Scaling Laws and Urban Systems

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Introduction

Towns and cities have expanded all over the world for more than eight thousands years The size of these permanent nodules of resident population ranges today from a few thousands to tens of millions inhabitants. The number of cities in a given territory is always in an inverse geometrical progression of their size (for instance, there are about 23 000 urban agglomerations larger than 10 000 inhabitants in the world, 2000 are larger than 100 000 inhabitants and about 200 above 1 million, after Moriconi-Ebrard, 1993). This persistent scaling behaviour has been questioned for more than one century, giving rise to a large variety of interpretations. Are the scaling effects observed in urban systems produced by the hierarchical organisation of societies? Are they generated by purely random processes, a mere product of laws of large numbers? Are they the possibly involuntary result of some not totally conscious social rules? Are they constrained by physical and/or social rules, according to some optimisation or ordering principles? Are they emergent properties linked with the historical process of urbanisation? Could they disappear after the end of the urban transition? As some of the answers that have been offered to these questions are highly contradictory, one might hope that a clarification could come from modelling and simulation, that can be very helpful tools for selecting among different hypothesis. On a theoretical level, urban theories, as in economics or sociology, could receive a better foundation by considering the geographical concept of “system of cities”. These two research directions might also help improve the policies that are intended to control urban systems, by informing the urban governance at local and regional scales about “spontaneous” trends.

As scaling processes have been analysed in other fields of knowledge, fruitful ideas can be borrowed from various disciplines. Scaling refers to a set of properties associated to a type of complex systems, which exhibit nonlinear relations (often formalised as power laws) among attributes of their subsystems, either in their spatial organisation or during their temporal development. In the social sciences, urban systems are good candidates for exploring the processes that give rise to such regularities. One should however be aware of the problem of transferability of models from natural sciences, whose basic requirements are not necessarily fulfilled in the case of social systems. The question of choosing a level of generality for the model is also very delicate. Scaling laws often are viewed as “over identified”: they can be generated by a wide range of distinct models. It is essential to select specifications that integrate in the model a significant part of the existing knowledge about towns and cities. As a good validation, the model should produce new, “surprising” results, not merely reproduce one particular empirical scaling relationship.

Establishing scaling laws about city systems is not an easy task because of problems of definition and measurement: comparable criteria of urban size or importance across different countries or for long periods of time are not always available, neither the evaluations of their possible constraints, as these linked with aggregate measures of space, time and distance, agglomeration economies and urban functioning costs, or related to some general social practices which would be observed at individual levels. We review here briefly the main directions which have been explored in the literature about urban systems for explaining their scaling properties. We also try to suggest what could be a manageable approach by taking into account the actual possibilities of measurement and availability of data for experiments.

1 Definitions and measurement

To define an urban entity is a difficult problem, because towns and cities can receive a variety of social meanings. They are usually considered as a permanent grouping of resident population on a small quantity of land, but no universal threshold of population density can be associated to the definition of a town for differentiating urban from rural settlements. Most of the time, urban features are associated to non-agricultural activities, but the urban portfolio has considerably evolved over time with further progress of social division of labour. Several factors have been claimed to be at the historical origin of towns and cities, either religious centres versus places for political and military control, or market places commercialising local surplus of agricultural activities versus central nodes in networks of long distance trade. The hypothetical purposes or advantages associated to the grouping of population and activities are considered to be of a social character (since proximity permits the maximization of social interaction and therefore, by increasing the probability of encounters, favours the emergence of innovation); economic (since urban agglomeration generate economies of different kinds (scale economies, market economies and sharing of urban infrastructures); or cultural (since symbolic representation and mimetic behaviour may explain why people gather in urban places). Probably all three of these explanations are at least partially valid, and this fact entitles urban settlements to be considered as complex systems!

Certainly in the end, a general explanation of urban systems would have to rely on a given social significance and on explicit social processes.. However, the research cannot embrace at once the entire semantic field of urban reality. Observations of regularities in urban systems have tempted some researchers to reduce it to simpler measures, especially of urban size. The size of a town or city is a rather synthetic indicator, which can be correlated with many aspects of city functions and urban life.

1.1 Urban growth and the measure of urban size

The most frequently used evaluation of the importance of a town or a city is its number of inhabitants. To determine this may appear as a simple problem of measurement, until it is realised that the resident population has to be counted within a physical geographical object, which is generally expanding over time. If towns and cities were strictly bounded by walls or inside an administrative border, the difficulty of the enumeration procedure would be limited to the usual problems of defining what a “resident” population should include. But usually, even if a strict delimitation did exist at some point in time, cities have been growing and the new buildings aggregated to the original urban node have spilled over the former walls or the once established administrative boundaries, expanding the urban agglomeration into several communes or counties (in some countries the territorial limits of the authority of municipalities are periodically revised, but this process is not so frequent and never follows exactly the physical expansion). Or, as today in many countries, the new urban buildings are sprawling into the surrounding countryside, functionally depending but no longer contiguously leaning on the original urban core.

