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S. G. Ortman
A. H. F. Cabaniss
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S. G. Ortman,[1,2,*], A. H. F. Cabaniss,[3,2] J. O. Sturm,[4] and L. M. A. Bettencourt[2]

[1] Department of Anthropology, University of Colorado Boulder, Boulder, CO 80309-0233, USA,

[2] Santa Fe Institute, 1399 Hyde Park Rd, Santa Fe NM 87501, USA,

[3] Department of Classics, University of North Carolina Chapel Hill, Chapel Hill, NC 27599, USA.

[4] Department of Anthropology, University of New Mexico, Albuquerque, NM 87131, USA.

[*] To whom correspondence should be addressed; E-mail: scott.ortman@colorado.edu

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A key property of modern cities is increasing returns to scale–the fact that many socio-economic outputs increase more rapidly than population. Recent theoretical work suggests this phenomenon is the result of general characteristics of human social networks embedded in space and, thus, is not necessarily limited to modern settlements. Here, we examine the extent to which increasing returns are expressed by archaeological settlement data from the Pre-Hispanic Basin of Mexico. We find three quantitative patterns which suggest that increasing returns were present and that they derived from the same processes that generate these returns in modern cities. In all cases scaling parameter values are consistent with expectations derived from theory. Our results thus provide evidence that the processes that lead to increasing returns in contemporary cities have characterized human settlements throughout history and do not require modern forms of political or economic organization.

1 Introduction

Many studies over the last few decades have demonstrated that average properties of contemporary urban settlements–from socio-economic outputs to land area to the extent of infrastructure–vary systematically and predictably with population size (*1–6*). For example, measures of the physical extent of urban infrastructure increase more slowly than city population, thus exhibiting economies of scale. On the other hand, various socio-economic outputs increase faster than population size and thus exhibit *increasing returns to scale*. Recent theory, building from comparative analyses of large datasets for many urban systems around the world, has proposed that these properties of modern cities take a simple mathematical form and emerge from a few general principles of human social organization (*2*). This view posits the primary role of cities in human societies as *social reactors*: larger cities are environments where a larger number of social interactions per unit time can be supported and sustained. This generic dynamics, in turn, is the basis for expanding economic and political organization, such as the division and coordination of labor, the specialization of knowledge, and the development of (hierarchical) political and civic institutions. Thus, larger cities anywhere, on average, magnify social interaction opportunities (*2, 3, 7*) and provide better matching complementarities (*8*) thereby increasing the productivity and scope of material resources and human labor (*2, 7, 8*).

An important aspect of these ideas is that the theoretical derivation of scaling relations does not invoke specific characteristics of modern economies, industrialization or global trade, but instead relies only on basic self-consistent characteristics of social networks embedded in space. Consequently, these models are potentially applicable to ancient (and even non-urban) societies and make a set of integrated and novel predictions for the structure and function of ancient settlements that can be tested using archaeological evidence.

We have previously examined the extent to which economies of scale characterized the Pre-Hispanic Basin of Mexico (BOM) (*7*). Here, we discuss settlement scaling theory, its relations to previous models of spatial economics, and its predictions in the context of archaeology. Then, we illustrate three ways in which scaling phenomena are expressed by data from the Pre-Hispanic BOM (Figure 1). First, we compare resident populations with settled areas to show that larger settlements are generally denser. Second, we compare the populations of political units with public monument volumes to illustrate that larger political units produced such structures at faster rates per capita. Third, we compare settlement populations with domestic mound areas and find that houses in more populous settlements were larger on average. In all cases, we observe patterns that are consistent with modern cities and with quantitative predictions of settlement scaling theory. Based on these results, we hypothesize that the same fundamental processes lie behind emergent scaling phenomena in contemporary and ancient societies, thus providing a unifying view of human settlements throughout history and a novel theoretical framework for the interpretation of archaeological data.

2 Results

2.1 Settlement Scaling Theory

We take human settlements to be, first and foremost, social networks embedded in space. Simple spatial models of settlements have a long history in geography and economics, starting with the von Thünen model (for the *isolated state*) to more recent related models that apply similar ideas to the structure of cities, due to Alonso and others (*9*). These models are most frequently invoked to parameterize different land uses as the result of the interplay between land rents and transportation costs in monocentric settlement geometries (*9*). Here, we retain some of the characteristics of these models but consider situations where circular symmetry is not necessary. We also do not specify the type or value of goods to be transported: the problem of what can be produced and spent is instead determined more abstractly as the result of socio-economic rates of interaction. This approach removes major stumbling blocks of traditional models of spatial economics because socio-economic outputs, transportation costs and settlement geometry are dependent on many contextual factors, such as a society's technology and culture. What is specified in our approach is the universal dependence of the rates at which goods are produced (and expended) on population size (elasticities). As such, the ideas developed here are more general than traditional models of land use economics and can be developed and tested in the context of ancient societies and archaeological data.

The first basic assumption of our theory is that settlements are containers within which a population interacts frequently, with internal social interactions far outnumbering external ones. From this very simple picture one can derive an expected scaling relation between total settlement land area and population. Consider a settlement with relatively low density of occupation that is not especially organized or planned spatially. We call such settlements *amorphous* and posit that their spatial extent is set by a simple form of spatial equilibrium where, on average, the benefits of social interaction balance movement costs for a given population size, N.

In this case, the cost of movement, c, is set by the energetic cost of walking ϵ (measured in cal/length) times the diameter of the circumscribing area L, which is proportional to the square root of the area, $c = c_0 \epsilon A^{1/2}$. This is true in many different geometries and does not require that the settlement be circular and transportation radial (*9*). The dimensionless constant c_0 is a simple number of order unity, as demonstrated by studies exploring movement in different urban forms (*10*). The average social benefits of interaction with others, y, are then estimated through the average productivity of each interaction, g, times the ratio of urban volume covered by a person in the settlement over its area, times the number of people in the settlement. If we parameterize these quantities by the distance at which interaction occurs, a_0 (a cross section in the language of physics), and the distance travelled over the given period, l, we obtain $y = (g a_0 l/A)N$. Equilibrium between social net benefits and movement costs, $c = y$, leads to $A = aN^{\alpha}$, where $a = (g a_0 l)/(c_0 \epsilon)^{\alpha}$ and $\alpha = 2/3$.

This simple picture needs to be elaborated as settlement densities increase and urban space becomes more structured and differentiated. The main feature of such changes is the appearance of spaces dedicated to flows, such as streets and waterways. As this happens, dwellings align along these transportation networks, as can be seen in maps of many ancient and modern cities. This change of settlement organization with size and density has been noted in archaeology by Flannery (*11*), who suggested it may be a general feature of human settlement growth. This spatial organization has a different geometry from the amorphous settlement and leads to different scaling exponents.

We extend the amorphous settlement model to larger and denser "networked" settlements by assuming that infrastructural space d is set aside on a per capita basis, proportional to the current overall density, such that $d = \rho^{-1/2}$, where $\rho = A/N$ (More complex and thorough models can also be developed that justify these simple derivations (*2*)). Thus, the total area of the infrastructural network, A_n, is proportional to the population times the square root of area over population, $A_n \sim A^{1/2}N^{1/2}$. Substituting $aN^{2/3}$ for A leads to $A_n \sim a^{1/2}N^{5/6}$, the relation observed for infrastructural quantities in contemporary metropolitan areas (*2, 6*).

