk premium must be negatively correlated. This result demonstrated that a risk premium, although small in size, may be important in understanding the departure from the unbiasedness hypothesis.

Using the covered interest parity, $f_t - e_t = i_t - i_t^*$, a finding that $\widehat{\beta_2}$ is less than zero is moreover regarded as a rejection of the uncovered interest parity relationship.[9] Specifically, a finding that $\widehat{\beta_2}$ is negative means that a positive interest differential is matched by an expected currency appreciation.

Table 3 summarizes other features of the data. One such feature is the high autocorrelation of forward premiums. For instance, the autocorrelation of the pound was as high as 0.928 per month. In comparison, the autocorrelation of the depreciation rates and the forecast errors are much smaller.

[9] See McCallum (1994) for an opposite view.

Table 3: Summary Statistics

Panel A Forward Premium					
	Canada	France	Italy	Japan	UK
Mean	-0.0035	-0.0061	-0.0156	0.0070	-0.0066
Standard Deviation	0.0039	0.0082	0.0112	0.0076	0.0075
Autocorrelation	0.917	0.876	0.879	0.952	0.943
Panel B Realized Returns from Currency Speculation					
Mean	0.0024	0.0128	0.0128	0.0021	0.0052
Standard Deviation	0.0116	0.0281	0.0281	0.0315	0.0289
Autocorrelation	0.316	0.455	0.455	0.451	0.459
Panel C Depreciation Rate					
Mean	-0.0011	-0.0029	-0.0029	0.0041	-0.0011
Standard Deviation	0.0101	0.0261	0.0261	0.0285	0.0268
Autocorrelation	0.186	0.403	0.403	0.349	0.382

Moreover, Table 3 shows that the depreciation rates are significantly more variable than forward premiums, casting doubt on the power of forward discounts in explaining future changes in the spot exchange rates. Table 2 and Table 3 also highlights that, for all the exchange rates studied, the depreciation rate is more volatile than the risk premium, which is itself more volatile than the forward premium. Bekaert (1996) labeled this regularity the volatility puzzle.

In summary: there are departures from the unbiasedness hypothesis; the expected excess returns from currency speculation are predictable, highly variable, and more volatile than the forward premium; the forward premium is highly autocorrelated; and exchange rates are highly volatile. The next section develops a model to explain these facts.

2. Economic Environment

2.1. Preferences and Forcing Processes

There are two countries, the home and the foreign, in which markets are incomplete. Ex-ante identical agents possesses only three assets to smooth their consumption: their country's money and two one-period riskless bonds. There is only one good which is perfectly traded on the world's market. In period t, residents of the home country are endowed with ξ units of commodity.[10] ξ can take on two values, ξ^h and ξ^l, denoting a high and a low income level, respectively. It is assumed that the probability of receiving a good or bad income shock depends on the aggregate income shock, z, which is assumed to follow a two-state Markov process. The probability of receiving a low endowment, given the aggregate income shock, is also assumed to be dependent on the previous realization of the idiosyncratic shock. The large number of infinitely-lived agents are risk-averse and care about their real money balances. They wish to maximize:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U_i(c_{i,t}, \frac{m_{i,t+1}}{p_t}) \right\}, \; 0 < \beta < 1, \tag{2.1}$$

where $c_{i,t}$ is the consumption at date t, by home agent i of the good, $m_{i,t+1}$ is the amount of domestic currency held from period t to $t+1$ by agent i, and p_t is the price level in terms of the good.[11] The home and foreign currencies are denoted by $\overline{M}$ and $\overline{N}$, respectively. Monetary policies evolve according to the following processes:

$$\overline{M}' = (1+g)\overline{M} \tag{2.2}$$

$$\overline{N}' = (1+g^*)\overline{N}, \tag{2.3}$$

where g and g^* are the stochastic money growth rates, which are also assume to follow two-state Markov processes. In order to smooth their consumption, agents trade nominal one-period riskless bonds denominated in home and foreign currencies, b and b^*, which promise to pay one unit of home and foreign currency,

[10] The structure of the foreign economy is identical to the home economy. Its full description in the text is thus omitted.

[11] For the reminder of the paper, the time subscript t and the agent i subscript will be dropped. A prime superscript will denote a time $t+1$ variable.

respectively. Monetary transfers, $g\overline{M}$ and $g^*\overline{N}$, are distributed in a lump sum manner to national residents. Claims to future monetary transfers or claims to future output are not traded.

