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Francisco Rodriguez C.

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# Inequality and Redistribution in Multiple Dimensions

Francisco Rodríguez C.  
University of Maryland<sup>1</sup>

**Abstract:** A three-class game of redistribution without a pure strategy equilibrium is studied under a model of adaptive politicians. This model allows us to integrate one person - one vote models of redistribution with the models which emphasize political influence so as to study when different social classes will form coalitions against each other and what types of redistributive policies are likely to emerge from that interaction. For a wide range of parameter specifications, we find that inequality is negatively related to redistribution.

## 1. Introduction

Why do some societies redistribute more than others? This paper argues that common explanations of the causes behind redistributive policies are incomplete because they have excessively relied on median voter theory. Median voter theory, as is well known, is restricted to the study of politics when preferences over issues can be reduced to a unidimensional scale. We will argue that redistributive conflict is inherently

multidimensional and that therefore median voter theory lacks applicability in this context.

To escape from median voter theory, this paper uses a computational approach based on the work of Kollman, Miller and Page (1992). Instead of studying how vote-maximizing Downsian parties would behave when voters have preferences over redistributive policies, we instead study how boundedly rational political actors behave. Our boundedly rational politicians are, however, limited by one very important constraint which is that of surviving politically. Thus we assume that an evolutionary process constantly weeds out unsuccessful politicians. Politicians also learn from their mistakes and imitate winners. We model the behavior of politicians in a manner that loosely resembles the learning dynamics of the genetic algorithm (Mitchell, 1996).

Our model has something particularly useful to say about the discussion between two styles of median voter models of redistribution: the *one person – one vote* median voter models (Meltzer and Richard, 1981, Alesina and Rodrik 1994, Persson and Tabellini 1994) in which the only way through which individuals can affect the political decisionmaking process is by voting, and the *political influence* models of redistribution (Rodríguez, 1997, Benabou, 1997) in which richer individuals can deploy their political power to influence politicians and thus enjoy political clout which is more than proportional to their proportion of the population. The empirical discussion between these models centers around the empirical issue of whether inequality is positively or negatively related to redistribution. One person one vote models hypothesize a positive relation whereas political influence models hypothesize a negative relation. However, as

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has been pointed out by Rodríguez (1997), political influence models can be best viewed as models of the supply side of redistribution (what motivates policymakers to adopt certain policies) whereas one person one vote models characterize the demand side of redistribution (what policies are preferred by voters). However, whether the supply or demand sides dominate depends in great part on whether individuals vote over a linear or a progressive tax rate. This is because with a totally progressive tax rate the effect of inequality on redistribution in demand-side models disappears. Typical demand side models solve this problem by assuming that voters are restricted to using a linear income tax rate.

Whether individuals vote over a linear or a progressive tax rate is an issue that is not satisfactorily dealt with when it is solved by assumption. But that is precisely what both one person-one vote and political influence models do, because of their inability to tackle multidimensional issues in a median voter context. In principle, we would prefer to have models of redistribution that are able to endogenously derive when the political equilibrium will be one with a progressive tax rate and when it will not. Such a question cannot be addressed within the framework of a median voter model, but it can be dealt with in the context of our model.

Section 2 characterizes in detail the multidimensional nature of redistributive problems and discusses the problems that arise when this multidimensionality is assumed away by arbitrarily restricting the policy space available to voters. In section 3 we present a computational demand-side model with adaptive politicians and show that usual demand-side intuitions about redistribution are confirmed in this setting: inequality is positively associated with redistributive spending. In section 4 we integrate this model

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support for the writing of this project. They must not, however, be held responsible for its many flaws.

with a supply-side model and find that the supply-side factors tend to dominate, making inequality negatively associated with redistribution. Section 5 concludes.

### 1. Why we should care about multidimensionality

Consider an extremely simple setup with three classes: the poor, the middle class, and the rich. Let them respectively have incomes  $y_l$ ,  $y_m$ , and  $y_r$ , and let each group represent 1/3 of the population. Suppose that they can be taxed with taxes (which can be negative, in which case they would represent a subsidy)  $t_l$ ,  $t_m$ , and  $t_r$ , which must satisfy a balanced budget constraint:

$$t_l y_l + t_m y_m + t_r y_r = 0$$

Then for any policy (that is, a vector  $t_l$ ,  $t_m$ , and  $t_r$ ), there is an alternative policy which would be preferred by a majority of voters. To see why, suppose that we raise the tax rate on one individual (say  $r$ ) by  $\epsilon > 0$ . Then we can lower the tax rates of  $l$  and  $m$  by an amount such that the additional tax rates collected by the tax on  $r$  is distributed equally among  $l$  and  $m$ . If the tax rate on  $r$  is already 1 and cannot be raised, then try the same experiment by lowering one of the tax rates which is less than 1 (by the balanced budget constraint, all 3 tax rates could never be 1).

The offshoot of this example is that the median voter model does not apply because there is no policy that is able to defeat all other policies. The basic reason behind this is that there is a basic multidimensionality in the problem: voters are voting over three tax rates (two, really, because the third one is determined by the balanced budget constraint). The concept of equilibrium used to solve political models cannot be used in this setting because such a concept requires finding a policy that can defeat all other challengers.

Typically, models of redistribution will solve this problem by restricting the tax rate to being either linear or steeply progressive. When the tax rate is linear, one can derive the usual result that inequality (as measured by the mean/median spread) is positively associated with redistribution: as the mean/median ratio rises the cost of higher taxes borne personally by the median voter falls, leading the median voter to vote for higher taxes. But if the tax rate is steeply progressive, no such result can be derived: when the tax rate is completely non-linear the median voter need not pay taxes and a fall in the median voter's income has no effect on the cost of redistribution and therefore on the equilibrium policy.

Ideally, one would like models of redistribution to be able to endogenously derive the progressivity of the tax schedule. But this is impossible within the framework of median voter theory. Progressive tax rates are tax rates that vary with the level of income, and thus look like the example with which we opened this section. For the reasons illustrated in that example, it is impossible for us to find an equilibrium tax rate using median voter concepts of equilibrium.

Our approach to this problem relies on studying a model with adaptive politicians. In our model, politicians propose policies. Elections measure up the policies of an incumbent politician against those of a challenger, and whoever receives a majority of votes has their policy implemented and becomes the incumbent. After the election is over, each of the members of a potential pool of challengers alters their strategies in one of two ways. They either have new ideas or they copy somebody else's ideas. When they copy somebody else's ideas, it is the incumbent's ideas, which have after all proved electorally successful.

It is easy to see that within such a setup policies will cycle endlessly without ever reaching a stationary state. This is precisely because there is no equilibrium in pure strategies in this game: there is always a policy that defeats the incumbent policy, and challengers will always, if only by chance, find it. But we can instead center on another question: what is the average level of tax rates around which the economy spends most of its time? What are the implied levels of redistribution given those tax rates? What kind of coalition do they represent?