For some administrative or political purposes, it may be relevant to identify a town or a city with its official, non-physical limits, and to consider each commune (municipality) as a separate and autonomous entity, even if one is an old centre and the next a contiguous suburb, while for a geographical or systemic study, a better definition is to aggregate within a single urban agglomeration all urban communes (municipalities) which are contiguous and whose urbanisation usually results from the historical growth around an older urban centre. This way of defining an urban agglomeration is recommended by UNO for producing comparable urban statistics. It reflects a concept of a town or a city as a spatially relatively isolated and autonomous entity. That means that a town is spatially organised in a coherent way by a principle of centrality and that it also has a consistency over time, through a relatively continuous aggregative spatial growth process. On the contrary, an urban commune (municipality) does not correspond to any coherent geographical concept. However, as urban definitions are far from being standardized despite UNO recommendations, too many urban studies are still relying on municipal statistics, because these are almost universally produced, while ignoring that such entities cannot be taken for consistent objects and that they do not offer comparable opportunities for urbanisation (communes or municipalities may have very different shapes and sizes, their surfaces are not comparable from one country to the next).

Of course several other types of urban forms complicate the definition: for instance, two or more urban agglomerations that have grown more or less independently but not far away from each other can merge, creating a conurbation. The spatial continuity of the built-up area admitted for the definition of an agglomeration (usually no more than 200 or sometimes 500 meters between two groups of buildings in a constructible area) has lost its significance with the enlarged range of spatial interaction made possible by the automobile, leading to alternative definitions of urban statistical areas (like the US SMSAs or the more recent French *aires urbaines*) that rely on a given threshold of labour force commuting to the centre. But not all countries define such more realistic or functional urban entities (commuting data are not available everywhere), and most of the existing urban data bases that can be used for comparative purposes (Moriconi-Ebrard, 1993), especially for historical times (Bairoch and al., 1988, de Vries, 1984), rely on the definition of urban agglomeration.

Table 1 Rank and population of cities according to three different delimitations

Central urban communes			Agglomerations			Daily urban systems		
Rank	Name	Population	Rank	Name	Population	Rank	Name	Population
1	Paris	2 125 246	1	Paris	9 644 507	1	Paris	11 174 743
2	Marseille	798 430	2	Marseille-Aix	1 349 772	2	Lyon	1 648 216
3	Lyon	445 452	3	Lyon	1 348 832	3	Marseille-Aix	1 516 340
4	Toulouse	390 350	4	Lille	1 000 900	4	Lille	1 143 125
5	Nice	342 738	5	Nice	888 784	5	Toulouse	964 797
6	Nantes	270 251	6	Toulouse	761 090	6	Nice	933 080
7	Strasbourg	264 115	7	Bordeaux	753 931	7	Bordeaux	925 253
8	Montpellier	225 392	8	Nantes	544 932	8	Nantes	711 120
9	Bordeaux	215 363	9	Toulon	519 640	9	Strasbourg	612 104

10	Rennes	206 229	10	Douai-Lens	518 727	10	Toulon	564 823

Further arguments may be advocated in favour of adopting the definition of urban agglomeration for comparative purposes: despite urban sprawl, the continuously built-up area still concentrates the major part of urban activities (since more central locations are observed for jobs than for resident population), and there is a rough proportionality between the size of this core and of the daily urban system, which makes the agglomeration rather well representative of the importance of the functional urban area (table 1). Noticeable exceptions are to be found in North America, where the presence of new "edge cities" located far away from an older centre may be blurring the concept of urban agglomeration or even of daily urban system. Furthermore, we shall see that, when available, the concept of functional urban area may today provide better historical continuity than the traditional concept of urban agglomeration (see figure 2 below).

1.2 Urban size and urban product

If the concept of an urban agglomeration is considered at the moment as offering the safest and most generally available measure of urban size for international and historical comparisons, the total population of the continuously built-up area (or, more frequently, of the set of administrative local units, e.g.., communes, within which it is located and within whose limits the population is enumerated) may not always be considered as a measure of the relative importance of the city with respect to all possible bases of comparison. For instance, when comparing towns or cities from developed and developing countries, the same population size may relate to very unequal amounts of urban product and wealth. For example, several orders of magnitude separate the economic size of Lagos or Manillia and Los Angeles, despite their comparable population size.