Finally, we assume that total socio-economic outputs are proportional to the total number of interactions that take place in a settlement, with technology and culture influencing only the productivity (and cost) of each interaction. Using this, one can derive the expected socio-economic output of a settlement Y, relative to others, by multiplying the per capita benefit of interaction by the population, $Y = yN = ga_0lN^2/A$, and then substituting the relation for the infrastructural area, A_n, for A. This simplifies to $Y = GN^2/A_n \sim N^2/N^{5/6} \sim N^{7/6}$, where $G = ga_0l$, again as observed for contemporary socio-economic quantities.

An important feature of these models is that their assumptions and input parameters are very general and are not specific to modern political or economic organization. Exponents are universal as they are set by the congruence of the geometries of spatial and social networks, under the assumption of an equilibrium between centripetal social interactions and centrifugal costs (*9*), as well as the requirement that such arrangements stay open ended relative to settlement size (*2*). Simple elaborations of these arguments further derive patterns of professional diversity (division of labor), labor productivity (*12*) and an urban area production function (*13*) consistent with those observed in modern cities.

2.2 Population and Settled Area

The population-area scaling relations discussed above apply to settlements for which it is reasonable to model the settled area as the container within which the resident population interacts on a regular basis. It proposes that such settlements tend to grow in ways that balance the costs of moving within the settlement with the benefits of the resulting social interactions. Thus, settlements whose spatial arrangements balance these costs and benefits should exhibit a specific and consistent overall average relationship between the resident population and settled area. Specifically, settlement scaling theory predicts that the exponent relating population to settled area for "interaction container" settlements should fall in the range between 2/3 and 5/6, with this exponent being closer to 2/3 among small, amorphous settlements and closer to 5/6 among larger, networked settlements. It also suggests the area taken up by an individual in the smallest such settlements derives primarily from travel costs (walking, in this context) and the average (energetic) benefit of social interactions. Technologies that reduce transportation costs or increase the effectiveness of interaction should increase this baseline area, but factors that influence the rate of energy capture by primary producers should not. This is because the movement and exchange of agricultural produce provides a stronger constraint on energy flows in social networks than agricultural production itself. Finally, our models predict the pre-factor of the scaling relation between population and settled area should be responsive to changes in within-settlement transport technology, but not to changes in agricultural productivity.

We test these expectations by comparing the populations and settled areas of BOM settlements dating from four Pre-Hispanic cultural periods (see Table 1). The Formative period (1150 BCE-150 CE) saw the beginnings of detectable settlements and the rise of local polities; the Classic period (150-650 CE), the political and economic dominance of Teotihuacan ($N \approx 100,000$); the Toltec period (650-1200 CE), the formation of a number of small competitive polities; and the Aztec period (1200-1519 CE), the unification of these into an empire centered on Tenochtitlán ($N \approx 200,000$) that was in place at the time of the Spanish Conquest. We also compare two size classes corresponding to ”amorphous” ($N < 5000$) and ”networked” ($N \geq 5000$) settlements and 1960 census data from the same area. For details of our data selection and grouping criteria, see (*7*) and the Supplementary Materials (SM). Our results (Table 1) are consistent with expectations. First, across four Pre-Hispanic periods, the exponent of the average scaling relation lies within the interval $2/3 \leq \alpha \leq 5/6$. Second, the results for ”amorphous” vs. ”networked” settlements illustrate the transition from $2/3$ to $5/6$ that follows from our models. Third, although maximum agricultural yields increased substantially over time, there is no clear evidence for change in the pre-factor of the average scaling relation across Pre-Hispanic periods. However, the pre-factor for the 1960 data is significantly larger. This is consistent with the fact that there were no major innovations in transportation technology during the Pre-Hispanic periods, whereas horse-drawn carts and automobiles appeared by the mid-20th century.

2.3 Polity Population and Monument Construction

Pre-Hispanic Mesoamerican societies are well-known for their monumental architecture, especially pyramids, administrative palaces and plaza-focused buildings. Several factors allow us to treat the total volume of public monuments in administrative and ceremonial centers as a measure of the total production of corvée labor pools drawn from the subject population over a period of time (see SM for details). Given these linking arguments, settlement scaling theory predicts public monument construction rates should on average be proportional to the population of subject settlements to the $7/6$ power, or more formally $Y = GN^2/A_n \sim N^2/N^{5/6} \sim N^{7/6}$.

In the SM we explain in detail our procedures for estimating the volumes of civic-ceremonial structures and the size of the subject populations that contributed labor in building these monuments during different cultural periods. Essentially, we combine archaeological studies of political organization and correlations between BOM survey data and ethno-historic sources to create groups of settlements, populations and monuments for specific political units and archaeological periods (*19, 33, 35–39*). The resulting dataset is presented as Table S2. These data are insufficient for a time-series analysis, but the average scaling relation between political unit population and public monument construction rates (total volume / years in period) across all periods (Table 2 and Figure 2A) indicates that the exponent of this relation, $1 + \delta \simeq 7/6$, as predicted by theory. These results suggest larger corvée labor groups generally produced more per person and per unit time than smaller groups, in precise analogy to the increasing returns to scale for socio-economic rates noted in contemporary cities (*2, 3*).

It is also apparent in Figure 2A that political units from different periods follow essentially the same scaling relation, characterized by a single pre-factor representing the baseline productivity of an individual. This suggests there was little change in the technology and energetics of public monument construction over time and that the remarkable concentrations of public monuments in political capitals can be attributed to the populations these authorities regulated and the returns to scale associated with the resulting corvée labor forces.

In analyzing the data this way, we do not mean to suggest that there was no variation in the annual labor tax, the proportion of population that performed corvée labor, the length of time over which monuments were built or the technology of monument design and construction. Undoubtedly there was variation in all these factors. Yet it is important to remember that these data vary over five orders of magnitude and as a result the overall scaling relationship is fairly robust to variation in these factors, the impact of which is summarized by the residuals of individual cases to the average scaling relation in Figure 2A (*5*). That the results are so well-behaved despite the wide chronological and demographic scope of the data suggests the dominant factor behind monument construction rates was in fact the size of corvée labor pools.

2.4 Settlement Population and House Area

Our theory predicts average per capita productivity should increase with settlement population to the power $\delta \simeq 1/6$, as the productivity of an individual in a social network is proportional to the number of interactions that person has with others. Stated more formally, $y = G\ N/A_n \sim N/N^{5/6} \sim N^{1/6}$, where N/A_n reflects the density of people with respect to the infrastructural area through which people move and $G = ga_0 l$, with $a_0 l$ reflecting the area covered by an individual's daily movements and g reflecting the average productivity of daily interactions (*2, 7*). Thus, the average productivity of an individual $y = y_0 N^{\delta}$, where $\delta = 1/6$. In turn, this implies the total productivity of a settlement is given by $Y = y\ N = y_0 N^{1+\delta}$. Per household production should scale the same way as per capita production because in most societies households are the basal units of production and consumption (see the SM for more details).

We treat the surface area of a domestic residence as a proxy for the productivity of the associated household unit (see the SM for background and justification). Given this association, settlement scaling theory leads to four expectations. First, following the typical distribution of wealth and income in contemporary societies (*40*), house areas should be log-normally distributed. Second, mean house area m should scale with settlement population N to the $1/6$ power, and thus the product of mean house area and settlement population, mN, should scale with settlement population to the $7/6$ power. Third, the product of mean house area and settled area per person, mA/N, should vary independently of settlement population. This is because these two measures represent the productivity of daily interaction and the area over which these interactions occur, respectively, and thus their product is an estimate of G, which should be independent of settlement size on average. Finally, the residuals to the average scaling relation between settlement population and mean house area should also be log-normally distributed. This is necessary because it is difficult to achieve a strong correlation between two variables when the dependent variable (in this case, mean house area) increases only slowly with respect to the independent variable (settlement population). In such cases it is important to evaluate the residuals from the best-fit line, as the residuals should be normally-distributed if the regression is capturing a valid relationship (*41*).