Our model is extremely useful for understanding whether one would expect a negative or a positive relationship between inequality and redistribution to emerge in the data. As we have discussed above, one person one vote models emphasize the demand side of redistribution and hypothesize a negative link between inequality and redistribution; political influence models emphasize the supply side and reach the opposite conclusion. Combining the demand and supply sides is impossible without placing unsatisfactory restrictions on the progressivity of the tax schedules available to voters and the conclusions depend on those restrictions. The only way to satisfactorily integrate the demand and supply sides is with a model that takes seriously the multidimensional nature of redistributive politics. We can thus use our model of adaptive politicians to predict whether the supply or demand sides are likely to dominate; that is, whether we are likely to see a positive or a negative effect of inequality on redistribution.

To analyze this issue we first set up a demand-side only model with adaptive politicians in the next section. In section 4 we look at a model that integrates political influence and thus combines the demand and supply sides of redistribution.

### 3. A one-person one-vote model of redistribution in multiple dimensions

This section sketches out a basic model of redistribution with adaptive politicians and three classes. Each class comprises 1/3 of the population. They are each subject to a different tax rate and the total of taxes collected are used to finance a lump-sum subsidy which satisfies the budget constraint:

$$y_l t_l + y_m t_m + y_r t_r = \frac{s}{3}$$

Politicians are characterized by their proposed tax policies, which are sets of tax vectors  $t_l, t_m, t_r$ . Individuals will choose among a vector of policies chosen by the incumbent politician and one chosen by a challenger. The incumbent politician is determined by history as the winner of the past election. The challenger politician is chosen at random from a pool of  $n$  challengers<sup>2</sup>.

The adaptive behavior of politicians is captured in how challengers change their strategies in response to election results. After an election, each of the  $n$  potential challengers will do one of two things: 1. With probability  $p_{\text{mute}}$  it will have a “new idea”; that is, it will replace one of its tax policies by a new tax randomly selected from a uniform distribution in the 0-1 interval. This is analogous to the “mutation” operator in the genetic algorithm. With probability  $p_{\text{cross}}$  it will decide to copy one of the taxes proposed by the politician who won the last election. This is analogous to the “crossover” operator in the genetic algorithm.

To run the model, we first generate an initial incumbent and an initial pool of challengers by selecting their tax rates from uniform distributions on the 0-1 interval. We

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<sup>2</sup> The C++ code for the program is available from the author upon request.

also generate incomes for the three classes (with distributional assumptions that ensure  $y_r > y_m > y_l$  as well as that average income is constant). The process is then iterated for 500 elections to ensure that the final tax rate is not affected on average by the distribution of initial policies<sup>3</sup>. The final tax rates are stored along with the income distribution and the process is repeated 9000 times.

A scatter plot of these 9000 points is given in Figures 1-4. They show the tax rates on the poor, the middle class and the rich as well as the average tax rate plotted against income inequality. The simple correlations and average values of the key variables are in Table 1.

There are several interesting insights from these figures. The first is that a progressive tax rate emerges endogenously from the model: the tax rates on the poor are lower than those on the middle class, which are in turn lower than those on the rich.

The rest of the results broadly serve to confirm the intuition of median voter style demand side models of redistribution. The tax rate on the rich and the average tax burden are steeply increasing in inequality. And although the tax rates on the poor and the middle class are only weakly correlated with inequality, the net resource transfers to each class are very highly correlated with inequality.

One interesting feature is that the model predicts that both the poor and the middle class tend to benefit from redistribution. It is interesting to note that there is no reason to expect this a priori: the middle class could just as well have united with the rich against the poor, or the poor and the rich could have formed coalitions to milk the middle class for resources. But on average it pays more to the poor to ally themselves with the middle

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<sup>3</sup> Our simulations indicate that the variances in tax policies across simulations tends to reach its minimum by election 200.

class: they are the ally that takes less resources to keep content and together they can take a big chunk out of the resources of the rich. The high resources of the rich make that group the most likely target of redistributive policies because an equal increment in tax rates on the rich is likely to produce much more resources to redistribute among other classes than the same increment on the poor or the middle classes' taxes. Adaptive politicians will thus be more likely to find changes in tax rates that harm the rich and satisfy both the poor and the middle class.

The pictures in Figures 5 and 6 can best illustrate this argument. These figures plot in the  $\{t_1, t_2, t_3\}$  space the indifference spaces of the three individuals over combinations of tax rates. Figure 5 plots these indifference spaces for an egalitarian income distribution. Individual 1's indifference surface is increasing in the  $t_1$ - $t_3$  space as well as in the  $t_1$ - $t_2$  space as individual 1 is willing to tolerate an increase in his tax rate if it is compensated by an increase in another agent's tax rate so as to leave the net resource transfer to 1 unchanged. Individual 1's utility increases in the direction in which  $t_1$  is lowered. Similar statements characterize the indifference surfaces of 2 and 3. By construction, the space over which the utilities of any two agents increase has equal area, as agents are assumed identical.

Figure 6 shows what happens when the egalitarian income distribution implicit in Figure 5 becomes unequal. In constructing Figure 6 we have assumed that individual 2 is richer than 1 and 3 is richer than 2. Note that individual 3's indifference surface has moved much more to the center of the three dimensional space. So have the surfaces for 1 and 2, but whereas this has increased the area in which policies improve both 1 and 2's utilities, they have shrunk the area over which the utilities of 3 coincide with any of the

other two agents. This is because in Figure 6a individual 1 and 2's utilities increase as one turns away from the  $t_1$ - $t_2$  axis but individual 3's utility decreases in this direction.

Adaptive politicians get new ideas by randomly sampling in the  $t_1$ - $t_2$ - $t_3$  space with a uniform distribution over each tax rate. In other words, every point in this space is equally likely to be tried out. If the zones of coincidence of interests between 1 and 2 have expanded and those between any of these agents and 3 shrunk then adaptive politicians are more likely to find policies that put together a coalition of 1 and 2. And that is indeed what is happening in the simulations: in 92% of the simulations, the ruling coalition is one of poor and middle class (6% of the time it is the poor and the rich and 2% of the time the middle class and the rich which are in power).

Figures 5 and 6 may raise doubts about whether the results we find are an artifact of the fact that the mutation process is defined on the space of tax rates. If the process whereby new ideas are imitated were instead selected on some other space, would this alter the results? Although we have no way of testing the results for any number of ways in which redistributive policies can be coded, Figure 7 shows the result of one straightforward alternative in which the political innovation process occurs over the space of net tax rates. In this case politicians generate ideas over two net tax rates (the third one being defined by the balanced budget constraint). As Figure 7 shows, the result that inequality is positively linked to redistribution in a demand-side model is maintained: the net tax rate of the poor is steeply decreasing in the level of inequality (which means that the net resource transfer to the poor is steeply *increasing*); the correlation is -.46. This suggests that our results are not an artifice of how the variables are coded. It appears

that, irrespective of how policies are coded, it will still be the case that greater levels of inequality are associated with greater political payoffs to taxing the rich.

