Nonequilibrium Fluctuation-Induced Phenomena in Josephson Junctions

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We show that nonequilibrium current fluctuations of a certain type can give rise to *net* voltages in superconducting tunnel junctions. This is the result of a fluctuation-induced net rate of change of the phase difference of the superconducting order parameter across the junction. Various exact expressions are derived for the mean voltage, and are evaluated explicitly in certain limits as well as by numerical simulations. These phenomena are due to an asymmetry in the spectral properties of the current noise which should be a fairly ubiquitous feature of nonequilibrium fluctuations in nature.

In the past year or so there has been a flurry of interest in fluctuation-induced transport phenomena [1] focused on the observation that nonequilibrium fluctuations can lead to transport in the presence of a spatial asymmetry. [2] It has also been pointed out that mean-zero noise of a more complicated asymmetric type can lead to similar phenomena *even in the absence of a spatial asymmetry.* [3] Preliminary attempts have been made to apply these phenomena to the operation of biomolecular motors, [4] as applications of new molecular separation techniques, [5] to condensed matter type systems, [7,8] and to understanding the kinetics of single ion channels. [9] All of this recent work has focused on the transport of particles along a given spatial axis. Here we show that these types of phenomena can appear in more general contexts by demonstrating that the "transport" of the phase difference of a quantum mechanical order parameter via an analogous process leads to new fluctuation-induced phenomena in superconducting electronic devices.

All of the previous work focused on completly symmetric nonequilibrium noise in the presence of a spatial asymmetry, except for [6], which treats slow asymmetric periodic driving. In the present case a complete symmetry of the underlying system (antisymmetry of the pair current across the junction as a function of the phase difference) is imposed by time reversal invariance, and the effect is due to a spectral asymmetry in the noise which we call *temporal asymmetry* to distinguish it from spatial asymmetry.

A typical superconducting (Josephson) junction consists of a junction shunted by a resistance R, and driven by a current $I(t)$ as pictured in Fig. 1. Such junctions are of interest not only in terms of their technological applications, but because of their ability to exhibit (singly and in arrays) a number of important types of nonlinear phenomena, including phase locking, bifurcations, chaos, solitonic excitations and pattern formation. [10] Josephson showed that electron pairs could tunnel through a narrow insulating material between two superconductors. [11] The pair current across the junction is given by $J_p = J_c \sin\phi$ where ϕ is the phase difference of the superconducting order parameter across the junction, and J_c is a critical current. The evolution of the phase difference of the current carrying state is described by the equation

$$\frac{\hbar}{2eR}\dot{\phi} + J_c \sin\phi = I(t), \tag{1}$$

where $I(t)$ is a driving current. [12] Here we will be primarily interested in totally unbiased driving $< I(t) >= 0$.

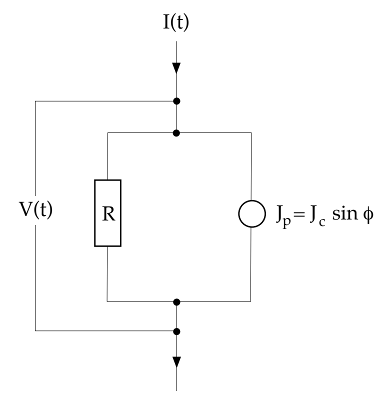

FIG. 1. **Resistively shunted Josephson Junction**

Of particular interest is the voltage V(t) across the circuit. The phase difference is related to the voltage according to

$$\phi(t) - \phi(0) = \frac{2e}{\hbar}\int_0^t V(s)\, ds \tag{2}$$

so that $V(t) = (2e/\hbar)\dot{\phi}$. We will show that, given the right conditions, the nonequilibrium fluctuations of $\eta(t)$ can lead to a fluctuation-induced net rate of change of the phase difference across the junction, and consequently a fluctuation-induced net voltage $\langle V(t)\rangle = (2e/\hbar)\left\langle \dot{\phi} \right\rangle$ where the <> indicate time averages.

As a first example where the mechanism should be readily grasped by the general reader we will consider the rather elementary case of "slow noise", that is, where the time scales of the noise are significantly slower than the principle relaxation time of the system. We set $I(t) = \xi(t) + f(t)$ where $\xi(t)$ is "fast" Gaussian white noise (either thermally or externally applied) with $< \xi(t) >= 0$,and $< \xi(t)\xi(s) >= 2D\delta(t-s)$, and the driving $f(t)$ is "slow noise" with mean zero $< f(t) >= 0$. Eq. 2 takes the form