So, another means of measuring in an objective way the importance of a city would be to estimate its gross local product. This is not easy either, since data of that kind are very rare. Usually, production statistics are collected at the place where headquarters of the firms are located and not at each place where the added value is actually produced. personal income cannot be used as a proxy, because such statistics are also very rarely produced at urban level. For measurement purposes, some estimation of urban size in economic terms has been carried out by disaggregating regional gross product (Prudhomme, 1996). This provides completely different rankings among the world largest cities than those measured by population (see table 2).

Table 2 The ten largest world cities after population size and production

City (1)	Population (millions)	City (2)	GDP(billions dollars)
Tokyo	28	Tokyo	854
New York	24	New York	448
Seoul	17	Los Angeles	326
Sao Paulo	17	Osaka	322
Mexico	17	Paris	318
Osaka	15	London	267
Los Angeles	13	Chicago	230
Jakarta	13	San Francisco	171
Mumbay	12	Dusseldorf	108

Manilla	12	Boston	107

After (1) Moriconi-Ebrard, 1993, (2) Prudhomme, 1996

One could imagine comparing cities as economic or ecological systems in terms of their costs of functioning or energetic balance. Once again, there is a lack of statistics about urbanisation costs, and much of the literature in this field has been theoretical rather than empirically founded. An interesting attempt has been made for comparing cities as ecological systems (MOB programme by UNESCO), but the results were deceptive, mainly because of the difficulty of bounding a city, measuring the ingoing and outgoing flows, and of estimating in the same units physical consumption and the products of human activities. In summary, then, population statistics remain a solid and universal basis for establishing scaling effects in urban systems, even if they have to be supplemented by other meaningful but partial information.

2 Observations about scaling in urban systems

Scaling laws as observed in natural or social systems are revealed by more or less constant relationships, either linear or non linear, between some attributes of subsystems, which cover usually several orders of magnitude within the system and persist over time. Even according to this very general definition, scaling effects in urban systems may refer to different observed regularities that have to be clarified. They could first refer to the effects of the scale of observation (or, in realist words, to the scale of organisation of urban systems) that lead to define the concept of urban system as including two main levels of aggregation: it may concern the city itself, as well as a set of cities belonging to the same regional or national territory, according to the famous formula by Brian Berry "cities as systems within systems of cities".

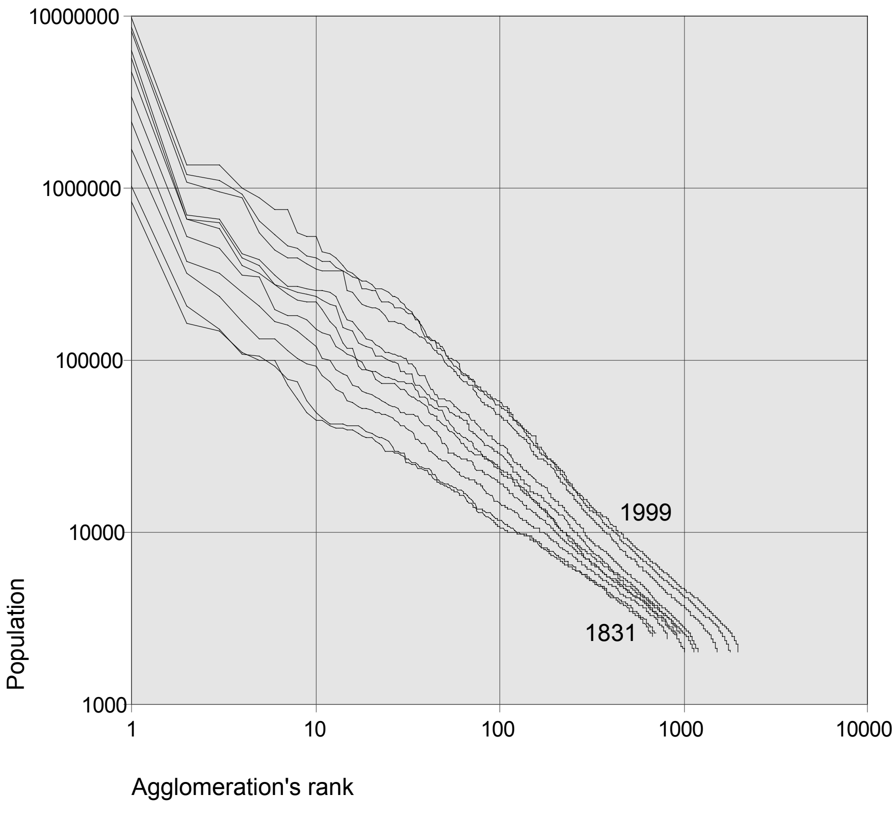
But the most frequent and well documented references in urban scaling are about the differences in town and city sizes within a system of cities. They have been summarized in different ways, either by statistical laws like Zipf's rank-size rule, or by central place theory. Less systematic but still strong regularities can be found at the city level between population size, urban surfaces and densities and travelling time, and perhaps this could be connected with observations made at the upper scale of the system of cities regarding the evolution over time of city size and spacing, in the context of an increasing speed of transportation. Finally, the existence of self-similar patterns at different scales, including mainly the reiterated scheme of centres and peripheries in the distribution of nodes and networks, could provide another synthetic view of urban scaling effects.