#### **4. Bringing political influence into the picture**

The model sets out in the previous section has concentrated on studying the demand for redistribution in the sense of understanding how voters' preferences over redistribution will change with inequality. We now integrate a model of the supply of redistribution in which politicians' motivations to carry out certain policies is taken into account. We follow Rodríguez (1997) by modeling agents not only as voters but also as potential contributors to politicians' campaigns. Agents can make an offer to politicians based on a promise of a tax favor in return when the politician is in power. These tax favors are meant to stand for a wide array of rent-seeking activities. The equilibrium of the bargain between politicians and campaign contributors is characterized in Rodríguez (1997); what is important for us is that only agents above a certain level of income  $y^*(t)$  will decide to enter the process of political bargaining, poorer agents will not try to buy off politicians as they do not have the necessary scale to do so. It is also important that  $y^*$  is declining in the tax rate that applies to the person: the more government threatens to take away property, the more incentive individuals have to try to buy them off.

We incorporate this model into our computational exercise by modifying our experiment so that whenever a tax rate is proposed that would make the income of the rich fall above the threshold  $y^*(t_r)$  then effectively no taxes are collected on the rich (it is assumed therefore that the rich were able to buy off all the politicians in this case). The

rich will now base their comparison of politicians not only on the net resource transfer but on how the policies adopted by politicians affect the price the rich must pay for those politicians: higher tax rates raise the bargaining power of politicians vis-à-vis campaign contributors. We assume for simplicity that  $y^*(t)$  is always large enough that only the rich will ever have an incentive to enter into bargains with the politicians

Figures 8-10 and Table 2 show the results of running this version of the model. The correlation between the average tax rate on the rich and inequality has now become a strong -.8. The tax structure is no longer progressive; although the middle class pay higher taxes than the poor they pay less than the rich. The average tax rate and the net resource transfer to the poor are also negatively correlated with the Gini coefficient.

This simulation provides encouraging results for supply-side models of redistribution in that they show that integrating the traditional demand side with a political-influence based supply model is likely to show results hypothesized in supply-side models; namely a negative correlation between inequality and redistributive policies. As Rodríguez (1997) has argued, this pattern seems to characterize the experience of OECD countries over the postwar period.

The simulations underlying Table 2 and Figures 8-10 are run under the assumption that  $t(1)=5$ ; under our parametric assumptions, this implies that if the tax rate were 75% only the top sixth of the rich (the top 1/18 of the total population) would choose to buy into political influence. It may be that this implies too much – or too little – political influence for the rich so that it is worth asking how sensitive our results are to changes in this parameter. These results are set out in Table 3, in which  $t(1)$  is varied from 2 to 6. At  $t(1)=7$ , we are back in the demand side case because there would never

be any reason for the rich to buy political influence; at  $t(1)=1$  the middle class would have an incentive to buy into the political market. Within that range, it appears that the result that inequality is negatively related to redistribution is quite robust: it seems to only weaken (and only partially so) towards the upper limit of this range.

## 5. Conclusions

This paper has analyzed the effect of studying redistributive politics when multidimensionality of the policy space is taken seriously. This multidimensionality makes political models intractable through the median voter concept of political equilibrium. To address this problem, we have looked at the behavior of adaptive politicians in this policy space.

Our results are encouraging for supply side models of redistribution, which emphasize the effect that the motivations of politicians above and beyond that of pleasing voters can have on redistributive variables. Under a wide range of parametric specifications, redistribution is seen to be negatively correlated with inequality, as predicted by supply-side models. They are also encouraging in establishing a practical issue which can be addressed with the modeling tool of adaptive politicians, going beyond Kolman, Miller and Page's contribution which was framed in a general multi-policy setting.

However, the results still point to quite a few gaps in our understanding of redistribution. One of them is that our models still produce tax rates which are much higher than observed; as pointed out by Rodríguez, 1997, introducing labor leisure choice or capital accumulation into the models does not solve this problem. Other gaps are more related to its incapacity to capture political mobilization and its effect on redistribution.

To the extent that the poor have been consistently observed to have lower participation rates than the rest of the electorate, their political clout is lessened; it would be useful to understand theoretically the effects that inequality and other variables could have on these participation rates.

One last question refers to the pervasiveness of coalitions which include the poor in our models. Even in the simulations of chapter 4, middle class – rich coalitions account for only 5 % of occasions (poor-rich coalitions occur 50% of the time, and poor-middle class coalitions 45%). But coalitions of the richest two thirds of the population are pervasive in the modern world; more often than not it is precisely the poor that get excluded from governing coalitions. Understanding this phenomenon would be of great usefulness.

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**Table 1: Average values and correlations with Gini, one man one vote model**

	Average Value	Correlation with Gini
Tax on Poor	47	-.01
Tax on Middle Class	63	.12
Tax on Rich	88	.35
Average Tax Rate	81	.58
Net Resource Transfer to Poor	24 %	.82
Net Resource Transfer to Middle Class	15%	.47
Net Resource Transfer to Rich	-40%	-.81

**Table 2: Average values and correlations with Gini, political influence model**

	Average Value	Correlation with Gini
Tax on Poor	57	-.29
Tax on Middle Class	82	.12
Tax on Rich	69	-.80
Average Tax Rate	63	-.60
Net Resource Transfer to Poor	23 %	-.45
Net Resource Transfer to Middle Class	13%	-.03
Net Resource Transfer to Rich	-36%	.61

**Table 3: Sensitivity of correlations to political influence threshold**

Threshold	4	6	8	10	12
Gini, Avg. Tax	-.54	-.55	-.59	-.6	-.04
Gini, Tax on Rich	-.4	-.61	-.73	-.8	-.51
Gini, Net Resource Transfer to Poor	-.39	-.40	-.42	-.45	-.71