$$\dot{\phi} + \omega_j \sin(\phi) = \zeta(t) + \eta(t), \tag{3}$$

where $< \zeta(t)\zeta(s) >= 2\tilde{D}\delta(t-s)$ with $\tilde{D} = (4e^2R^2/\hbar^2)D$, $\eta(t) = 2eRf(t)/\hbar$ and $\omega_j = 2eRJ_c/\hbar$. The evolution of the probability density for ϕ is given by the associated Fokker-Planck equation

$$\partial_t \rho(\phi) = \partial_\phi \left[\Psi(\phi, t) + \tilde{D}\partial_\phi \right] \rho(\phi) \tag{4}$$

where $\Psi(\phi, t) = -\omega_j \cos\phi - \eta(t)\phi$. We first note that the r.h.s. of Eq. 4 is just $-\nabla j$, where j is the probability current of the phase. The steady-state current (for constant η) is found by imposing periodic boundary conditions $\rho_s(\phi) = \rho_s(\phi + 2\pi)$, and normalizing $\int_0^{2\phi} \rho_s(x)\, dx = 1$. [13] This yields an exact expression for the mean rate of change of ϕ,

$$\left\langle \dot{\phi} \right\rangle = \frac{2\pi[1 - \exp(-2\pi\eta/\tilde{D})]}{\frac{1}{\tilde{D}} \oint dy \oint dx\ e^{-\Psi(y)/\tilde{D}} e^{\Psi(x+y)/\tilde{D}}}. \tag{5}$$

For $\eta(t)$ which changes on time scales much slower than the principle relaxation time of the system, the net voltage is found by averaging

$$\langle V \rangle = \frac{2e}{\hbar} \lim_{\tau\to\infty} \frac{1}{\tau} \int_0^\tau \left\langle \dot{\phi}(t) \right\rangle\, dt.$$

Any net voltage will be due to a net bias in the "hopping" of the phase difference through angles of $\pm 2\pi$. Since the effect is nonlinear in the applied force $f(t)$, a force with mean zero can give rise to a net voltage if the force is applied asymmetrically in time. Since $< \dot{\phi} >$ is an antisymmetric nonlinear function of $\eta(t)$ voltage can be expanded in a series in the odd moments of $\eta(t)$,

$$\langle V \rangle = \sum_{n=1}^{\infty} c_{2n+1} \left\langle \eta^{2n+1} \right\rangle.$$

Therefore there will be a net voltage whenever any odd moment $< \eta^{2n+1} > \neq 0$. This happens *even though the net current is zero*, and therefore is a fluctuation-induced effect.

The net voltage in Eq. 5 can be evaluated by steepest descents when $D, \eta << 2\omega_j$. The result is

$$< V(t) >= \left(\frac{8e^2RJ_c}{\hbar^2}\right) e^{-J_c\hbar/eRD} \sinh\left(\frac{\pi\hbar f(t)}{2eRD}\right)$$

As we have mentioned, the averaged voltage can be expanded in terms of the odd moments of $f(t)$,

$$< V >= \left(\frac{8e^2RJ_c}{\hbar^2}\right) e^{-J_c\hbar/eRD}$$

$$\times \sum_{n=0}^{\infty} \left(\frac{\pi\hbar}{2eRD}\right)^{2n+1} \frac{< f^{2n+1} >}{(2n+1)!}.$$

This asymmetry can also be viewed in terms of the spectral properties of the noise. Any noise with a uniform distribution of phases, such as Gaussian noise, will be symmetric, and will not give rise a net voltage. *If the phase distribution of the spectrum is asymmetric the noise will give rise to a net voltage.*

Here the main features introduced by the temporal asymmetry are the interplay of the lower potential barriers in the positive direction relative to the negative direction (for this particular driving) and the corresponding shorter and longer times respectively the force is felt. These types of competitive effects appear ubiquitously in systems where there is an interplay between thermal activation and dynamics.

For arbitrarily fast noise the net voltage will be nonvanishing whenever any odd cumulant is nonvanishing, $\langle \eta^{m_1}(t_1)\dots\eta^{m_N}(t_n)\rangle \neq 0$, where $\sum_{i=1}^n m_i$ is odd. As our principle example we consider the only example of non-trivial noise that (to our knowledge) can be treated analytically, and can also exhibit temporal asymmetry. This is a system which is driven by telegraph $I_n(t)$ (Dicotomous) noise

$$\dot{\phi} + w_j \sin\phi = \eta(t) \tag{6}$$

where $\eta(t) = (2eR/\hbar)I(t)$. The telegraph noise has two states,

$$\eta_+ = \sqrt{\frac{D}{\tau}\left(\frac{1+\epsilon}{1-\epsilon}\right)}, \quad \eta_- = -\sqrt{\frac{D}{\tau}\left(\frac{1-\epsilon}{1+\epsilon}\right)}. \tag{7}$$

The transition probabilities w_+ from the plus to the minus state and w_- from the minus to the plus state are

$$w_+ = \frac{1+\epsilon}{2\tau}, \quad w_- = \frac{1-\epsilon}{2\tau}. \tag{8}$$

This noise has mean zero $< \eta(t) >= 0$ and correlation function

$$\phi(t) =< \eta(t)\eta(0) >= \frac{D}{\tau}\exp\left(-t/\tau\right). \tag{9}$$

There are a number of stochastic processes with precisely this correlation function, but not all of them will give rise to a net voltage in the junction. This noise is temporally asymmetric in the same sense as our first example. The noise spends a different amount of time in each state on the average, yet the the average force is still zero.