2.1 Zipf's rank-size rule

The same word, "urban settlement", (or *ville* in French, *Stadt* in German) is used for designating entities which may vary by several orders in magnitude, a few thousand inhabitants for the smallest up to several tens of millions for the largest. City sizes are not distributed uniformly between these extremes of 10^3 and 10^7, but on the contrary they follow a rather strict statistical law inside a given territory. Zipf had expressed such a regularity as an inverse geometric progression between the population P_i of a city and its rank R_i in a national set of towns and cities, giving an approximate size of one half of the largest city population for the population of the second city and one third for the third one, and so on. This "rank size-rule" formulated as $P_i = P_1 / R_i$ has been generalised as a Pareto-type distribution of the number of cities according to their size, $P_i = K / Ri^{\alpha}$, where the parameter K has a value close to P_1 and α is around 1. (Note that this α parameter has a value which is inverse of the parameter of a Pareto distribution which would be usually formulated as the number of units R_i having a size larger than P_i: $R_i = A / P_i^{a}$; then, contrary to the Pareto corresponding parameter a, the higher the absolute value of α, the larger the size inequalities within the observed distribution).

This model has been fitted many times to more or less correctly measured population series of towns and cities (figure 1). When adjusted to the population of the urban agglomerations (towns and cities over 10 000 inhabitants) of each state in the world (including all those having at least 30 urban agglomerations), the estimated values for the parameter α range between 0.7 and 1.3 (Moriconi-Ebrard, 1993). The variation among different countries is rather small: the standard deviation of measured α values is only 0.138.

Figure 1 The hierarchical differentiation in urban systems:
Rank-size distribution of French agglomerations (1831-1999)



The fit of the Pareto model is rather good, even if in many cases better fits can be found to other types of asymmetrical distributions (Quandt, 1964, Guérin-Pace, Lesage, 2001). Usually, when settlements of smaller sizes are included, the lognormal distribution provides better fit than the Pareto model (Baker, 1969, Robson, 1973, Pumain, 1982). Very often, the upper part of the size distribution, corresponding to the largest urban settlements, does not fit very well to any model: these cases of urban primacy (one to up to eight cities per state whose size exceeds the expected values) were detected a long time ago (Jefferson, 1939) and seem to be a generality rather than an exception. For two thirds of the world's states, the mean value of the ratio P1/P2 is significantly larger than the one that would correspond to the model. When this " primacy index " is computed, as the ratio between the population of the largest and second largest city, it is found that in most states of the world it is much larger than the value of two which would correspond to Zipf's rank size rule (for a Pareto distribution with a value of 1.3 for the parameter α, the expected ratio would be 2.5) and the mean value for all countries of the world taken together is 5.2 (Pumain, Moriconi, 1997).

Several remarks have to be made, because the literature on Zipf's law is full of ill-founded conclusions, mainly due to small samples of observations and a lack of accuracy in empirical data (for instance, Rosen and Resnick, 1980, repeated in Alperovitch, 1993). It is true that whatever the part of the world and the period of observation, since the 10 000 years when towns first emerged, the model of settlement size distribution has always been reasonably well approximated by a Pareto or lognormal distribution. This has been demonstrated by Fletcher (1986), based on available data on early urban settlements as determined by archaeologists.

During historical times, the inequality in city sizes has been increasing (Roehner, 1991). Empirical evidence from historical data (Bairoch, Batou, Chèvre, 1988, de Vries, 1984) shows a clear evolution from values for the α parameter of around 0.7 in many countries before the XIXth century, to values significantly higher (at least 0.9 and often larger than 1) for distributions observed since the middle of XXth century. However, the direction of the most recent evolution is by no means clear: between 1950 and 1990, the value of the α parameter has steadily increased in 19 countries, steadily decreased in 9 others, while it followed no regular evolution in the remaining 21. There is a rather general trend to a diminution of the contrast in city sizes in the more developed countries, but there are exceptions (USA, France, Japan, Russia, Hungary and Greece). The diversity in evolutionary paths is still larger in third-world countries.