The primary data for this analysis consists of surface areas of domestic mounds (including elite residences and palaces) recorded in BOM survey reports, with additional data added for specific sites from the literature (see the SM for details). Using these data, we computed m for ”interaction container” settlements that are at least 1 hectare in area, possess well-preserved architectural remains, and are associated with at least two measured domestic mounds. Then, we estimated the total production of that settlement as $Y = mN$, and we estimated G as $G = mA/N$. The resulting dataset is presented in detail as Table S3.

Once again there are insufficient data for time-series analysis but patterns in the pooled data are consistent with expectations. First, the distribution of domestic mound areas across all settlements (Figure 3A) is approximately log-normal, with most deviations from log-normality deriving from expedient rounding of mound dimensions by fieldworkers (e.g. $5 \times 5m = 25m^2$, $20 \times 20m = 400m^2$, $30 \times 30m = 900m^2$ and so forth). Second, the average scaling relation between N and m and N vs. $Y = mN$ (Table 2 and Figure 2B) show that mean household productivity scales with settlement population as expected. Third, our measure of G is independent of settlement population (Table 2 and Figure 2B, inset). These results indicate that per-household productivity was higher in larger settlements, with the degree of increasing returns offset by the economy of scale in population density. Finally, the distribution of standardized residuals from the average scaling relation between m and N is approximately log-normal (Figure 3B), given deviations derived from expedient rounding by fieldworkers. Thus, the observed relationship appears valid, despite the modest correlation between the two variables.

These results suggest the average productivity of households in the BOM generally increased with settlement size in the manner predicted by settlement scaling theory. However, as was the case for public monument construction rates, the data from all periods appear to follow the same scaling relation, and are reasonably well-characterized by a single pre-factor that applies across periods. This suggests the intrinsic productivity of individuals y_0 did not change appreciably over time. Yet, these results also suggest the average wealth of households could have varied depending on the distribution of settlement sizes during different periods. In this way the material conditions of life could have improved for some households, even if there were no significant improvements in individual productivity. We consider this issue in more detail below.

3 Discussion

We have outlined an approach to the analysis of ancient societies rooted in complex systems concepts and applied this framework to data from the Pre-Hispanic Basin of Mexico to investigate the degree to which scaling phenomena observable in modern urban systems also characterized this ancient society. Our analysis requires number of assumptions, linking arguments and data selection criteria, and it is important for future research to investigate the validity of these aspects of our analysis. It is also important to recognize that, even if the scaling phenomena we investigate prove to be universal, scaling relationships may not be universally observable due to the nature of archaeological and historical data. Nevertheless, settlement scaling theory should apply to any society due to its basic underlying assumptions, and the BOM surveys represent one of the most systematic documentations of an ancient non-Western civilization ever accomplished. Thus, the fact that the archaeological record of this society can be shown to exhibit the same fundamental relationships between population, infrastructure and socio-economic outputs predicted by theory, and observed in modern urban systems, is a striking and exciting result.

Settlement scaling theory proposes that one can use a variety of aggregate quantities as measures of infrastructure, socio-economic outputs, social network connectivity and their evolution over time in any society. In this case, we have shown that, in the Pre-Hispanic BOM, larger population aggregates used space more efficiently, produced public goods more rapidly, and were more productive per household. Further, the congruence of these results with theory suggests that the benefits of scale across all these domains ultimately derive from the properties of strongly-interacting social networks embedded in structured spaces. This reinforces our view that human settlements of all times and places function in the same general way by manifesting strongly-interacting social networks, thus magnifying rates of social interaction and increasing the productivity and scope of material resources, human labor and information sharing (*2, 7, 12*).

In addition to the constancy of scaling exponents, our analyses reveal a remarkable consistency in scaling pre-factors across cultural periods. These pre-factors either do not change in time-series analysis, or the data from multiple time periods follow a single scaling relation with the same pre-factor. Our theory predicts that scaling pre-factors should change in cases where transport costs or the average productivity of individual interactions changed over time. Such changes could arise from technological innovations such as beasts of burden or wheeled vehicles; changes in information technology such as currency or literacy; or any number of factors that facilitate the flow of goods and services through a social network. The system-level effects of such innovations should be perceptible through scaling analyses of quantities measured before and after their appearance, as our comparison of the Pre-Hispanic and 1960 census data illustrate. Given this, the fact that such changes are not evident across the Pre-Hispanic periods suggests one of the two sources of modern economic growth–increases in baseline productivity–

was limited in this society (*13*). Agricultural production per unit of land clearly improved over time (*33, 46*) and this had a significant impact on the maximal size of settlements and their spatial distribution, but the overall productivity of an individual working alone appears to have been relatively constant. As a result, any changes in per capita economic output over time are most likely traceable to changes in the size and density of social networks. This may have impacted average household "income" during periods when sociocultural and political institutions allowed larger fractions of the population to live in larger settlements, but changes in the material conditions of life would have depended on the economies and returns associated with social coordination as opposed to increases in fundamental productivity. This is potentially a striking and important realization with wide-ranging implications for our understanding of human civilization.

If in fact larger social networks are intrinsically more efficient and productive, there is at least the potential for the majority of individuals in a society to benefit from increases in the scale of social coordination. Whether individuals actually do depends on the way in which the system-level benefits of scale are distributed among the individuals comprising these networks. Our analyses suggest the material conditions of life did improve for that fraction of the BOM population that lived in larger settlements, but the baseline area per person, and roofed space per person, did not change much over time for small settlements that were presumably devoted to food production. This suggests there was little change in the conditions of life for primary producers in this society, regardless of the way the benefits of scale were distributed among the more urbanised population. During periods of larger-scale social coordination, the BOM population was more productive and more efficient overall, but disparities in production (income) were high in all periods, with 40-50% of total house area encompassed by the top decile of households. This is comparable to levels of income disparity seen in the contemporary US (see Table S4 and (*44*)). The conclusion we draw from these patterns is that the (latent) benefits of scale for human groups are universal and are likely responsible for the long-term increase in human group sizes world-wide (*14*), but there are fewer constraints on how these benefits are distributed among individuals in groups. Thus, we suspect one of the most fundamental dynamics in early civilizations was the tension between the benefits of scale and the allocation of these benefits. In this particular case, it appears that reinvestment of surpluses toward innovations that increased baseline productivity was quite rare, and this limited intensive economic growth in the long run. Such possibilities reinforce our views that the archaeological record presents a vast archive of information on the determinants of socio-economic development, and settlement scaling theory provides a useful framework for examining these processes. We thus believe settlement scaling theory offers a means through which archaeology can make a broader contribution to the social sciences and perhaps even to contemporary policy (*14, 43, 45*).

Materials and Methods

Data Sources: We use settlement data from archaeological surface surveys conducted in the epicenter of Pre-Hispanic Mesoamerican civilization. These surveys took place between 1960-1975, prior to the destruction of many sites by the expansion of modern Mexico City. Figure 1 shows the location of our study area, the surveyed areas relative to Mexico City when the surveys took place, and the distribution of Pre-Hispanic settlements for the Formative Period. We compiled a database of information for some 4,000 archaeological sites resulting from these surveys, beginning with existing digital compilations (*15, 16*) and adding information from the original survey reports (*17–25*). We also added data for a few important sites, some of which were outside the survey area, based on information in the literature (*26–31*). In addition, we tabulated the dimensions of civic-ceremonial mounds reported in the survey volumes or in other sources (*27, 28, 32–34*); and we associated Aztec-period settlements with native political units based on ethno-historic information summarized by Hodge and others (*35–37*). The resulting database contains information on the settled area, population, time period, location, functional classification, political affiliations, and architectural remains of every recorded settlement. For additional details and background on this database, see (*7*) and the SM.