The system is described by the set of equations [14]

$$\partial_t \rho_+ = -\partial_x[\omega_j \sin\phi + \eta_+]\rho_+ - w_+\rho_+ + w_-\rho_- \quad (10)$$

$$\partial_t \rho_- = -\partial_x[\omega_j \sin\phi + \eta_-]\rho_- - w_-\rho_- + w_+\rho_+ \quad (11)$$

where $\rho = \rho_+ + \rho_-$. The net voltage can easily be found

$$\langle V\rangle = (4\pi e/\hbar)\mu(\phi)[\omega_j \sin\phi - D\partial_\phi W(\phi)]\rho_s(\phi). \quad (12)$$

$$\mu(\phi) = [1 + \tau\omega_j \cos\phi]^{-1}, \quad \theta = 2\epsilon/\sqrt{1-\epsilon^2}$$

$$W(x) = \left[1 - (\tau/D)\omega_j^2 \sin^2\phi(x) - \theta\sqrt{\tau/D}\omega_j \sin\phi\right].$$

Again imposing periodic boundary conditions and normalizing [13] we obtain

$$\langle V\rangle = \left(\frac{4\pi e}{\hbar}\right)\frac{1-\exp(\Delta/D)}{N}, \quad \Delta = \oint dy \frac{\omega_j \sin y}{W(y)} \quad (13)$$

$$\Psi(x) = -\int^x dy \frac{\omega_j \sin y}{W(y)} \quad (14)$$

$$N = \frac{1}{D}\oint dy \oint dx \frac{e^{-\Psi(y)/D} e^{\Psi(x+y)/D}}{\mu(x+y)W(y)} \quad (15)$$

To order $\sqrt{\tau/D}$ we have

$$\tilde{V} = \frac{\epsilon}{\sqrt{1-\epsilon^2}}\frac{\sqrt{\tau/D}}{I_0^2(w_j/D)} \quad (16)$$

where $\tilde{V} = \hbar < V > /2e\omega_j^2$, and I_0 is a modified Bessel function. The current is positive if $\epsilon > 0$ and negative if $\epsilon < 0$.

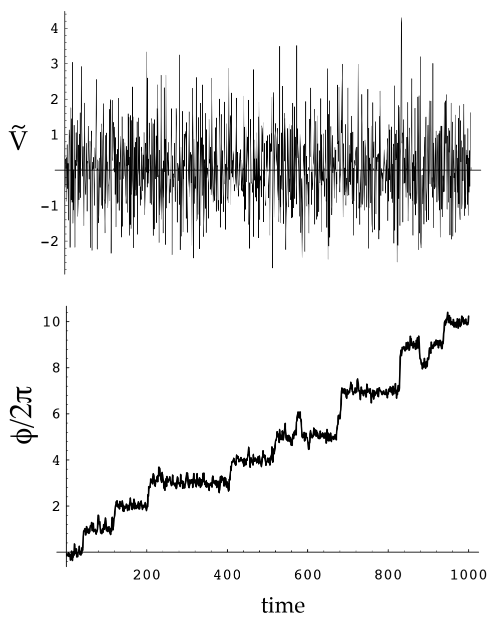

FIG. 2. **Time series of (a) the renormalized voltage $\tilde{V}$, and (b) the phase difference for $\tau = 0.01$, $\epsilon = 0.9$ and $D = 1.5$. The voltage time series has been averaged over a time which is longer than the correlation time of the driving.**

Fig. 2 shows time series of the voltage and the phase. The phase steps with consistent bias in the positive direction and gives rise to a net voltage. The voltage is essentially white noise with a small net part (about 0.1 in this figure) and a rather substantial variation about this mean. Fig. 3 shows that the mean voltage is a peaked function of the noise strength. This peak is due to a cooperative effect between the effects of the noise and the dynamics of the Josephson junction. For very small noise the phase will not jump at all and there will be no net voltage. For very large D the effects is to "wash out" the effects (both positive and negative phase jumps) and the mean voltage again falls off. A similar effect is responsible for the phenomenon of stochastic resonance. However, this particular phenomenon is due solely to the property of temporal asymmetry in the noise. Fig. 4 shows how the mean voltage depends on the temporal asymmetry parameter ϵ. The net voltage vanishes when the asymmetry vanishes ($\epsilon = 0$).