Figure 1: Tax on Rich and Inequality

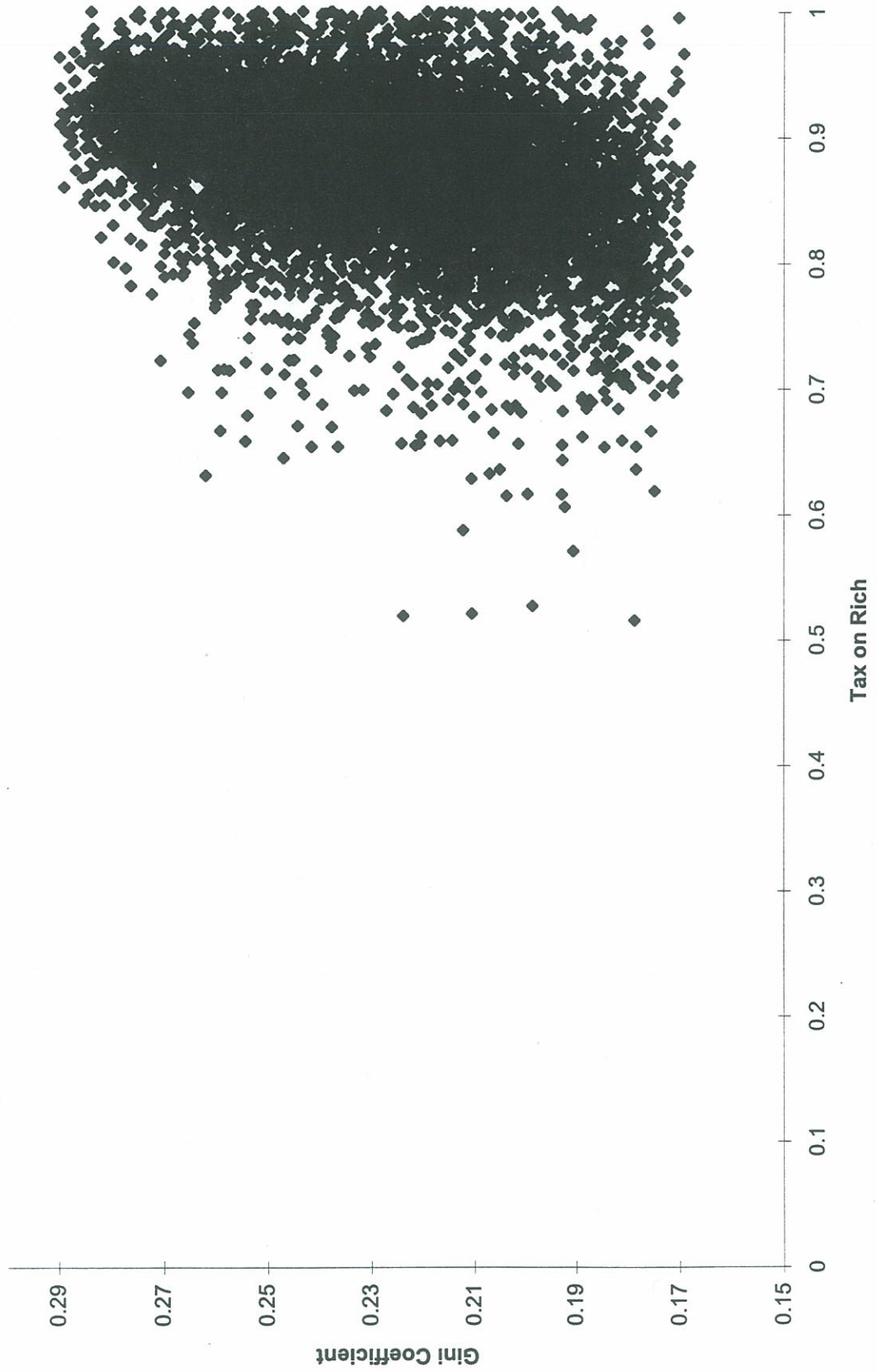


Figure 2: Tax on Middle Class and Inequality

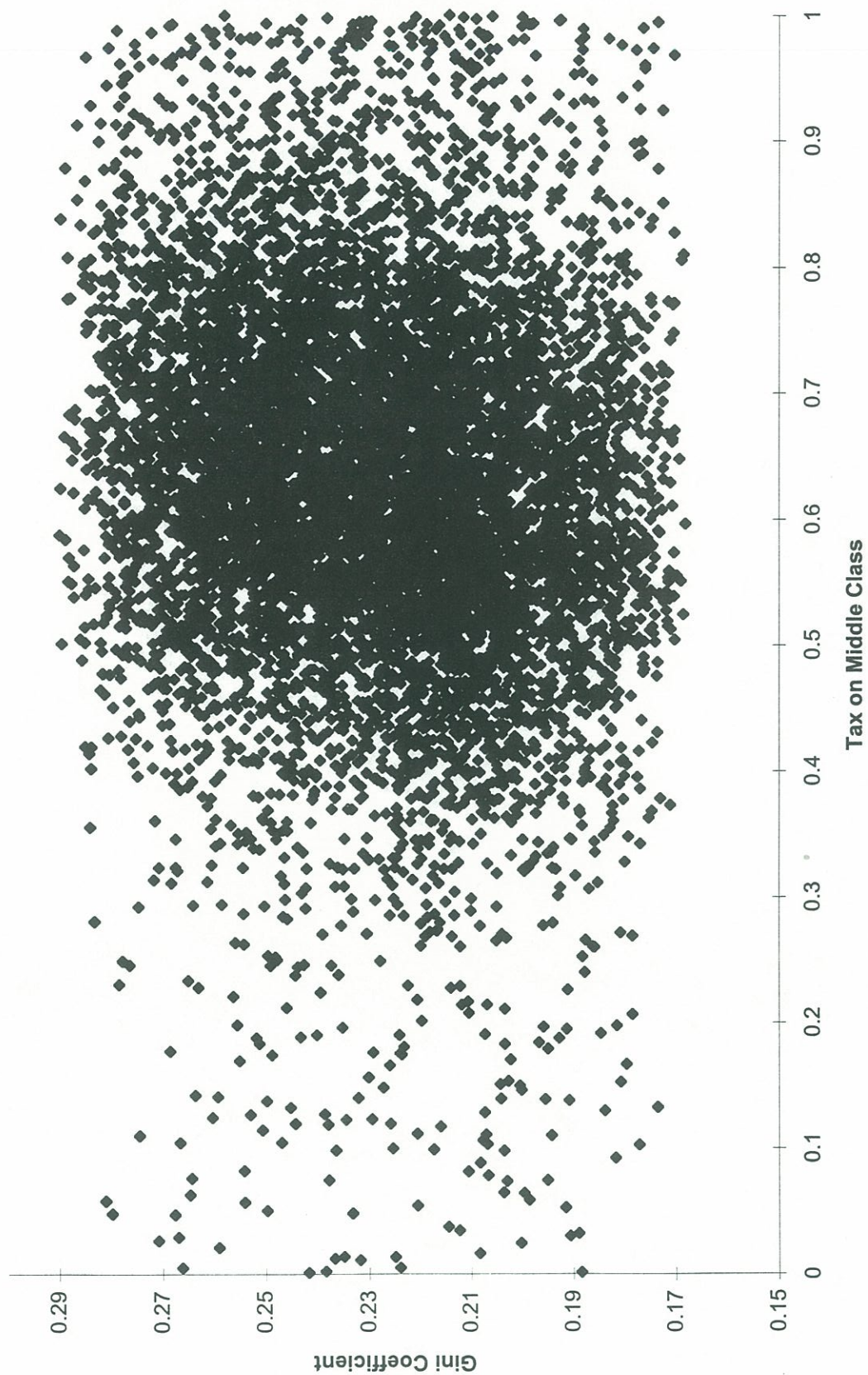


Figure 3: Tax on Poor and Inequality

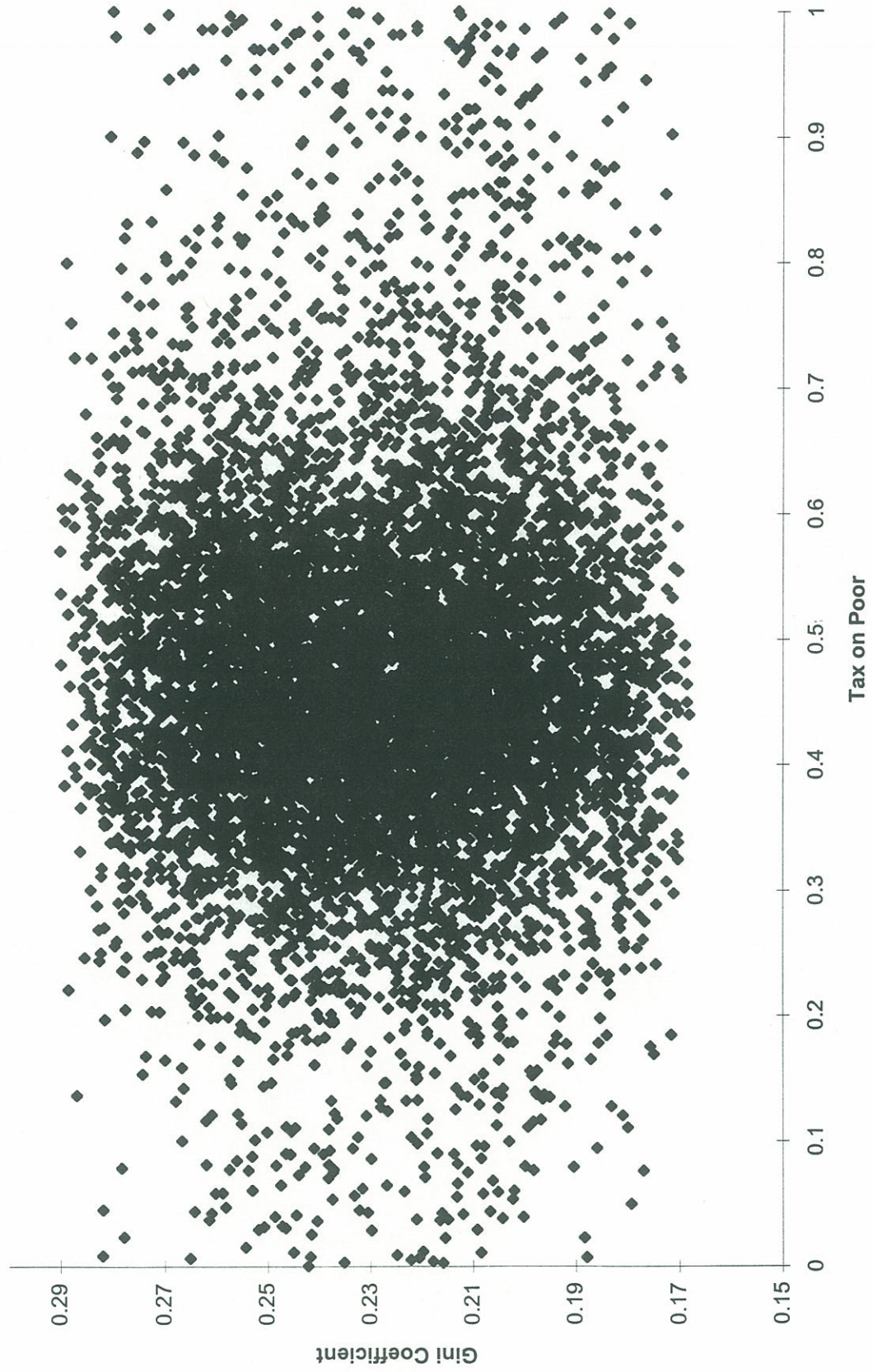


Figure 4: Average Tax Rate and Inequality