Site Population and Parameter Estimation: A potential problem with the BOM survey data is that the method used to estimate population for most sites was not independent of the settled area (*17, 33*). This method involved: 1) determining the extent of the surface artifact scatter for each period of occupation by mapping its boundary on low-altitude aerial photos; 2) assigning each scatter to one of a series of artifact density classes based on the observed potsherd density within the scatter; and 3) multiplying the extent of the scatter for each period by a population density derived from associations of surface potsherd densities with population densities of various settlement types in 16th and 20th century records from the area. This method ensures that there will be a relationship between the settled area and settlement population. Fortunately, we have previously shown that the estimates produced by this area-density method are nearly identical to those produced using house-counting methods that rely on the count or surface area of residential mounds at well-preserved sites in lieu of settled area (*7*). Thus, there is a basis for viewing the BOM population estimates as reasonably accurate in both a relative and an absolute sense. In all analyses, we use ordinary least-squares (OLS) regression of the log-transformed data to estimate scaling exponents, pre-factors and confidence intervals for these parameters.

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Acknowledgements

3.1 General

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3.3 Author contributions

SO and LB designed the research; SO, JS and AC compiled data; SO, LB and AC performed analysis; SO, LB, AC and JS wrote the paper.

3.4 Competing interests

The authors declare no competing interests in the production of this paper.

3.5 Data and materials availability

All data analyzed in this paper are available either within the Supplementary Materials or at www.tdar.org.

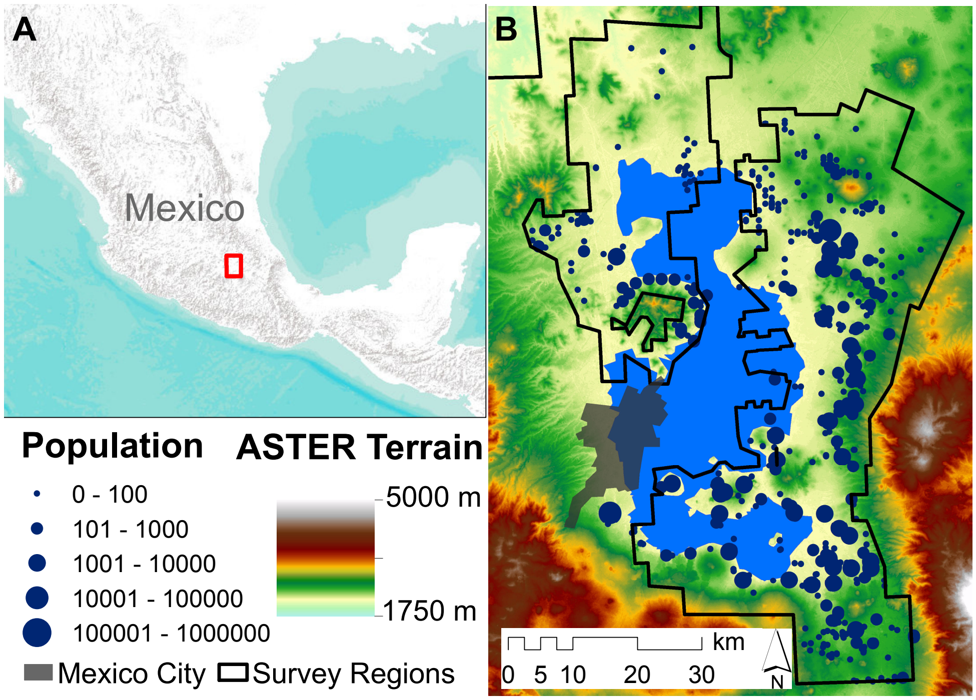

Figure 1: The Basin of Mexico. A: Location within Mexico. B: Settlements dating to the Formative period (outline shows surveyed area; circle size is proportional to population; colors denote elevation; gray area shows the extent of Mexico City in 1964). Today, settlement covers the entire basin and the lake has been drained. See the Supporting Materials for imagery sources.

Group	Sites	$a[ha]$	95% C.I.	α	95% C.I.	r^2
Formative (1150 BCE-150 CE)	230	.195	.160-.238	.711	.673-.749	.855
Classic (150-650 CE)	272	.221	.174-.279	.632	.583-.681	.707
Toltec (650-1200 CE)	484	.210	.180-.244	.718	.684-.753	.777
Aztec (1200-1519 CE)	546	.177	..156-.201	.764	.734-.793	.830
1960 Census	181	.445	.250-.945	.641	.552-.729	.532
Amorphous ($N < 5,000$)	1510	.237	.217-.259	.671	.651-.691	.741
Networked ($N \geq 5,000$)	22	.109	.009-1.303	.853	.598-.1.109	.709

Table 1: Population-Area scaling analysis results. The yields(kg/maize/ha) of the most productive agricultural strategies for the Pre-Hispanic periods are: Formative, 700; Classic and Toltec, 1,400; and Aztec, 3,000 (*46*)

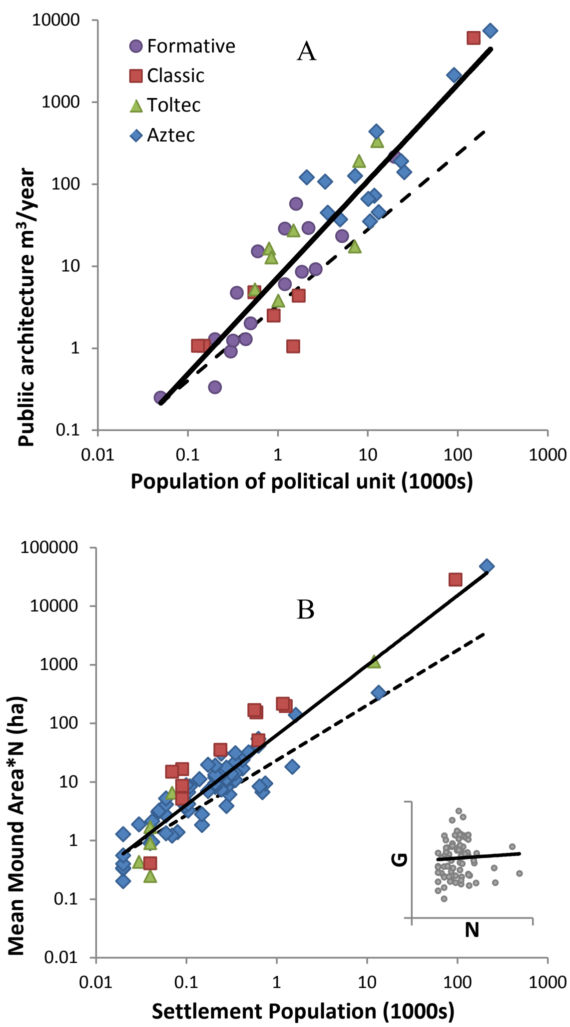

Figure 2: Super-linear scaling of socio-economic rates with population. A. political unit population vs. public monument construction rates; B. settlement population vs. total domestic mound area. Symbols denote time periods, solid lines show power law fits from OLS regression of the log-transformed data, and dashed lines represent proportionate (linear) scaling. Inset shows the independence of average G on N, where $G = A/N$*(Mean Domestic Mound Area), see also Table 2.