The variable e will denote the nominal exchange rate at date t, expressed in units of foreign currency. Since the consumption good is perfectly traded and there are no transport costs, the law of one price holds:

$$e = \frac{p}{p^*}. \tag{2.4}$$

2.2. The Agent's Problem

Each agent in the home country faces the following budget constraint:

$$c + qb^{'} + q^*b^{*'} + \frac{m'}{p} \leq \omega \tag{2.5}$$

where b', $b^{*'}$, m' are the new holdings, at time t, by an home resident, of home bonds, foreign bonds, and home currency, respectively. The real prices of home and foreign bonds are given by $q = p_b/p$ and $q^* = ep_b^*/p$, where p_b and p_b^* are the home and foreign nominal bond's price. The wealth of agent i at time t is represented by ω, which follows the law of motion given by:

$$\omega' = \xi' + \frac{b'}{p^{'}} + \frac{e^{'}}{p^{'}}b^{*'} + \frac{m'}{p^{'}} + \frac{g'\overline{M}}{p^{'}}, \tag{2.6}$$

An agent's wealth at time is the sum of his endowment of the good, his return on his previous period bond and money holdings, and a lump-sum government transfer.

Agents also face the following borrowing constraints:

$$b^{'} \geq \overline{b}, \tag{2.7}$$

$$b^{*'} \geq \overline{b^*}, \tag{2.8}$$

where $\overline{b}$ and $\overline{b^*}$ are either negative or zero.

The aggregate state of the world is given by the aggregate income shocks, z_t and z_t^*, the monetary shocks, g_t and g_t^*, and by the joint distribution (across countries) of agents over their individual wealth and employment status, Γ_t. The agent's wealth and his current employment status represent his individual state variable. We let $s = (z, z^*, g, g^*)$. The aggregate laws of motion are given by the Markov chains for the aggregate income shocks and the money growth rate shocks and by an endogenous function H: $\Gamma^{'} = H(\Gamma, s)$. $P(\Gamma, s)$, $P^*(\Gamma, s)$, $q(\Gamma, s)$, and $q^*(\Gamma, s)$ represent the equilibrium pricing functions for the good and for the bonds.

The dynamic programing problem thus becomes:

$$V(\omega, \xi; \Gamma, s) = \max_{c, b^{'}, b^{*'}, m^{'}} \left\{ U(c_i, \frac{m'}{P(\Gamma, s)}) + \beta E[V(\omega^{'}, \xi^{'}; \Gamma^{'}, s^{'}) | \xi, s] \right\} \tag{2.9}$$

$$\text{subject to } c + q(\Gamma, s)b^{'} + q^*(\Gamma, s)b^{*'} + \frac{m'}{P(\Gamma, s)} \leq \omega \tag{2.10}$$

$$\omega' = \xi' + \frac{b'}{P(\Gamma', s')} + \frac{e^{'}(\Gamma^{'}, s^{'})}{P(\Gamma^{'}, s^{'})} b^{*'} + \frac{m'}{P(\Gamma', s')} + \frac{g'\overline{M}}{P(\Gamma', s')} \tag{2.11}$$

$$b^{'} \geq \overline{b} \tag{2.12}$$

$$b^{*'} \geq \overline{b^*} \tag{2.13}$$

$$\Gamma' = H(\Gamma, s, s') \tag{2.14}$$

2.3. Equilibrium

2.3.1. Definition

A recursive equilibrium consists of: a set of individual decision rules and value functions, $B(\omega, \xi; \Gamma, s)$, $B^*(\omega, \xi; \Gamma, s)$, $M(\omega, \xi; \Gamma, s)$, and $V(\omega, \xi; \Gamma, s)$;a set of pricing functions $P(\Gamma, s)$, $P^*(\Gamma, s)$, $q(\Gamma, s)$ and $q^*(\Gamma, s)$; and a law of motion H, such that *(i)* $B(\omega, \xi; \Gamma, s)$, $B^*(\omega, \xi; \Gamma, s)$, $M(\omega, \xi; \Gamma, s)$, $V(\omega, \xi; \Gamma, s)$ solve the dynamic programing problem above, *(ii)* $B(\omega, \xi; \Gamma, s)$, $B^*(\omega, \xi; \Gamma, s)$, $M(\omega, \xi; \Gamma, s)$ are such that the bond and money markets clear:

$$\sum_{i=1} b_i = 0, \tag{2.15}$$

$$\sum_{i=1} b_i^* = 0, \tag{2.16}$$

$$\sum_i m_i' = \overline{M}, \tag{2.17}$$

and,

$$\sum_i n_i' = \overline{N}, \tag{2.18}$$

and the law of motion H is consistent with individual behavior.