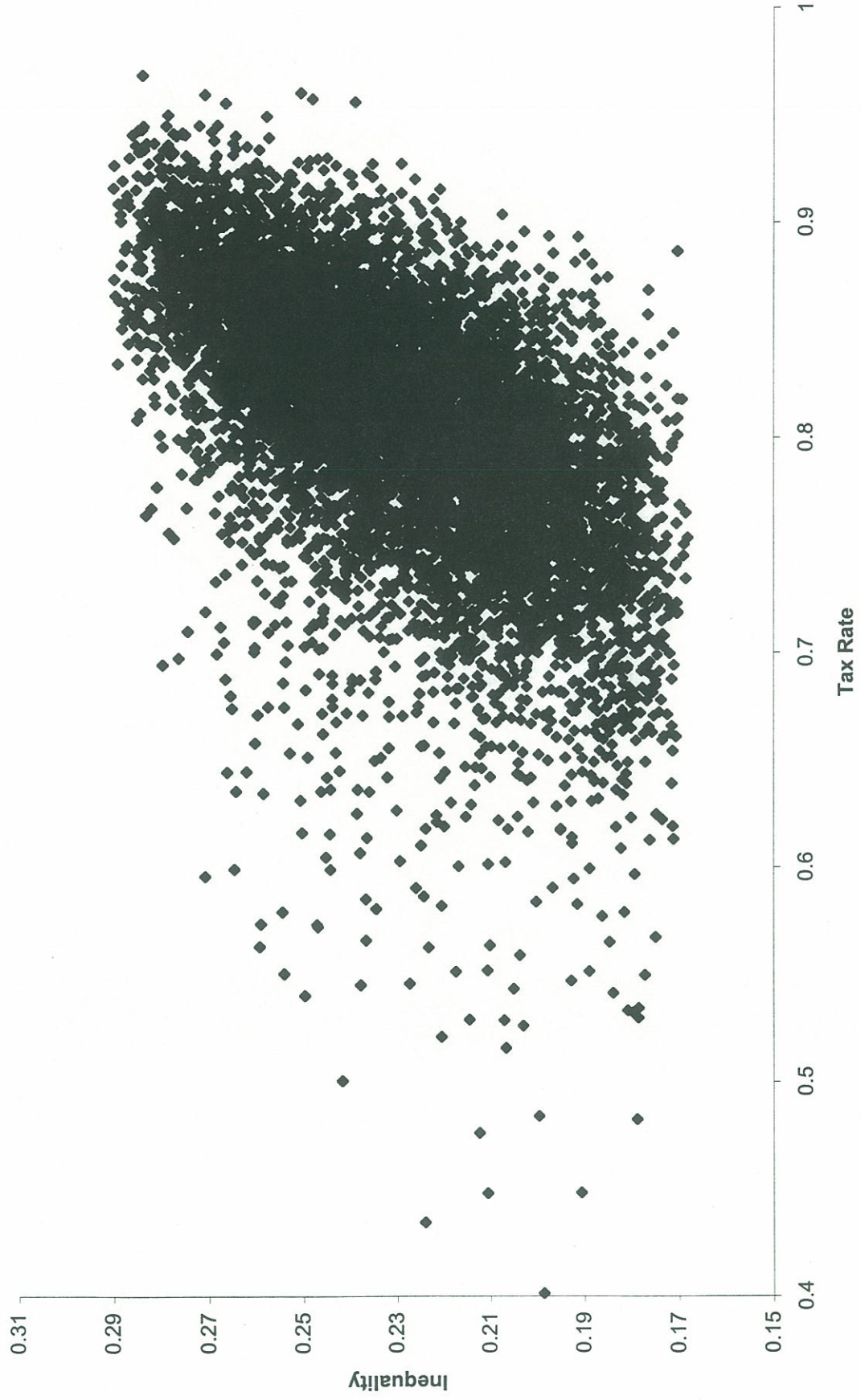


Figure 5a: Preferences over Tax Rates; Egalitarian Distribution

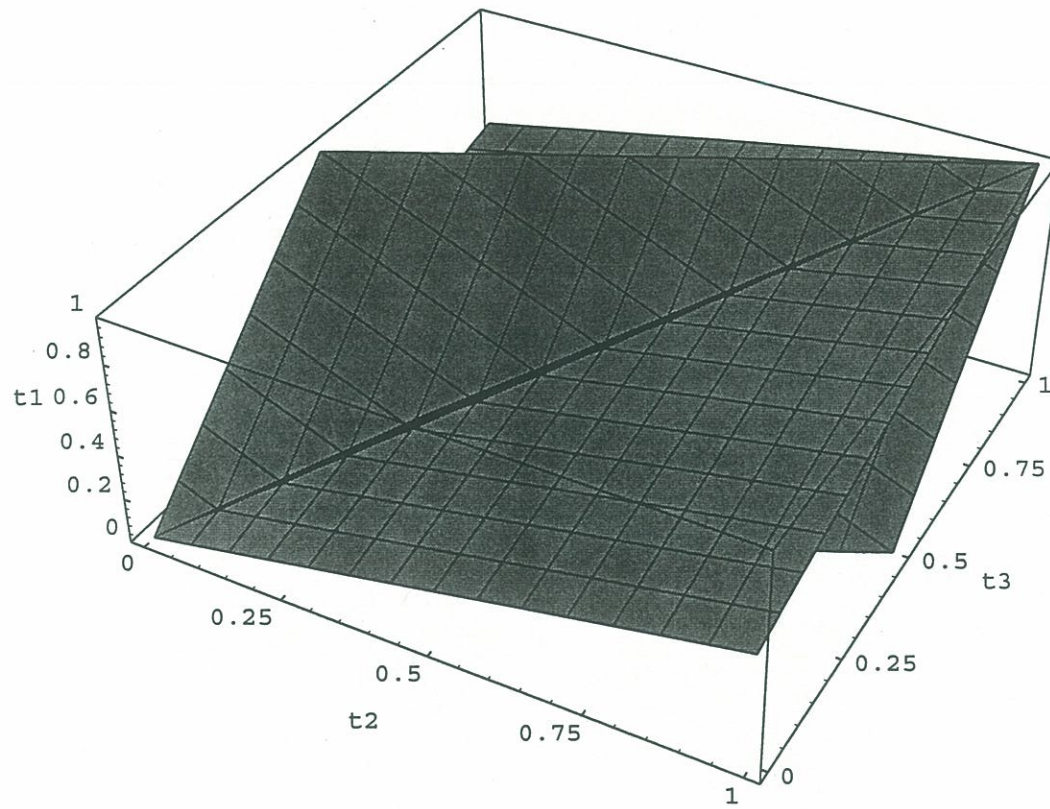


Figure 5b: Preferences over Tax Rates; Egalitarian Distribution

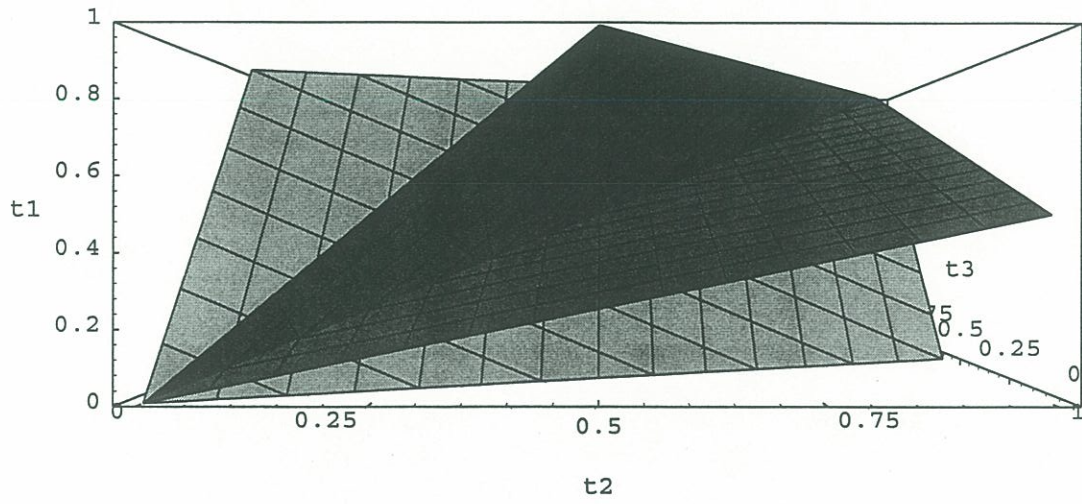