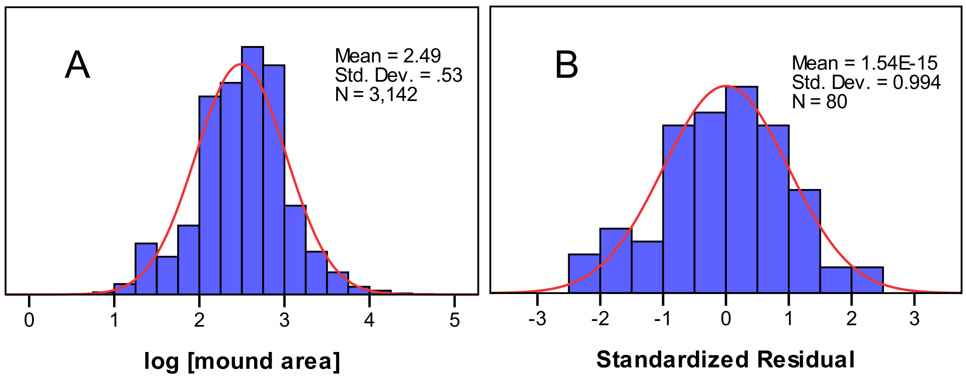

Figure 3: Histograms of Domestic-mound areas. A. distribution of log-transformed domestic mound areas across all sites; B. distribution of residuals from OLS regression of log [settlement population] vs. log [mean house area]. Note that both distributions are approximately normal (K-S (lilliefors) test results are $P < .001$ for A, $P = .2$ for B).

Dependent Variable	Sample	Pre-Factor	95% C.I.	Exponent	95% C.I.	r^2
Civic mound volume/year	48	$Y_0 = .0021$	.0006-.0070	$1 + \delta = 1.177$	1.028-1.327	.852
Domestic-mound area (m)	80	$y_0 = 168.6$	93.3-304.8	$\delta = .190$	.083-.298	.863
$Y = m * N$	80	$Y_0 = 168.6$	93.3-304.8	$1 + \delta = 1.190$	1.083-1.298	.863
$G = m * A/N$	80	$G_0 = 28.71$	12.88-63.97	$\gamma = .037$	-.108-.182	.003

Table 2: Estimated scaling parameters for socio-economic outputs with population. For the first analysis, the independent variable is the population of the political unit; for all others, the independent variable is the settlement population.

Supplementary Materials for:

Settlement Scaling and Increasing Returns in an Ancient Society

Scott G. Ortman
Department of Anthropology
University of Colorado Boulder
Boulder, CO 80309-0233

Andrew H. F. Cabaniss
Department of Classics
University of North Carolina Chapel Hill
Chapel Hill, NC 27599

Jennie O. Sturm
Department of Anthropology
University of New Mexico
Albuquerque, NM 87131

Luís M. A. Bettencourt
Santa Fe Institute
1399 Hyde Park Rd.
Santa Fe, NM 87501

Materials and Methods

We examine scaling relations in the ancient world using settlement data from archaeological surface surveys conducted in the Basin of Mexico (BOM), the epicenter of Pre-Hispanic Mesoamerican civilization. These surveys took place between 1960 and 1975, prior to the destruction of many sites by the expansion of modern Mexico City. Figures 1 and S1 show the location of our study area, the surveyed area relative to Mexico City when the surveys took place, and the distribution of Pre-Hispanic settlements for two cultural periods. We compiled a database of information for some 4,000 archaeological sites resulting from these surveys, beginning with existing digital compilations (*1, 2*) and adding information from the original survey reports (*3-11*). We also added data for a few important sites outside the surveyed area (Cuicuilco, Tenochtitlán/Tlatelolco, Tenayuca, Xochicalco) and for Teotihuacan based on information in the literature (*12-17*). In addition, we tabulated the dimensions of civic-ceremonial mounds reported in the survey volumes or in other sources (*13, 14, 18-20*) and we associated Aztec-period settlements with native political units based on ethno-historic information summarized by Hodge and others (*21-23*). The resulting database contains information on the settled area, population, time period, location, functional classification, political affiliations, and architectural remains of every recorded settlement. For more details and background on this database, see (*24*).

The BOM survey data have two great advantages for settlement scaling research. First, surveyors recorded data across the entire range of settlement sizes, from the smallest farming hamlets to the largest cities, in a common format. Second, the survey methods used allowed the population densities of settlements to vary. As a result, it is possible to examine scaling relations across five orders of magnitude in settlement population in a settlement system that spanned the urban-nonurban divide. However, a potential problem with the BOM survey data is that the method used to estimate population for most sites was not independent of the settled area (*3, 19*). This method involved: 1) determining the extent of the surface artifact scatter for each period of occupation by mapping its boundary on low-altitude aerial photos; 2) assigning each scatter to one of a series of artifact density classes based on the observed potsherd density within the scatter; and 3) multiplying the extent of the scatter for each period by a population density derived from associations of surface potsherd densities with population densities of various settlement types in 16th and 20th century records from the area. This method ensures that there will be a relationship between the settled area and settlement population. Fortunately, we have previously shown that the estimates produced by this *area-density* method are nearly identical to those produced using *house-counting* methods that do not rely upon settled area, but are instead based on the count or surface area of residential mounds at sites where such remains are well-preserved (*24*). Thus, there is a sound basis for viewing the population estimates derived from the BOM surveys as reasonably accurate in both a relative and an absolute sense.

The BOM surveys assigned remains dating to each archaeological phase at each settlement to a separate site record. We assigned these records to one of four chronological periods dating from initial colonization of the Basin up to the Spanish Conquest, following the chronology in the most recent publications of these data (*4, 7*). The Formative period (1150 B.C.E.-150 C.E.) saw the beginnings of detectable settlements and the rise of local polities; the Classic period (150-650 C.E.), the political and economic dominance of Teotihuacan (ca. 100,000 people); the Toltec period (650-1200 C.E.), the formation of a number of small competitive polities; and the Aztec period (1200-1519 C.E.), the unification of these into an empire centered on Tenochtitlán-Tlatelolco (ca. 200,000 people). Collectively, these four chronological groups span more than three millennia and capture the development of an ancient urban system that was independent of its old-world counterparts.

Population and Settled Area

Data Selection

BOM surveyors classified each site into a series of settlement types based on the spatial extent, density, and character of the surface remains. Because our theory suggests population-area scaling relations arise from the interactions of residents within settlements, this analysis excludes site types that do not conform to the "interaction container" model, namely: 1) sites lacking permanent residential populations, such as isolated ceremonial centers, quarries and salt mounds; and 2) dispersed sites consisting of isolated residences interspersed with farmland. We also exclude sites with settled areas less than 1 ha from this analysis due to limits in the precision of the recorded data. Approximately 1,500 settlements in our database meet these criteria. The resulting dataset is available at http://www.tdar.org. In addition, we tabulated the populations and settled areas of BOM settlements in the 1960 census of the area to provide a point of modern comparison (*24*).

Polity Population and Monument Construction

Linking arguments

Several factors make it possible to treat the total volume of public monuments in administrative and ceremonial centers as a measure of the total production of the subject population over a period of time. First, ethno-historic evidence suggests these monuments were constructed primarily using corvée labor, a type of labor tax in which each household owed the polity a certain number of days of labor per year (*13, 22*). Second, cross-sections of these buildings indicate that their final volumes accumulated over a period of time, as new phases of construction encased the existing structure (*13, 18, 25*). Finally, a recent study of the energetics of monument construction at Teotihuacan found that as much as 85 percent of the total cost involved in building these monuments derived from transporting earth and stone to the construction site (*18*). As a result, it is reasonable to view public monument volumes as resulting from the piling of earth and stone by subject populations over a period of time.