Such observations clearly maintain the city sizes distribution within the sphere of attraction of Levy's stable laws, whereas other distributions of inequalities in social systems have evolved towards the attraction domain of the normal law. M. Barbut (1998) has demonstrated that the distribution of incomes shifted towards the attractor of the Gaussian model about in the middle of the 1930's in France and about the same time for other developed countries which had undertaken policies of social redistribution. Similarly, the inequalities in the size distribution of firms were reduced by anti-trust regulations.

Over time, there is no indication either of any convergence towards more or less regularity in the shape of the size curve. This contradicts an hypothesis made by Berry (1964) and repeated many times since (for instance, Haggett, 2001). According to these authors, the existence of a primary city would reflect an earlier state of urbanism, and the size distribution should evolve towards a more regular pattern over time. On the contrary, the persistence of systematic deviations from the Pareto model seems to be the rule, especially in the upper part of the city size distribution. This aspect may be considered as relatively minor in the scaling property of systems of cities, which gives a good fit to power laws for the largest part of the size distributions. But it may be taken into account by noticing that urban systems never are completely isolated from their environment, and that the larger the cities are, the broader is the range of their relations. Perhaps the largest cities of each country should be considered as being parts of wider territorial systems or networks, which would make their frequently exceptional size more understandable (see below). However, some very large urban systems, as in United States, have very regular size distributions.

In any case, another regularity has to be mentioned. Although it may be rather loose , there is a definite relationship in the magnitude of the size of the largest city of a country (P1) and the total urban population of this country (PT). The proportion of the urban population which is concentrated in the major city varies between 10 and 30%. This proportion tends to be larger in smaller countries than in the large ones. An adjustment of that very much improves the fit is to use the non-linear relationship : $P1 = k\ PT^{0.8}$ (Moriconi-Ebrard, 1993). This is a direct consequence of the general Pareto shape of all national city size distributions (Gibbs, 1963).

2.2 Central place theory

Christaller's central place theory succeeded in summarizing a set of interesting regularities related to city size that had been observed long ago. These regularities refer to the correlation between the total population of a city and the number and diversity of urban functions it performs (especially the services offered to the resident population), as well as the total number of consumers it attracts around it, and the spacing between cities of the same hierarchical level. Though it has been tested and rather well verified in a wide variety of territorial and economic environments (even on systems of periodic markets, for instance by Skinner), and is still valid in may respects, central place theory has lost some of its explanatory power with increasing urbanisation, mainly because cities are no longer just central places for surrounding rural populations. The theory does not take into account the presence of non-service urban activities, even if services to the resident population may still concentrate more than half and often two thirds of a city's labour force.

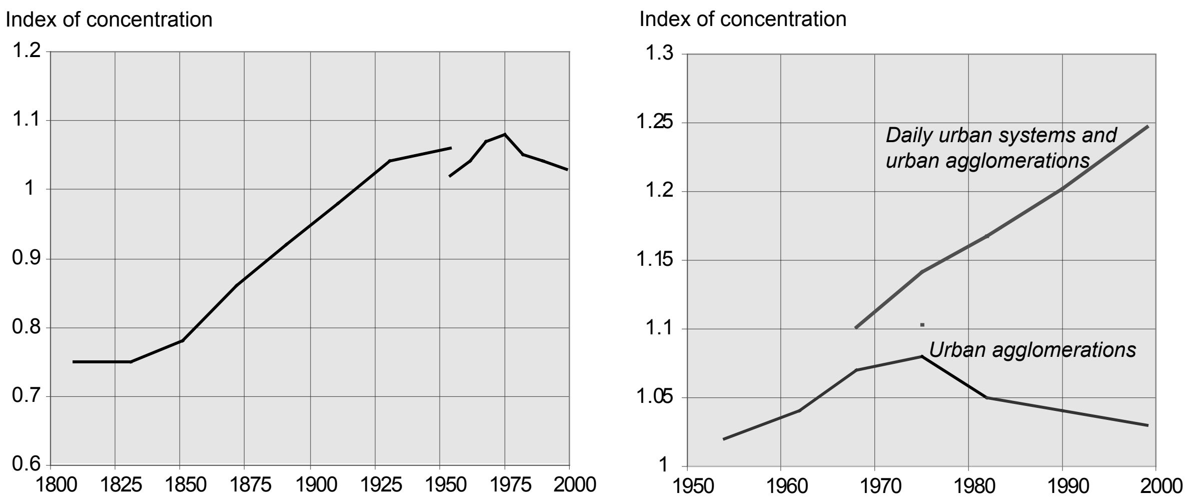
Even though central place theory gives an interesting insight in the explanation of inequalities in city sizes and how they correlate to many aspects or indices of a hierarchy of urban activities, it is difficult to use these relationships to search for scaling laws in urban systems. International comparisons would require that the "levels" that are identified among the cities of one country or regions and the corresponding portfolio of urban activities should be made comparable. As nomenclatures of urban activities are not defined in the same way in different countries, the statistical shape of the relationship between them and the size of cities has never been investigated systematically in comparative studies. There are also differences in income level, as well as cultural differences, that introduce important variations in the ratio between the number of residents of a city and the number of jobs in services supplied to them.