Figure 6a: Preferences over Tax Rates; Unequal Distribution

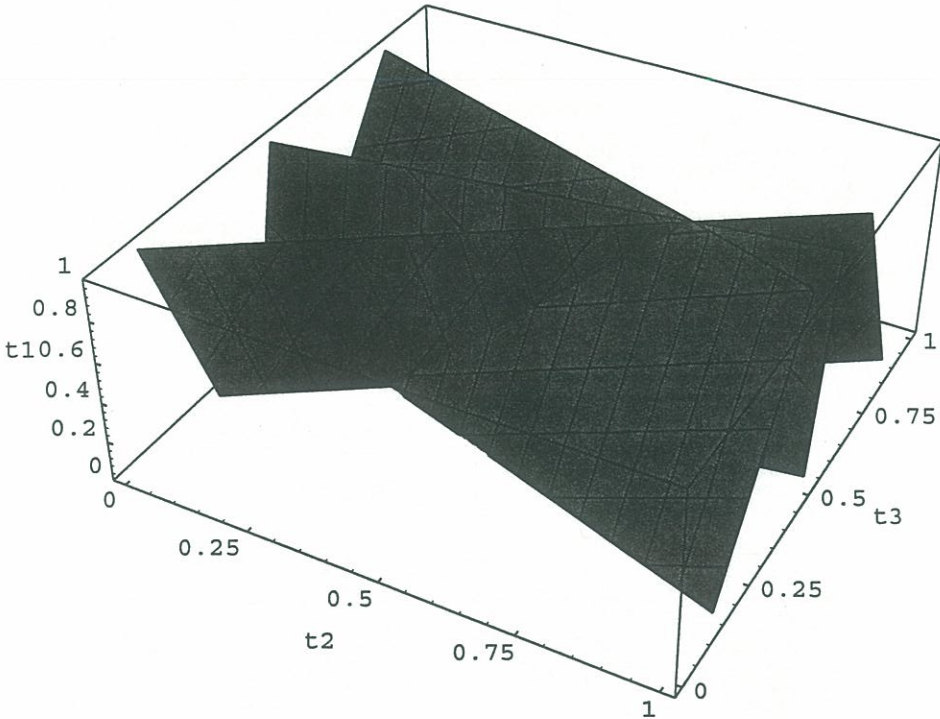


Figure 6b: Preferences over Tax Rates; Unequal Distribution

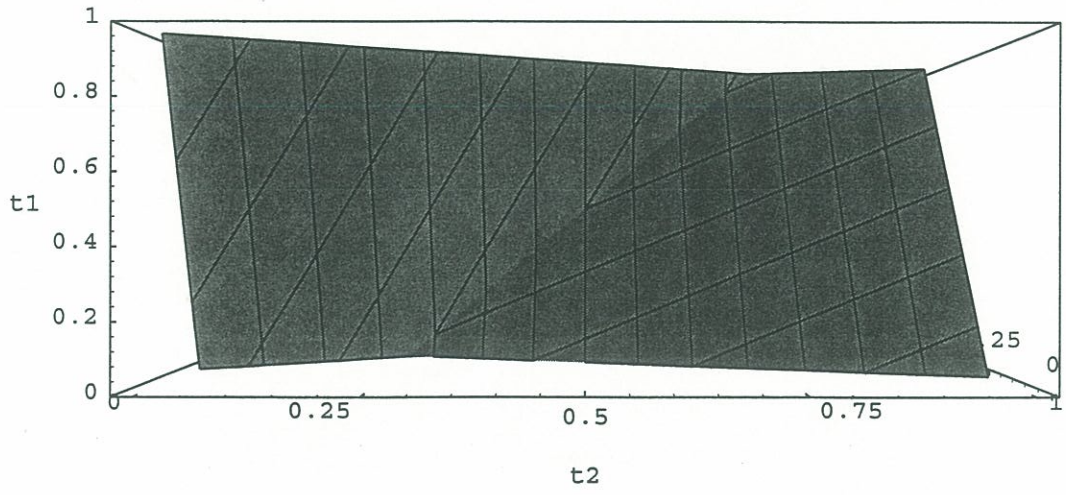


Figure 7: Net Resource Transfer to Poor and Inequality, Alternative Parameterization