To relate public monuments to settlement scaling theory, several additional assumptions are necessary. First, following ethno-historic records, we assume corvée labor forces had some freedom to self-organize: "The building of the temples and the houses of the lords and public works was always a common undertaking, and many people worked together with much merriment . . . Each worked a little and did what he could, and no one hurried or mistreated him for it . . . Thus they went about their work, cheerfully and harmoniously" (Zorita, in *13:127*). Such statements suggest the labor forces involved in monument construction were mixing populations of the type that produce emergent scaling properties. Second, we assume labor forces derived from the total populations subject to specific administrative centers. During certain periods, the political organization of the BOM was characterized by multi-level administrative and decision-making hierarchies (*19, 21, 22, 26, 27*). We assume the composition of labor forces reflected this multi-tiered organization. For example, "building the Great Temple in Tenochtitlán was organized by a division of labor based on regional state or provincial affiliation. When this temple was being enlarged by Motecuhzoma I, Texcoco and the Aculhua communities built the front, Tlacopan and the Tepaneca the back, Chalca the left side, and the

Xochimilca the right. The Otomi area furnished sand and the Tierra Caliente area supplied lime" (*22*). Finally, we assume corvée labor forces represent a relatively consistent fraction of subject populations (for example, all males within a certain age range), and that the sizes of the labor force varied much more substantially than the average labor tax imposed by political authorities during different archaeological periods.

Data Compilation

Many settlements with civic-ceremonial architecture were inhabited during multiple periods. In such sites, BOM surveyors examined the types of potsherds on the tops of mounds, or eroding out of the sides, to estimate periods of construction and use. We simply carried forward these field judgments in this analysis. We also estimated the volumes of civic-ceremonial structures by modeling them as rectilinear pyramids, cones, or rectilinear solids, depending on the recorded dimensions. For rectilinear solids $V = l \times w \times h$; for pyramids and cones, when $h > 1m$, $V = 2/3\, bh$, with $b = base\ area$; otherwise $V = b$. When no height is recorded, we conservatively assume $h = 1m$. The largest concentrations of civic-ceremonial architecture in our database come from Teotihuacan, Texcoco and Tenochtitlán-Tlatelolco. The data for these specific sites derive from Murakami (*18*) for Teotihuacan; Parsons (*3:361-362*) and Smith (*13:22*) for Texcoco; and a variety of sources for Tenochtitlán-Tlatelolco (*13, 14, 20*). Table S1 presents our tabulation of public monuments at Tenochtitlán-Tlatelolco, the Mexica capital.

We estimate the average size of the labor force that built these monuments by combining population estimates in BOM survey records with current understandings of the political organization of each period. The oldest recorded public buildings date from the Late Formative (500-250 BCE) and Terminal Formative (250 BCE – 150 CE) periods. For these periods, we assume a strong correlation between settlements and political units, and thus infer that public monuments were built primarily by the inhabitants of the settlements in which they occur. During the subsequent Classic (150-650 CE) period, the entire BOM population was subject to Teotihuacan, and we therefore assume the entire BOM population contributed labor to the monuments there. In addition, we assume the smaller concentrations of civic-ceremonial structures outside the Teotihuacan Valley were built by the residents of immediately-surrounding settlement clusters (*5*). During the Early Toltec (650-900 CE) period political organization appears to have fragmented into a series of independent city-states (*27, 28*). Accordingly, we infer monuments dating to this period were built by the inhabitants of the sites in which they occur along with the residents of immediately-adjacent smaller settlements. We have not included monuments from the Late Toltec (950-1200 CE) period due to uncertainties surrounding the political organization of this period. Finally, for the Aztec period we first assign all settlements to the *altepetl* or city-state to which they belonged based on Hodge's correlations between the BOM survey data and ethno-historic sources (*21-23*); then we assign these *altepéme* to their associated triple alliance political unit (Mexica and Aculhua). Following these groupings, we infer the total population of each *altepetl* contributed labor to the monuments within those settlements; and we assume the monuments at Tenochtitlán-Tlatelolco were built by citizens of the Mexica confederacy, and the monuments at Texcoco by the Aculhua confederacy. The resulting dataset of total monument volumes and subject populations is presented as Table S2.

Settlement Population and House Area

Linking arguments

Several studies have suggested that the distribution of house sizes is a reasonable proxy for the distribution of wealth in ancient and preindustrial societies (*29-34*). In general, the distribution of house sizes follows a log-normal distribution, such that a histogram of log-transformed house sizes follows a normal, bell-shaped curve. Income distributions in contemporary societies are also typically log-normal (*35*), thus supporting the idea that domestic mound areas offer a useful proxy for production levels in archaeological contexts.

In many societies, larger residences house larger domestic groups, which can include extended families, lineages, corporate groups, servants and retainers (*33*). The clearest examples from the Pre-Hispanic BOM are the apartment compounds of Teotihuacan, which clearly housed large corporate groups (*17, 36, 37*); but one would expect servants and retainers to have lived with elite residents in the largest domestic structures dating to other periods as well. Given the difficulties involved in specifying the relationship between domestic space and people across an ancient complex society, we adopt a functional definition of the household as the basal unit of production and consumption (*38*) and associate this unit with domestic mounds recorded in the BOM surveys. From this perspective, domestic mound areas are a proxy for the relative production levels of basal socioeconomic units. Even if the number of people comprising these basal units varied with the socio-economic status and position of residents in the political hierarchy, the range of such variation (say, 10 to 100 persons) and its scaling effect would have been small relative to aggregate effects derived from overall settlement populations (10 to 200,000 persons) and their associated social interaction networks. We thus expect summary statistics for house areas sampled across a settlement to be driven primarily by properties of the social networks in which all households are embedded.

An interesting question with respect to comparisons of archaeological data with modern economic data is whether house size is a better proxy for income inequality or wealth inequality. On the one hand houses are forms of capital investment that can be inherited, and are thus a form of wealth. On the other, most households in ancient societies stored the food and other goods they produced directly in their residences, thus suggesting house area is a function of the income of the resident group. Although it would seem that house areas are responsive to different aspects of inequality depending on the social position of the occupants, we suspect they are a better overall proxy for income than for capital accumulation, for three reasons. First, in modern societies capital ownership tends to be highly concentrated, such that nearly all wealth is owned by the top half of households (*39*), but the lower half of households still live in a dwelling, with the size of the dwelling being related to household income via rents or mortgages. Second, in modern societies income derives from both labor (wages) and wealth (rents, interest, dividends), and it seems reasonable to suggest that the ruling elites of ancient societies had incomes of both types (direct production, rents, taxes and tribute). Third, the long-term wealth to income ratios of ancient societies were likely somewhat lower than is typical of modern societies due to relatively low savings rates and high demographic rates (*39*). For these reasons it seems that house areas in ancient pre-capitalist societies are more comparable to incomes in modern societies.

Data Compilation

The primary data for this analysis consist of domestic mound areas recorded in BOM survey reports (*3-11*). The dimensions of domestic mounds are somewhat larger than those of the original house that produced the mound, but excavations suggest house floor areas are generally proportional to domestic-mound areas (*11, 40*), so the distribution of domestic-mound areas should mirror the underlying distribution of house areas. In order to examine mean domestic-mound areas from the full range of settlement sizes in Central Mexico, we included additional data for 156 domestic mounds at Xochicalco, an Epi-Classic (Early Toltec) city of ca. 10,000 people some 50km south of the BOM; the areas of 14 apartment compounds at Teotihuacan, a few of which are referred to as "palaces" (*17, 18*); and 56 house floor areas tabulated from ethno-historic sources for Tenochtitlán (*13, 16*). For the Tenochtitlán data, we multiplied the recorded house-floor area by 4 to convert these areas into estimates of domestic-mound area. This conversion factor derives from excavations at Cihuatecpan, an Aztec-period settlement at which excavated house areas are approximately one-fourth the area of the unexcavated mound (*40*) across all house sizes. We also included the three imperial palace areas from Table S2 in our data for Tenochtitlán-Tlatelolco because they were elite residences as well as administrative buildings. Palaces from Texcoco were also included, using data from (*13*).