2.3.2. Characterization

A solution to the agent's problem satisfies:

$$U_1(c, \frac{m'}{P(\Gamma, s)})q(\Gamma, s) \geq \beta E\left[U_1\left(c', \frac{m''}{P(\Gamma', s')}\right)\left(\frac{1}{P(\Gamma', s')}\right)|\xi, s\right], \tag{2.19}$$

$$U_1(c, \frac{m'}{P(\Gamma, s)})q^*(\Gamma, s) \geq \beta E\left[U_1\left(c', \frac{m''}{P(\Gamma', s')}\right)\left(\frac{e'(\Gamma', s')}{P(\Gamma', s')}\right)|\xi, s\right], \tag{2.20}$$

$$U_1\left(c, \frac{m'}{P(\Gamma, s)}\right)\left(\frac{1}{P(\Gamma, s)}\right) = U_2\left(c, \frac{m'}{P(\Gamma, s)}\right) + \beta E\left[U_1\left(c', \frac{m''}{P(\Gamma', s')}\right)\left(\frac{1}{P(\Gamma', s')}\right)|\xi, s\right], \tag{2.21}$$

$$b' \geq \overline{b}\,(\mu \geq 0), \tag{2.22}$$

$$b^{*'} \geq \overline{b^*}\,(\mu^* \geq 0), \tag{2.23}$$

where U_i denotes the derivative of the utility function with respect to the ith argument, and μ and $\mu^*$are the multipliers of equations (2.12) and (2.13), respectively. Equations (2.19) and (2.20) hold with equality when the agent is not at the borrowing constraints. In this event, the disutility of investing $q(\Gamma, s)$ and $q^*(\Gamma, s)$ in the home and foreign bonds this period should be equal to the expected discounted utility of next period's payoffs, at the margin. If the borrowing constraints are strickly binding however, these equations show that there is an upward pressure on bond prices, as the marginal utility of receiving $q(\Gamma, s)$ and $q^*(\Gamma, s)$ units of currency is greater than the expected discounted disutility of repaying the debt.

From equations (2.19) and (2.20) and the definition of q and q^*, we can derive the nominal home and foreign bond prices:

$$p_b \geq \beta E\left[\frac{U_1(c^{'}, \frac{m^{''}}{p^{'}})}{U_1(c, \frac{m^{'}}{p})}\left(\frac{p}{p^{'}}\right)|\xi, s\right], \tag{2.24}$$

and

$$p_b^* \geq \beta E\left[\frac{U_1(c^{'}, \frac{m^{''}}{p^{'}})}{U_1(c, \frac{m^{'}}{p})}\left(\frac{e^{'}}{e}\right)\left(\frac{p}{p^{'}}\right)|\xi, s\right]. \tag{2.25}$$

In order to derive an expression for the foreign exchange risk premium, one first needs to define the forward exchange rate. By covered interest parity, $\frac{f}{e} = \frac{p_b^*}{p_b}$, and the bonds' prices from equations 2.24 and 2.25, the forward price of foreign exchange must be given by:

$$f \lesseqgtr \frac{E\left[U_1(c^{'}, \frac{m^{''}}{p^{'}})(\ e^{'}\ /\ p^{'})|\xi, s\right]}{E\left[U_1(c^{'}, \frac{m^{''}}{p^{'}})(1\ /\ p^{'})\ |\xi, s\right]}. \tag{2.26}$$

Defining $\lambda = \frac{U_1(c^{'}, \frac{m^{''}}{p^{'}})}{U_1(c, \frac{m^{'}}{p})}\frac{p}{p^{'}}$ and using the definition of covariance, the risk premium, defined previously as the expected return from currency speculation, can therefore be expressed as:

$$rp \equiv E\left[\left(e^{'} - f\right)|\xi, s\right] \lesseqgtr -\frac{cov[\lambda; (e^{'} - f)|\xi, s] + E[\lambda(e^{'} - f)|\xi, s]}{E[\lambda|\xi, s]}. \tag{2.27}$$

The last term in the numerator will be zero if the credit constraints do not bind. In this case, since the IMRS is always expected to be positive, the above expression says that the risk premium is proportional to the covariance between the IMRS and the return from currency speculation. Since neither terms need to be time-invariant, the risk premium can vary over time and possibly explain the departures from the unbiasedness hypothesis. As previously mentioned, variations in the risk premium require that the IMRS be significantly volatile.