2.3 Urban size, spacing and speed of transportation

A simpler theory of urban systems that would include scaling effects comes from considering them as forms of spatial organisation that optimise in some way the space-time budget for daily activities of their inhabitants. At the scale of a single city (a daily urban system), the speed of transportation is rather slow, but the concentration of many activities at short distances ensured by the high density allows for a good level of generalised accessibility. Over time, the speed of transportation has slightly increased, from 4km/h to about 16 km/h in the dense central parts of urban cores, until 60 to 80 km/h in the better equipped suburbs. Zahavi (1974) has advanced the hypothesis that where a gain of transportation speed has been possible, it has not been used to reduce transport time; on the contrary, the mean commuting time tends to remain constant (about one hour per day) and commuting distances are increased. This has been roughly confirmed by observations about the recent evolution of commuting times and distances in many countries. As an example, a recent survey estimated that the daily time budget dedicated to transport varies very little from one country to another (among developed countries): from about 55 minutes (as in Hungary and France) to one hour and 24 minutes (in UK) or 1h23 (Belgium), 1h15 (Denmark). Statistics for historical times are more difficult to provide, but most estimates are around one hour per day (for example, this estimate is consistent with observed patterns in the extension of intensively used agricultural land around villages in rural Europe). If we admit that the time devoted to commuting has remained about constant during historical times, then it is the speed of transportation that explain why the diameter of urban areas has been expanding, mainly first with tramways and railways, then with motorcars, increasing for instance the available surface of a city like Berlin by a factor of 100 between the "city on foot" (2km radius) and the "city by car" (20 km). Meanwhile, the mean urban densities have decreased (Marchetti, 1991).

At another spatial scale, it is the length of the intervals between two neighbouring towns or cities that seems to be regulated by the speed of the transport system over long periods of time. As the mean available speed has increased (from less than 20 km/h with horses and coaches to 500 km/h with planes), the number of necessary steps along itineraries has decreased, the weakest towns located in intermediary positions between more important centres has declined, at least in relative terms, and the average spacing between the main urban centres has increased (Bretagnolle, 1999).

Figure 2 Evolution of urban concentration in France



On the left is plotted the evolution of the absolute value of the slope α in the equation $P_i = K / Ri^{\alpha}$ *adjusted to the population of French agglomerations at all census dates from 1831 to 1999. There seem to be a reversal in the historical trend toward stronger hierarchisation, but when urban entities are delimitated according to a more sensible definition (aires urbaines, right side), the continuity of the concentration trend appears. Source: Bretagnolle, Paulus, Pumain, 2002.*

This trend is one part of the explanation of the reinforcement of urban hierarchies since the beginning of the nineteenth century and the industrial revolution of the transportation systems (figure 2). As at the same time the general income and production were increasing, enlarging and diversifying the demand for urban services, towns and cities have grown in size and number. Such an increase in the size and number of towns and cities has been especially observed during the period of intense urbanisation from the beginning of the XIXth century in developed countries and from the middle of XXth century in the developing ones. But as the increase in general wealth of population (or GNP) did not occur at as high a rate as the increase in transportation speed (inducing the apparent shrinking of space), the result is a higher contrast in city size distribution over the past two centuries. More precise measured evidences of this process can be provided.