Using these data, we computed the mean of domestic-mound areas for 80 settlements that are at least 1 hectare in area, possess well-preserved architectural remains, and are associated with at least two recorded domestic mounds. Then, we multiplied this area by the population of the settlement to create a measure of Y, the total production of that settlement, and we multiplied the area per person at that settlement by the mean domestic mound area to calculate a measure of G, $G = Y\frac{A}{N}$. This dataset is presented as Table S3.

It is important to acknowledge the limitations of these data. For many smaller settlements it is reasonable to treat the available domestic-mound dimensions as a representative sample of the total houses at those settlements. There are also a few larger, especially well-preserved settlements (TE-CL-8, IX-A-26, TX-A-24, Xochicalco) where the distribution of domestic-mound areas in the available sample likely reflects the real distribution across the population of houses. But for many settlements we do not have a representative sample of domestic mound areas, and we therefore expect the sample means to have significant errors relative to the population means. However, so long as these errors are unstructured, and so long as we compare data from a reasonable number of settlements spanning the range of variation in the data, we would still expect scaling analysis to be able to recover the average underlying relationship between household production and settlement population.

An interesting result of our approach is that it enables a direct comparison of the share of income that went to the top decile of households in the Pre-Hispanic Basin of Mexico and in 20th century Europe and the United States. Table S4 presents summary statistics for measured domestic mounds dating from the Classic, Toltec and Aztec periods. These data show that, although median house area (and most likely household composition) declined through time, the distribution of house area across the population was relatively consistent, with 40-50% of domestic-mound area occurring in the top decile of house areas and 10-15% occurring in the bottom half. The share of house area in the top decile is comparable to the share of income in the top decile in contemporary US society, whereas the share of house area in the bottom half is most comparable to the share of wealth in the bottom half of contemporary US society (*39*). In light of the discussion above concerning the interpretation of house-area distributions, these data suggest either that disparity of income was more extreme in the Pre-Hispanic Basin of Mexico than in contemporary US society, or that disparity of wealth was less extreme.

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Figure S1. The Basin of Mexico. A: Location within Mexico. B: Settlements during the Formative period (circle size is proportional to population; colors denote elevation; outline marks surveyed area; gray area is the extent of Mexico City in 1964). C: Settlements during the Aztec period. During the latter period settlement expanded into the shallows of the lake. Today, modern settlement covers the entire basin and the lake has been drained.

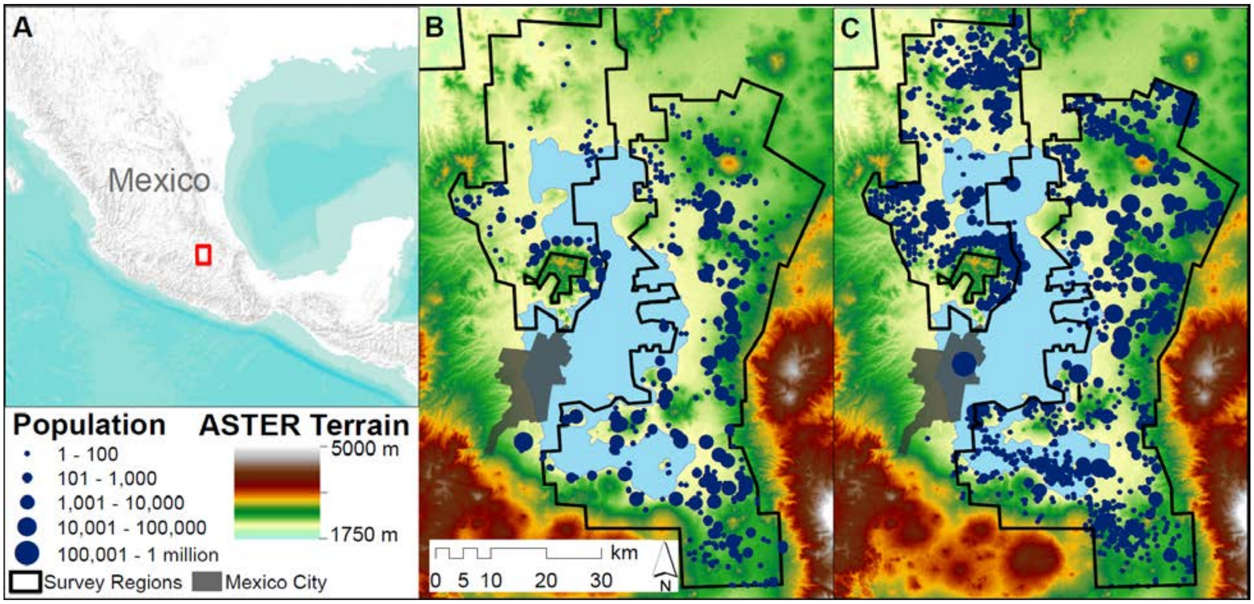

Table S1. Civic-ceremonial structures at Tenochtitlan-Tlatelolco. All dimensions are in meters.

Structure	Comments	Height	Length	Width	Area	Volume
Huitzilopoctli Temple	The great temple of Huitzilopoctli, data from Aguilar-Moreno (2005).	30	80	100	8000	160000
Palace 1	Motecuhzoma Ilhuicamina palace. Area from Smith 2008:Table 4.3.				7950	7950
Palace 2	Axayacatl Palace. Area from Smith 2008:Table 4.3.				11439	11439
Palace 3	Motecuhzoma Xocoyotzin Palace. Area from Smith 2008:Table 4.3.				25425	25425
Tenochtitlan Precinct	Square platform for the great temple, data from Sanders 2008:75.	11.5	400	400	160000	1840000
Tlatelolco Precinct	Area from Sanders (2008:69), est. 2m high on average, following Aguilar-Moreno (2005).	2	200	325	65000	130000
Tlatelolco Temple	Temple of Tlatelolco. Smith 2008:102 indicates this was the same size as the Great Temple.	30	80	100	8000	160000
Tezcatlipoca Temple	Temple of Tezcatlipoca, data from Aguilar-Moreno (2005).	20	40	60	2400	32000

Table S2. Civic-ceremonial architecture volumes and associated subject populations.