3. Solution Algorithm

This section describes the solution method. The reader who is only interested in the economic results can skip this section without any loss of continuity.

The main problem associated with heterogenous-agents models is that the wealth distribution matters for the determination of prices, i.e., it is part of the set of state variables. Since this distribution is a high-dimensional object, an approximation is needed to successfully solve these models numerically. Typically, authors have either assumed that there are only two types of agents in the economy or that there is a continuum of (types) of agents and no aggregate uncertainty. Without these assumptions, approximating the wealth distribution becomes more complicated. Basically, only two methods solve environments with a continuum of agents and aggregate uncertainty: parametrized expectations and the procedure proposed in Krusell and Smith (1995, 1997).

The algorithm adapts the method of Krusell and Smith (1997) to a monetary economy context. The method concentrates on finding stationary equilibria. The idea is to assume that agents perceive prices as depending on a limited set of moments I of the wealth distribution. Krusell and Smith (1995,1997) find that, in a one-sector neoclassical growth model, the mean of the distribution alone suffices to generate accurate approximations. This result is due to the similarity in the marginal propensities to save across different agents. The strategy will be to apply their result to the present framework. Here the mean of the distribution corresponds to the sum of the agents' endowment value and money supply in each country. Since there is no capital and since money is a veil in this economy, the prices in this context are simply given as functions of the aggregate income shocks and the money growth rate shocks: $\widetilde{q}(s)$, the home real bond pricing function, $\widetilde{q^*}(s)$, the foreign real bond pricing function, $\widetilde{P}(s)$, the home pricing function of the good, and $\widetilde{P^*}(s)$, the foreign pricing function of the good, where $s = (z, z^*, g, g^*)$. The algorithm approximates the four functions by:

$$\widetilde{P}(s) = a_j \text{ if } s = s_j, \; j = 1, 2..., J \tag{3.1}$$

$$\widetilde{P^*}(s) = c_j \text{ if } s = s_j, \; j = 1, 2..., J \tag{3.2}$$

$$\widetilde{q}(s) = d_j \text{ if } s = s_j, \; j = 1, 2..., J \tag{3.3}$$

$$\widetilde{q^*}(s) = k_j \text{ if } s = s_j, \; j = 1, 2..., J, \tag{3.4}$$

where J is the number of possible states.

Thus, the following problem is solved:

$$V(\omega; s) = \max_{c,b',b^{*\prime},m'} \left\{ U(c, \frac{m'}{\widetilde{P}(s)}) + \beta E\left[V(\omega^{'}; s^{'}) | \xi, s \right] \right\} \tag{3.5}$$

$$\text{subject to } c + \widetilde{q}(s)b^{'} + \widetilde{q^{*}}(s)b^{*'} + \frac{m'}{\widetilde{P}(s)} \leq \omega \tag{3.6}$$

$$\omega' = \xi' + \frac{b'}{\widetilde{P}(s')} + \frac{e(s^{'})}{\widetilde{P}(s')}b^{*'} + \frac{m'}{\widetilde{P}(s')} + \frac{g'\overline{M}}{\widetilde{P}(s')} \tag{3.7}$$

and subject to (3.1)-(3.4), (2.12), and (2.13), and where $e(s) = \frac{\widetilde{P}(s)}{\widetilde{P^{*}}(s)}$.

Step 1: Generate random shocks for z, z^{*}, g, g^{*}, ξ, and ξ^{*}

Step 2: Discretize the state space and restrict H to a finite set of moments H_I with chosen parameters. In the case here, the functions are given by equations (3.1)-(3.4).

Step 3: For each economy, solve (3.5) using value function iteration.

Step 4: In order to verify that the market clears use the value function from Step 3 and solve problem (3.8) below. That is fix prices, derive the optimal decision rules for all agents, and iterate on prices until the markets clear. Thus,

(a) Fix an initial wealth/employment distribution for a large number of agents and initial values for the aggregate shocks.