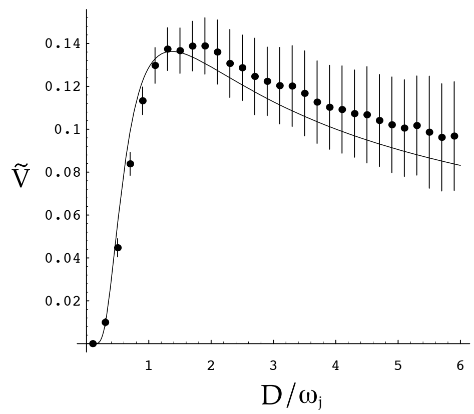

FIG. 3. $\tilde{V}$ **vs. noise strength** D **for** $\tau = 0.01$ **and** $\epsilon = 0.9$. **The solid line is the theoretical prediction and the dots and error bars are the results from numerical simulations.**

We have examined just two cases here, but obviously there are nearly unlimited examples of temporally asymmetric noise. However, this lack of symmetry makes their mathematical treatment diffucult, and we assume that it is this difficulty which has lead to the situation that the effects of this type of noise have been largely ignored.

The dichotomic noise used above is a special case of a multi-state "Kubo-Anderson" process [16] (sometime called the "Kangaroo processes"). There are also types of continuous noise which can have temporal asymmetry. Shot noise, which is also of great importance in quantum electronics, is of this latter type. Mean zero shot noise which is temporally asymmetric can be produced if the frequency and amplitude distribution is slightly different for positive and negative fundamental pulses.

While in the previous work the energy was extracted in the form of a net transport of particles, here the nonequilibrium effects of the driving are translated into the more generic form of a net voltage. In previous papers ([2,4,5,7]) only temporally symmetric noise was examined, and the effect vanished in any symmetric situation, as is the case with Josephson junctions. Since many periodic structures in condensed matter physics are also symmetric, we feel that our observations here might have significant practical implications. For example, temporally asymmetry voltage fluctuations should give rise to net currents in superionic conductors. These system typically have periodic and symmetric internal fields, and the effect would be precisely analogous to the situation discussed here. We also believe that new information can be obtained by driving biological ion channels with temporally asymmetric voltage fluctuations. [9]

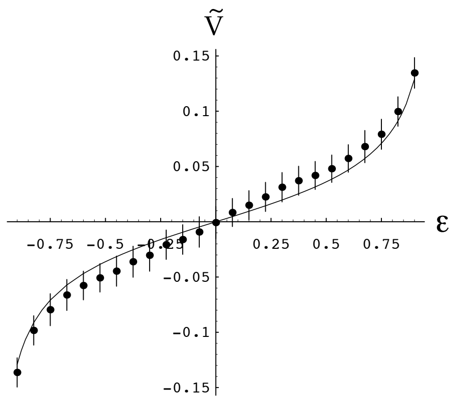

FIG. 4. $\tilde{V}$ vs. temporal asymmetry parameter ϵ for $\tau = 0.01$ and $D = 2.0$. The solid line is the theoretical prediction and the dots and error bars are the results from numerical simulations.

Temporal asymmetry and spatial asymmetry in fact relate to the problem of nonequilibrium transport in *precisely the same way.* In both cases a net effect arrises due to an interplay between the strength of a fluctuation, the time it acts, and underlying dynamics. In the case of a spatial asymmetry a fluctuation to the right with a given strength which lasts a given time will tend to take the system over the right-hand barrier while the same fluctuation with sign reversed does not lift the system over the left-hand barrier. In the case of temporal asymmetry the probabilities of the fluctuations to the right and left are different, so a net effect arrises in the absence of a spatial asymmetry. What both of them show is that even a subtle asymmetry in the *shape* of the potential or in the *shape* of the spectral properties of the noise will give rise to an effect even when the net force due to each vanishes. Previously all the emphasis has been put on spatial asymmetry, but we believe that temporal asymmetry deserves to be put on equal footing with spatial asymmetry as one of the principle elements which, in combination with the nonequilibrium time correlations, can give rise to a net effect.

At this point it appears that the basic principles behind fluctuation-induced transport type phenomena in over-damped ohmic systems are well understood. The lesson to be learned from all this is that any kind of broken symmetry will usually allow energy to be pumped out of a "bath" as long as the system as a whole is not in equilibrium. This occures at the expense of the increased entropy of the bath, and is the essence of the "surprising" effects observed in [2–5]. From the standpoint of fundamental physics this observation is the main significance of an understanding of fluctuation-induced transport.

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