The regularity of the spacing between towns and cities is not as regular and geometrical as predicted by the hexagonal geometry of classical central place theory. However, when mapped for instance for the whole set of European agglomerations having 10 000 inhabitants or more, the distances between neighbouring towns may reveal very consistent spatial patterns at different geographical scales and levels of the urban system. Figure 3 reproduces maps drawn by C. Rozenblat (1995) where a straight line connects two neighbouring agglomerations if they are separated by a distance of less than 25 km (figure 3a). The resulting pattern shows clearly three main types of urban systems in Europe: a central part with much higher densities (high number of connections), a western part (mainly France and Spain) more contrasted (parts of the territory, as coast lines or main valleys, exhibit concentrations while in other parts the towns are much more apart from each other and there are no connections), and an eastern part with a high regularity in spacing (connections appear in all regions). Combined with size, these three types would rejoin the typology established by Etienne Juillard who labelled respectively as Rhenish, Parisian and Peripheral such urban networks. But the point is that the general pattern is maintained, if one raises the minimum distance to less than 50 km (figure 3b), and less than 100 km for the subset of agglomerations larger than 100 000 inhabitants (figure 3c). So a simple analysis by a cartographic filtering method illustrates the evidence of the hierarchical and spatial organisation of urban systems that is at the basis of their scaling properties. Obviously history can explain the three different types of spatial patterns, that roughly correspond to distinct political and administrative organisation of the territories (more centralised in France and Spain for centuries, also in United Kingdom but there the urban system was more disturbed by the industrial revolution; Germany and Italy have been unified much later, that facilitated the persistency of rather large close regional capitals; the regularity of the eastern side may reflect the systematic organisation that was imposed by a late colonisation through settlements from the XIIIth century on…to give only a few clues).

The evolution of the European urban system also demonstrates a high consistency over time, as illustrated by (Bretagnolle et al., 1998). The effects of increasing demography and income explains the general expansion of the system, in the number of cities and in their size, but hides the structural transformations of the system. Taking into account the relative size of cities, either by computing the share of their population in the total population of the system, or by mapping solutions, put forward the impressive accentuation of city size inequalities, which affects all parts of the system, whatever the region or the hierarchical level under consideration. Space-time contraction due to the speeding-up of transportation means is considered as an important factor responsible for that reorganisation process. The possible spatial transformations of urban systems have then to be observed, not in a physical space, but in a time (or cost-)-space which is relevant for defining the real possibilities of interaction between cities. Following works by historians (de Vries, 1984), we suggested to introduce in measurements of classical urban population potentials one parameter which accounts for the general and local modifications in the accessibility of cities. The maps of urban population potentials which are produced illustrate in a synthetic way the spatial effects of differential growth in a system of cities. The evolution of the spatial configuration of European urban system shows a regular and slow adaptation of its structure to major changes in energy sources and trade roads.

2.4 Fractality and urban systems

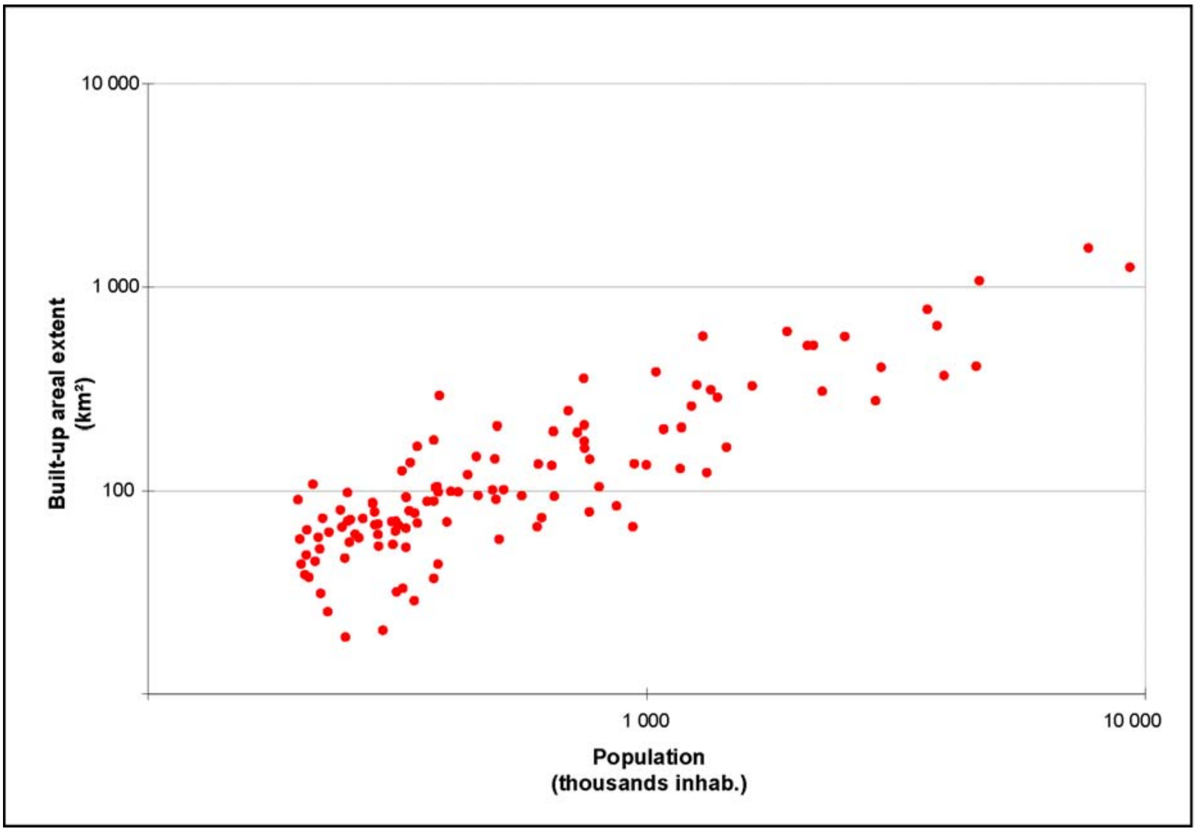
Fractal measures have been applied to urban systems, first at the level of systems of cities (on theoretical central place systems: Arlinghaus, 1985, or on real distributions: François et al., 1994, Le Bras, 1998), and second, at the level of the internal structure of urban areas, in particular with respect to the the spatial distribution of built-up surfaces (Frankhauser, 1993, Batty, Longley, 1994).