(b) Solve problem (3.8) and iterate on prices until all markets clear.

$$\widetilde{V}(\omega; s, \widehat{q}, \widehat{q^{*}}, \widehat{P}, \widehat{P^{*}}) = \max_{c,b',b^{*\prime},m'} \left\{ U(c, \frac{m'}{\widehat{P}}) + \beta E\left[V(\omega^{'}; s^{'}) | \xi, s \right] \right\} \tag{3.8}$$

$$\text{subject to } c + \widehat{q}b^{'} + \widehat{q^{*}}b^{*'} + \frac{m'}{\widehat{P}} \leq \omega \tag{3.9}$$

$$\omega' = \xi' + \frac{b'}{\widetilde{P}(s')} + \frac{e(s^{'})}{\widetilde{P}(s')}b^{*'} + \frac{m'}{\widetilde{P}(s')} + \frac{g'\overline{M}}{\widetilde{P}(s')} \tag{3.10}$$

and subject to (3.1)-(3.4), (2.12), and (2.13).

(c) Using the decision rules from (3.8) derive the new wealth/employment distribution. From the Markov processes get new aggregate shocks.

(d) Repeat the procedure for a large number of times, discarding the first part.

Step 5: For each state of the world, compute the mean and the variance of the simulated prices. If the mean of each simulated price series is close to the initial guess, given some convergence criteria, and the variance of the simulated series is small, then the algorithm has converged. Otherwise, update the initial guesses and go back to Step 3.

3.1. Algorithm for the Most Restrictive Constraints

This section describes the numerical method to solve the model when agents cannot borrow. This algorithm is simpler since in equilibrium nobody will hold bonds and a world economy in which the home and the foreign currencies are the only assets can be studied. In this case, it is sufficient for the algorithm to approximate the following pricing functions:

$$\widetilde{P}(s) = a_j \text{ if } s = s_j, j = 1, 2, ..., J \tag{3.11}$$

$$\widetilde{P^*}(s) = c_j \text{ if } s = s_j, j = 1, 2, ..., J \tag{3.12}$$

The following problem can thus be solved:

$$V(\omega; s) = \max_{c,m'} \left\{ U(c, \frac{m'}{\widetilde{P}(s)}) + \beta E \left[V(\omega^{'}; s^{'}) | \xi, s \right] \right\} \tag{3.13}$$

$$\text{subject to } \ c + \frac{m'}{\widetilde{P}(s)} \leq \omega \tag{3.14}$$

$$\omega' = \xi' + \frac{m'}{\widetilde{P}(s')} + \frac{g'\overline{M}}{\widetilde{P}(s')} \tag{3.15}$$

and to equations (3.11) and (3.12). The solution to this problem is then approximated using the procedure outlined in the previous section. Notice that the bond prices can be retrieved by computing the agents' subjective valuation of the bonds' payoffs, which are given by the agents' expected nominal intertemporal marginal rate of substitution. The highest subjective price determines the bond's price. At this price, all agents would like to borrow, except the one determining the bond price. This agent is therefore indifferent at zero bond holdings.

4. Parametric Specifications

This section chooses the parameters' values, selects the transition probabilities for the idiosyncratic shock process and estimates the processes for aggregate shocks.

4.1. Preferences

The utility function is assumed to be of the following form:

$$\frac{\left[\frac{c^{1-\phi}}{1-\phi}+b\frac{(\frac{m'}{p})^{\omega}}{\omega}\right]^{1-\sigma}}{1-\sigma} \tag{4.1}$$

Following Chari, Kehoe and McGrattan (1996), $\phi = 0.03$, $\omega = 0.97$ and $b = 0.025$. The discount rate is set to 0.9967, i.e., the rate of time preference is 0.33 percent per month. The paper reports the results for different coefficient of relative risk aversion.

4.2. Shocks

The aggregate income shocks are assumed to follow a VAR process of order one:

$$\begin{pmatrix} z_{t+1} \\ z_{t+1}^{*} \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} \begin{pmatrix} z_{t+1} \\ z_{t+1}^{*} \end{pmatrix} + \begin{pmatrix} e_{t+1} \\ e_{t+1}^{*} \end{pmatrix}. \tag{4.2}$$

The VAR process is then transformed to a two-state Markov process using the Tauchen and Hussey (1991) method. The VAR process is estimated with the variables in level using Canadian and US GDP. The data are quarterly and cover the period 1973:2 to 1996:2. Table 4 and 5 report the results of the estimation and the approximating Markov chain, respectively.