Polity	Period	Time Span (yrs)	Volume (m^3)	Political Unit Population	Notes
CH-LF-5	LF	250	5817	5200	
CH-TF-16	TF	400	11667	2200	
Cuicuilco	TF	400	85333	20000	Data from Sanders et al. (1979:99).
IX-TF-4	TF	400	3414	1850	
IX-TF-3	TF	400	133	200	
IX-TF-5	TF	400	22922	1600	
IX-TF-8	TF	400	100	50	
TX-TF-1	TF	400	3672	2650	Includes TX-TF-2
TX-TF-14	TF	400	800	500	
TX-TF-36	TF	400	2400	1200	

TX-TF-4	TF	400	6066	600	
TX-TF-51	TF	400	11467	1200	
TE-TF-101	TF	450	579	200	Tezoyuca hilltop center.
TE-TF-23	TF	450	576	440	Tezoyuca hilltop center.
TE-TF-26	TF	450	409	300	Tezoyuca hilltop center.
TE-TF-33	TF	450	557	320	Tezoyuca hilltop center.
TE-TF-34	TF	450	2125	350	Tezoyuca hilltop center.
Teotihuacan	CL	500	3020450	150000	Population is the total Classic-period BOM population.
CH-CL-12	CL	500	525	1485	Large hamlet; population is total of Cluster 1 from Parsons et al. 1982:334.
CH-CL-24	CL	500	2176	1710	Small dispersed village; population is total of Cluster 3 from Parsons et al. 1982:334.
IX-EC-17	CL	500	533	150	Small dispersed village.
IX-EC-18	CL	500	2400	550	Large dispersed village; time span includes Late Classic component (IX-LC-3).
IX-EC-7	CL	500	1250	900	Local center; time span includes Late Classic component (IX-LC-1).
TX-EC-26	CL	500	533	130	Large hamlet; population includes adjacent TX-EC-27.
CH-ET-31	ET	250	4083	800	Large nucleated village.
CU-ET-21	ET	250	3183	850	Regional Center.
TE-ET-21	ET	250	945	1015	Large Nucleated Village.
TX-ET-18	ET	250	83567	12775	Regional center; includes TX-ET-17 as per Parsons 1971:75 and IX-ET-1 as per Blanton 1972:86.
TX-ET-23	ET	250	1298	560	Small dispersed village; includes TX-ET-22.
TX-ET-4	ET	250	4320	7200	Regional center; includes TX-ET-5 as per Parsons 1971:70.
TX-ET-7	ET	250	47951	8000	Regional center.
ZU-ET-12	ET	250	6803	1500	Local center.
Tenochtitlan-Tlatelolco	AZ	320	2366814	231375	Population is Mexica portion of Triple Alliance.
Texcoco	AZ	320	681333	90975	Population is Acolhua portion of Triple Alliance.
Ixtapalapan	AZ	320	11819	4933	
Ixtapalucan	AZ	320	38607	2106	
Tenanco	AZ	320	40105	7265	
Tenayuca	AZ	320	140000	12500	Population is the average of other

					regional centers (Hodge 1997:220).
Amecamecan	AZ	320	11250	10585	
Tuexotla	AZ	320	44500	25590	
Chimalhuacan	AZ	320	14507	13290	
Coatepec	AZ	320	34317	3370	
Coatlinchan	AZ	320	23100	11890	
Tepetlaoztoc	AZ	320	60473	23525	
Cuitlahuac	AZ	320	21000	10125	
Xochimilco	AZ	320	14241	3595	

Table S3. Mean domestic-mound areas and settlement populations. All mound areas are in square meters.

Site	Population	Area (ha)	Mean Domestic-Mound Area	Sum of Mound Areas	Area of Largest Mound	Number of Mounds
ZU-LC-10	40	2.4	102	203	112	2
TE-CL-90	70	1.2	2133	8530	3330	4
TE-CL-3	90	1.5	568	1135	745	2
TE-CL-7	90	1.5	948	4738	2700	5
TE-CL-78	90	1.5	1825	3650	1850	2
TE-CL-32	240	4.4	1462	16085	3300	11
TE-CL-40	570	9.5	2925	14625	5000	5
TE-CL-31	600	10	2556	46008	20000	18
TE-CL-8	630	10.5	815	56209	3650	69
TE-CL-30	1170	19.5	1829	36583	7000	20
TE-CL-73	1260	21	1562	68744	5360	44
Teotihuacan	95597	2253	2942	35309	4875	14
TX-ET-12	30	3	144	576	144	4
TX-LT-18	40	2	425	850	625	2
TX-LT-31	40	2	62	248	132	4
ZU-LT-117	40	2.1	225	675	225	3
ZU-LT-81	70	3.3	935	4677	3025	5
Xochicalco	11984	245	950	148197	10658	156
CH-AZ-257	20	1.5	198	396	209	2
CH-AZ-280	20	2.6	165	823	350	5
CH-AZ-63	20	1	170	340	196	2
XO-AZ-14	20	1.5	635	1270	1000	2
XO-AZ-59	20	1.5	275	550	400	2

ZU-AZ-214	20	1	101	304	144	3
IX-A-68	30	1.7	625	2500	625	4
ZU-AZ-88	40	2	225	450	225	2
TE-AZ-3	42	7.4	225	1350	225	6
TE-AZ-31	42	11	506	1519	506	3
TE-AZ-15	49	8	625	3125	625	5
XO-AZ-37	50	4	632	1895	1400	3
XO-AZ-49	50	5	649	3244	1250	5
TE-AZ-5	60	1.5	863	5175	1925	6
TX-A-34	60	6	219	1973	625	9
XO-AZ-47	60	8.8	446	3122	1150	7
ZU-AZ-84	60	3.1	683	2050	1600	3
TE-AZ-50	70	4.7	173	693	216	4
TX-A-32	80	4	173	1212	625	7
TE-AZ-16	98	17	400	5600	400	14
TE-AZ-45	100	38	875	8750	1633	10
TE-AZ-14	105	26	306	3675	306	12
TE-AZ-32	105	22	400	3600	400	9
TE-AZ-78	105	12	636	6994	1406	11
CU-AZ-254	110	2.8	764	8400	1575	11
TE-AZ-77	140	24	792	10293	2500	13
TX-A-26	150	10	185	2591	441	14
TX-A-39	150	10	122	1834	400	15
TE-AZ-28	175	28	1109	17750	2500	16
TE-AZ-61	175	24	400	4400	400	11
TE-AZ-25	210	37	625	11875	625	19
TE-AZ-58	210	135	900	17100	900	19
TE-AZ-73	210	74	450	7644	900	17
TE-AZ-74	210	33	422	7175	900	17
TE-AZ-75	210	36	390	6236	1800	17
TE-AZ-82	210	21	566	11880	1406	22
TE-AZ-18	245	28	1241	31025	2500	25
TE-AZ-27	245	35	529	11644	2025	22
TE-AZ-51	245	14	301	7524	1024	25
TE-AZ-39	280	101	400	11600	400	29
TE-AZ-40	280	18	624	13100	900	21
TE-AZ-41	280	9	288	6630	550	23
TE-AZ-46	280	15	139	1806	139	13
TE-AZ-85	280	43	376	11269	1225	30
TX-A-27	300	17	203	5472	1650	27
TE-AZ-43	315	18	437	8739	891	20

TE-AZ-29	350	44	306	11331	306	37
TE-AZ-36	350	72	625	20000	625	32
TE-AZ-57	350	161	874	24475	1225	28
TE-AZ-63	350	60	400	9600	400	24
TE-AZ-62	420	47	400	14000	400	35
TE-AZ-71	420	99	564	15794	1600	32
TE-AZ-38	490	88	650	1300	1000	2
TE-AZ-101	630	102	862	37947	5625	44
TE-AZ-103	630	103	652	42378	3516	66
TX-A-38	650	65	129	3355	400	26
TX-A-31	700	50	96	4873	375	51
TX-A-59	750	75	125	1878	400	15
TX-A-30	1500	100	120	10889	3025	91
IX-A-26	1630	90	845	4226	1456	5
TX-A-24	13500	450	244	27521	1600	114
Tenochtitlan	212500	1350	2227	131398	25425	59

Table S4. Domestic-mound area distributions in the Basin of Mexico through time.

Period	Measured Mounds	Total Area	Med -ian	90^{th} %-ile	100^{th} %-ile	Share in Top 10%	Share in Bottom 50%
Classic (150-650 CE)	398	420706	490	2755	20000	0.45	0.11
Toltec (650-1200 CE)	355	258772	446	1536	10658	0.42	0.16
Aztec (1200-1519 CE)	2389	1377190	306	1000	33750	0.50	0.13