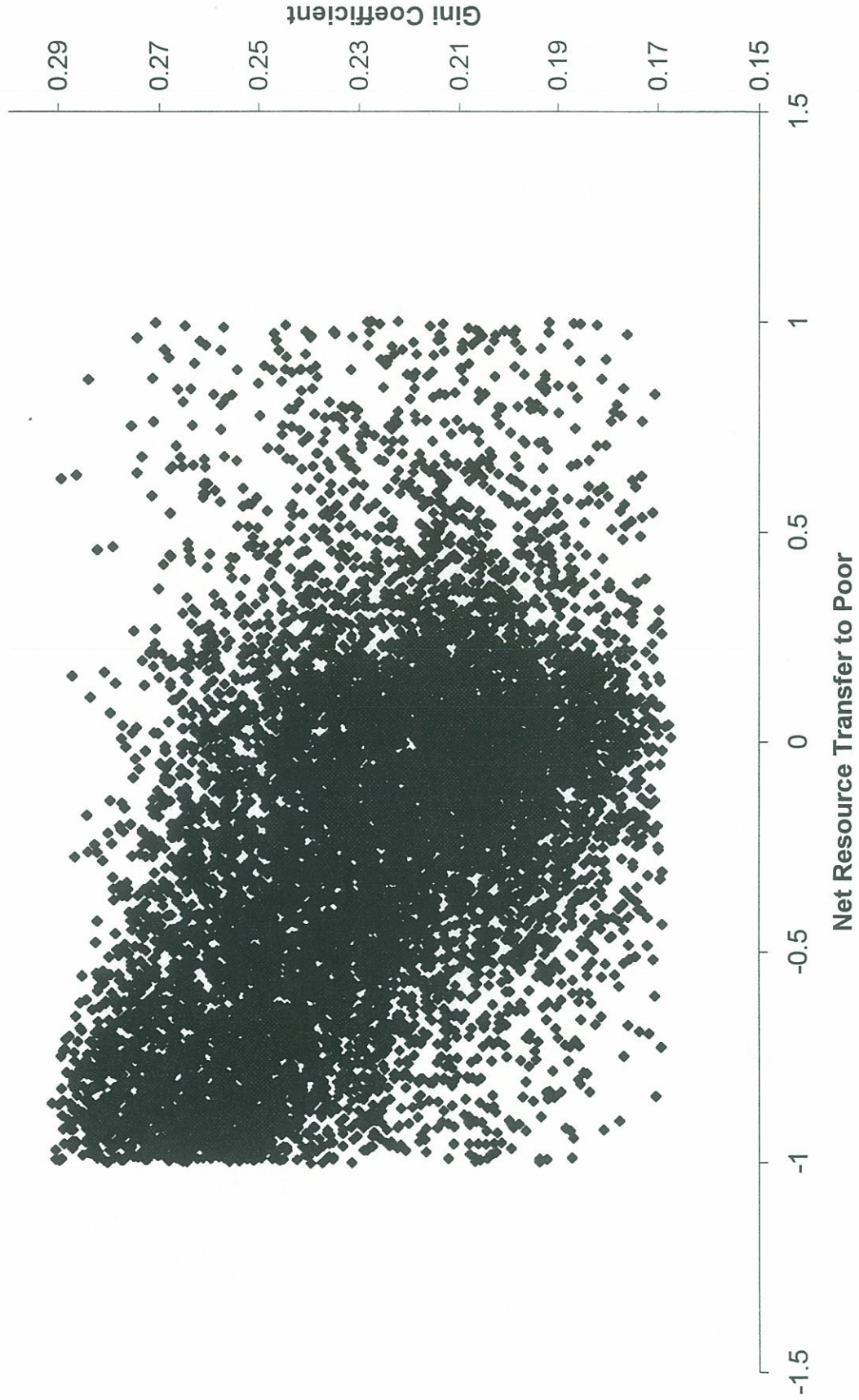


Figure 8: Tax on Rich and Inequality, Political Influence Model

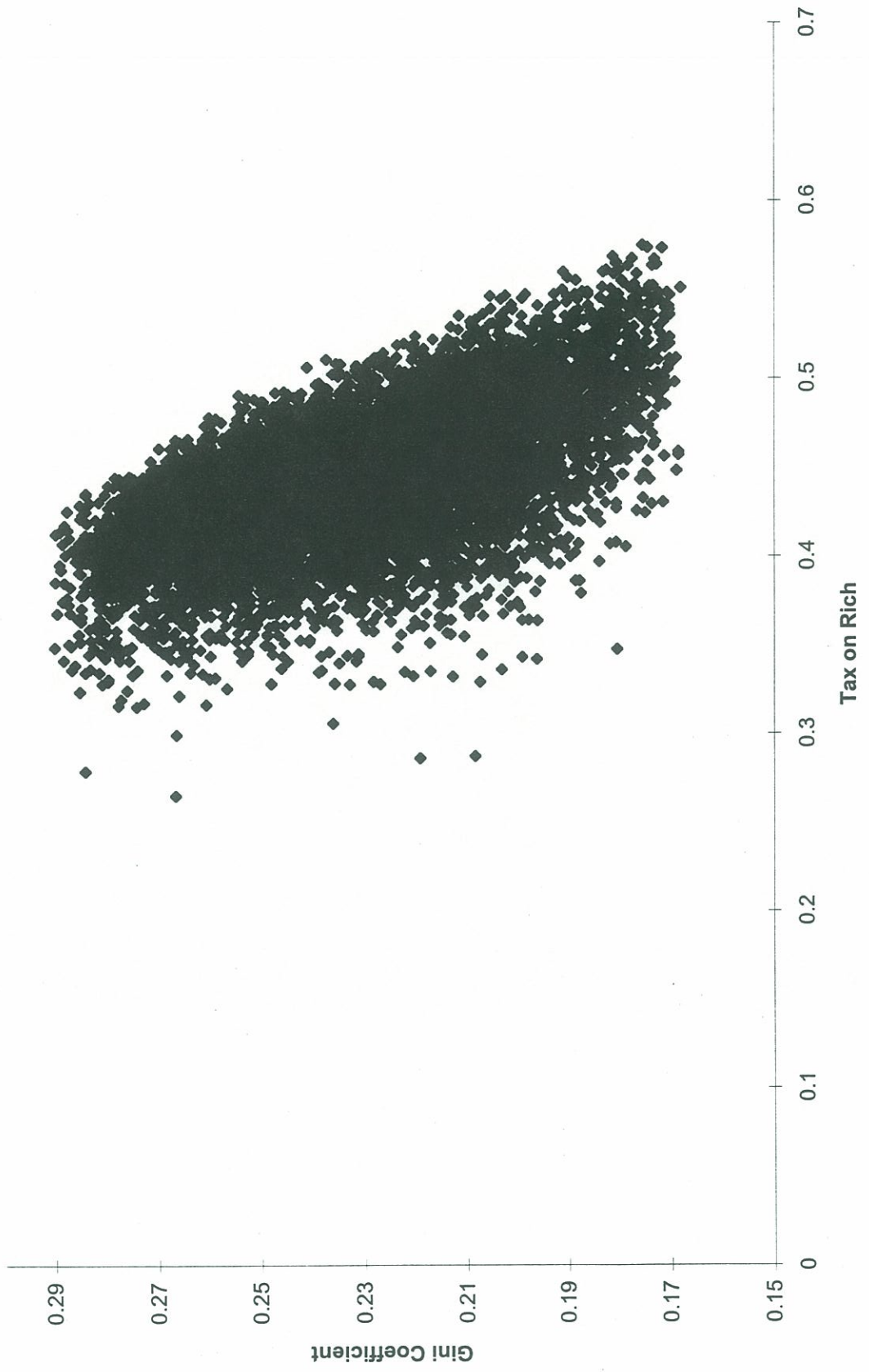