Table 4. Coefficient Estimates from the VAR Process

$\widehat{c_1}$	$\widehat{c_2}$	$\widehat{a_1}$	$\widehat{a_2}$	$\widehat{a_3}$	$\widehat{a_4}$
9.4356	27.4384	1.0267	-0.0089	0.1570	0.9531
(1.766)	(10.613)	(0.021)	(0.007)	(0.127)	(0.041)

Table 5. Transition Probability Matrix for the Aggregate Shocks

	$(z_g z_g^*)$	$(z_g z_b^*)$	$(z_b z_g^*)$	$(z_b z_b^*)$
$(z_g z_g^*)$	0.7637	0.1135	0.1069	0.0158
$(z_g z_b^*)$	0.0961	0.7797	0.0136	0.1106
$(z_b z_g^*)$	0.1106	0.0136	0.7797	0.0961
$(z_b z_b^*)$	0.0159	0.1069	0.1135	0.7637

The money growth rates are assumed to follow independent AR(1) processes:

$$g_{t+1} = \rho_h g_t + \varepsilon_{t+1} \tag{4.3}$$

$$g_{t+1}^* = \rho_f g_t^* + \varepsilon_{t+1}^*. \tag{4.4}$$

Table 6 reports the coefficients' estimate for the two processes, using US and Canadian M1s for the period 1973:2 to 1996:2.

Table 6. Coefficient Estimates from the Monetary Processes

$\widehat{\rho_h}$	$\widehat{\rho_f}$
0.8192	0.5368
(0.0601)	(0.0889)

The AR(1) processes are then transformed to two-state Markov chains using the Tauchen and Hussey (1991) method.

Table 7. Transition Probability Matrix for the Home Monetary Shock

	(g_h)	(g_l)
(g_h)	0.8373	0.1627
(g_l)	0.1627	0.8373

Table 8. Transition Probability Matrix for the Foreign Monetary Shock

	(g_h^*)	(g_l^*)
(g_h^*)	0.7453	0.2547
(g_l^*)	0.2547	0.7453

As in Imrohoroglu (1989), the transition probabilities are selected such that the US (Canadian) unemployment rate is 4 (6) percent in good times and 8 (10) percent in bad times; and the average duration of unemployment is 1.5 (1.6) in good times and 2.3 (2.5) in bad times.

5. Results

This section studies the case of no borrowing in the bond markets.[12]This framework will therefore provide an upper bound on the model's potential to explain the forward discount puzzle.

In order to put the results in perspective, it will first be useful to study the case in which there is no idiosyncratic risks. Notice that this is not equivalent to having complete markets however, as aggregate risks cannot be fully insured across countries. Table 9 reports the results for different level of risk aversion. It is obvious that incomplete markets *per se* cannot increase the volatility of the risk premium. For all level of relative risk aversion, the model generates risk premiums that are nearly constant. As a result, excess returns from currency speculation are unpredictable: the slope coefficient in a regression of the excess returns from currency speculation on the forward premium is zero for all cases. Moreover, although risk and forward premiums are autocorrelated in the model, the serial correlation is still insufficiently large to match the data. Overall, the results are similar to the ones of complete-markets frameworks of Macklem (1991) and Bekaert (1994).

[12]You can also think of this case as having a small amount of borrowing allowed, but for numerical simplicity we model it setting the borrowing constraints to zero.

Table 9. Sample Moments and Implied Moments from the Case with no Bond Borrowing and no Idiosyncratic Shocks

Moment	Data	Model		
		$\sigma = 1$	$\sigma = 2$	$\sigma = 3$
$E(rp)$	-0.0023	-4$\times 10^{-6}$	-5$\times 10^{-6}$	-5$\times 10^{-6}$
$\sigma(rp)$	0.0057	3$\times 10^{-6}$	4$\times 10^{-6}$	4$\times 10^{-6}$
$\rho(rp)$	0.917	0.5847	0.5834	0.5842
$E(fp)$	-0.0033	6$\times 10^{-4}$	0.0001	0.0001
$\sigma(fp)$	0.0039	0.0004	0.0004	0.0005
$\rho(fp)$	0.928	0.5108	0.5439	0.5540
$\sigma(e' - e)$	0.0101	0.0316	0.0316	0.0316
$\widehat{\beta_1}$	-1.4593	0.0004	0.0005	0.0007
	(0.1540)	(0.0002)	(0.0003)	(0.0002)
$\frac{\sigma(IMRS)}{E(IMRS)}$	0.2930	0.0002	0.0004	0.0005

rp=risk premium, fp=forward premium, $\widehat{\beta_1}$ is the estimated slope coefficient from $e' - f = \alpha_1 + \beta_1(f - e) + u'$, and standard errors are in parentheses.

Table 10 summarizes the case in which agents face both aggregate risks and idiosyncratic risk. Notice that the standard deviation of the risk premium increases significantly. With a coefficient of relative risk aversion of three, the standard deviation is now 0.03 percent compared to 0.05×10^{-2} percent when there is no idiosyncratic risk: an increase by approximately a factor of 100. The results are similar for different coefficients of relative risk aversion. However, although the standard deviation of the risk premium increases, it is still only 5 percent of that in the data. Backus et al. (1993) obtain similar results by introducing time-nonseparable preferences: with moderate habit persistence their risk premium is approximately 15 percent of the estimated risk premium volatility in the data.

Table 10. Sample Moments and Implied Moments from the Case with no Bond Borrowing

Moment	Data	Model		
		$\sigma = 1$	$\sigma = 2$	$\sigma = 3$
$E(rp)$	-0.0023	-0.0002	-0.0022	-0.0041
$\sigma(rp)$	0.0057	$8{\times}10^{-5}$	0.0001	0.0003
$\rho(rp)$	0.917	0.6912	0.7021	0.7489
$E(fp)$	-0.0033	-0.0020	-0.0072	0.0193
$\sigma(fp)$	0.0039	0.0063	0.0163	0.0301
$\rho(fp)$	0.928	0.7048	0.7706	0.7596
$\sigma(e' - e)$	0.0101	0.0322	0.0572	0.0785
$\widehat{\beta_1}$	-1.4593	-0.0775	-0.1140	-0.25
	(0.1540)	(0.0008)	(0.0023)	(0.0079)
$\frac{\sigma(IMRS)}{E(IMRS)}$	0.2930	0.0131	0.0372	0.0536

rp=risk premium, fp=forward premium, $\widehat{\beta_1}$ is the estimated slope coefficient from $e^{'} - f = \alpha_1 + \beta_1(f - e) + u^{'}$, and standard errors are in parentheses.

Note however that, contrary to a framework with habit persistence, the increase in the variance of the risk premium does not come at the cost of generating negatively autocorrelated forward premiums. In all cases, the forward premium has an autocorrelation of approximately 0.7, which is still lower than the serial correlation of 0.917 observed in the data.

Increases in the standard deviation of the risk premium materialize in more negative slope coefficient estimates. However, it is clear that the model still fails to match the slope coefficient, $\widehat{\beta_1}$, estimated from the data. Compared to a coefficient of -1.4593 the largest estimated slope coefficient is -0.25. As Bekaert et al. (1994) note, one explanation is that, while necessary, it is not sufficient to raise the standard deviation of the risk premium to explain the predictability of excess returns from currency speculation: the slope coefficient is also a function of the forward premium and the expected rate of depreciation, which are endogenous variables. These variables will also be affected by changes in the underlying structure of the economy. This can be seen from the definition of β_1 and noticing that the forward premium can be decomposed into a risk premium and the expected rate of currency depreciation:

$$\beta_1 = \frac{cov(e^{'} - f; f - e)}{var(f - e)} = \frac{cov(rp; E(\Delta e^{'})) - var(rp)}{var(E(\Delta e^{'})) + var(rp) - 2cov(rp; E(\Delta e^{'}))}, \quad (5.1)$$

where $E(\Delta e^{'})$ denotes the expected change in the exchange rate. Therefore, estimates of β_1 that are less than -1 imply that:

$$var(rp) > var(fp) \quad (5.2)$$

and

$$var(rp) > cov(rp; E(\Delta e^{'})) > var(E(\Delta e^{'})), \quad (5.3)$$

Table 11 reports the components of β_1 for the different levels of relative risk aversion in Table 10.

Table 11. Decomposition of the Slope Coefficient

	$\widehat{\beta_1}$	$var(rp)$	$var(E(\Delta e'))$	$cov(rp; E(\Delta e^{'}))$
$\sigma = 1$	-0.0775	8×10^{-5}	0.0063	-0.0004
$\sigma = 2$	-0.1140	0.0001	0.0162	-0.0013
$\sigma = 3$	-0.25	0.0003	0.0297	-0.0048

With more risk-averse agents, both the volatility of the risk premium and that of the expected rate of depreciation increase and the covariance between the risk premium and the expected depreciation rate becomes more negative in such a way that, the estimate of β_1 also become more negative. However, note that (5.3) is violated: the model instead generates

$$var(E(\Delta e^{'})) > var(rp) > cov(rp; E(\Delta e^{'})).$$

Moreover, the risk premium and the expected rate of depreciation are negatively correlated. Thus, these statistics, in part, underlie the failure of the model in explaining the predictability of excess returns.