Figure 9: Tax on Middle Class and Inequality, Political Influence Model

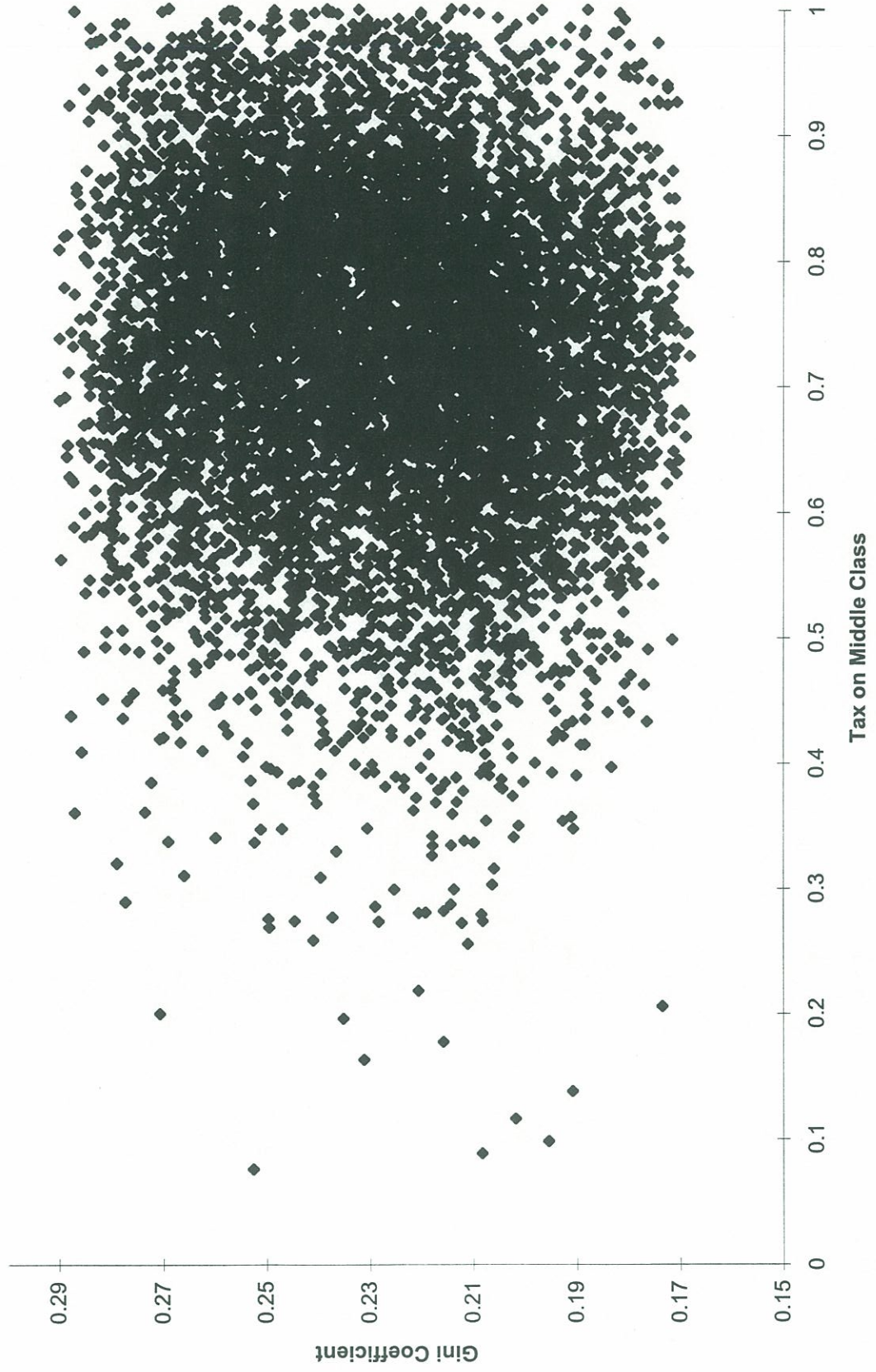


Figure 10: Tax on Poor and Inequality, Political Influence Model