5.1. Hansen-Jagannathan Bounds

The test proposed in Hansen and Jagannathan (1991) provides another mean to better understand the results. The test imposes a lower bound on the standard deviation of the model's IMRS. Consider an unconstrained agent in the home country. The Euler equation must satisfy:

$$E\left[\lambda'(e' - f)|\xi, s\right] = 0. \tag{5.4}$$

When no borrowing is allowed, the agent determining the bonds' price is instead considered. Hansen and Jagannathan (1991) show that the Sharpe ratio imposes a lower bound on the standard deviation of the IMRS to its mean (i.e., the market price of risk), that the model must satisfy. In the present case, the lower bound can be derived as:

$$\frac{\sigma_\lambda}{E\lambda} \geqslant \frac{E(e' - f)}{\sigma(e' - f)}$$

Backus et al. (1993) estimate the Sharpe ratio for currency speculation for the Canadian dollar to be 0.293 per month. Bekaert and Hodrick. (1992) estimate bounds as high as 0.776 when US equity investments and German, Japanese, and UK equity and foreign exchange investments are jointly considered.

When σ equals three, the model is able to substantially increase the market price of risk to about 20 percent of that estimated in the data. Storesletten et al. (1997) obtain comparable results, although Krusell and Smith (1997) can generate significantly higher market price of risk. Table 5 shows that as the market price of risk increases, so does the standard deviation of the risk premium. To increase the volatility of the risk premium to empirically plausible values, the economies would need to be more risky. In a similar fashion, Backus et al. (1993) show that, with habit persistence, the standard deviation of the risk premium increases as the market price of risk rises. To generate a standard deviation of expected returns from currency speculation that is half its estimated value in the data, they need preferences that exhibit strong habit persistence, so that agents are very sensitive to small changes in consumption.

6. Conclusion

This paper investigates the effects of incomplete markets and borrowing constraints on the foreign exchange risk premium. Although the model can generate significantly higher risk premium volatility when the borrowing constraints are very restrictive, the standard deviation of the risk premium still remains much lower than that in the data. This conclusion is similar to most other models attempting to raise the variability of the IMRS in order to generate more volatile risk premiums. The presence of idiosyncratic risk is shown to be important: the variability of the risk premium falls to a much lower level when idiosyncratic shocks are removed. The Hansen-Jagannathan bound helps us understand these results. The cases in which the risk premium is more volatile also display a higher market price of risk. Contrary to models incorporating habit persistence, the increase in the variance of the risk premium does not come at the cost of generating negatively autocorrelated forward premiums. This is important since forward premiums display high serial correlation.

The model, however, is unable to bring about risk premiums that are more volatile than forward premiums. This is symptomatic of models explaining the excess returns from currency speculation by the presence of a time-varying risk premium. As a result, the departures from the unbiasedness hypothesis are relatively small. Successful models of the risk premium will need mechanisms that can simultaneously raise the volatility of the risk premium while keeping that of the forward premium relatively constant.

The analysis suggests that excess returns from currency speculation are unlikely to be uniquely explained by the presence of a risk premium in an incomplete-markets economy. An interesting approach would be to study the forward discount puzzle in a framework allowing for both idiosyncratic risks and learning. It is often argued that the resolution of the puzzle will involve both a risk premium and systematic forecast errors (e.g., Lewis (1995); Marston (1997)). Although theoretical models with Bayesian or different types of learning are by now more common in certain economic fields, they remain relatively unexploited in international finance. Designing models including both risk-averse agents and learning provides therefore a challenging project for future work.

7. Appendix

7.1. Numerical Implementation

A combination of the Newton-Raphson and simplex algorithms (see Numerical Recipes) are used to solve the value function and the optimal decision rules in (3.5) and (3.8) above. 30 grid points for individual wealth are used when solving (3.13). Cubic splines are used to interpolate between grid points. To solve for the market-clearing prices, the algorithm uses 2000 agents. Since the decision rules turn out to be linear, the state space is discretized using 20 grid points for individual wealth, and a simple bi-linear interpolation scheme computes the optimal decision off the grid. The market-clearing prices are found using the Newton-Raphson algorithm (it is often useful, however, to try a simple bisection routine to obtain good starting values). The two economies are simulated for 1100 periods, of which the first 100 periods are discarded.

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