The urbanised space is actually implemented in a fractal way: it is fragmented, self-similar in the sense that the same patterns, like the dilution from a centre towards a periphery, can be observed at several scales, and hierarchical, including many more small centres than large ones, as well as a many more small narrow roads than very large ones, or many more small urban squares than very large plazas. The same differentiation processes have produced more or less regular gradients in building, population and activities densities around the centres towards peripheries, at local scales as well as at regional or higher scales, according to various degrees of accessibility.

The available measurements that have been computed for a variety of cities throughout the world are not always strictly comparable, since they partly rely on the resolution of the maps that were used for estimating them. Moreover, since urbanism is not the same kind of phenomenon over time, urban structures are probably multifractal; a medieval centre did not have the same factor of dilution of its buildings in surrounding space than does a contemporary suburb. Differences have also been observed between European or Asiatic cities, which tend to have lower values for the fractal dimension of their built-up areas (between 1.4 and 1.8), and Australian or North-American cities, which are closer to homogeneous distributions (dimensions between 1.8 and 2). However, from available measures, it seems that these fractal dimensions of intra-urban structures always have values between 1 and 2. This corresponds to a more or less fragmented occupation of land by the buildings, that remains however consistent and reveals the effect of a principle (coined by concepts like centrality or urban field) that organises the process of urbanisation (Batty, Longley, 1984, Frankhauser, 1993, Batty, Xie, 1996).

Recent comparisons among European cities built-up areas (Guérois, 2003, see figure 4) imply another interesting theoretical interpretation about the concept of urban agglomeration: the spatial limits that are defined for urban agglomerations coincide with the area that is spatially organised according to a fractal law. The external limit of the urban form inside which the buildings are spatially distributed in a fractal way is closer to the geographical limit of the urban agglomeration than of the daily urban system defined after links created by commuting. Moreover, it has been demonstrated that such a limit has moved further away from the centre over time, following the spatial delimitation of the urban agglomeration (Frankhauser, 1999). The spatial expansion of cities then consists in the progressive integration of urban built-up space into a fractal structure. A new and more universal definition of cities could perhaps be suggested, as portions of geographical space where the habitat obeys an ordering principle that is a fractal law.

Figure 4 Population and built-up surface of European agglomerations



M. Guérois, 2003
Sources : EEA (1996), Eurostat (1991), Megrin (1991), Moriconi-Ebrard (1994)

Whereas most often the spatial distribution of built up areas or population in rural space does not differ significantly from homogeneity, the analyses of the interurban distribution of buildings or population have shown fractal dimensions which are usually below 1 (François et al., 1994, Guérois, 2003). This could be attributed to another way of occupying space, more diluted, for less frequent relations, as mentioned above in a space-time interpretation of urban systems.

Such observations confirm our conception of two distinct levels in the spatial organisation of urban systems: 1) the intra-urban level, or the "urban field", including as a functional organising principle the accessibility to the centre (not exceeding 30 minutes time in radius, and inducing variations in density levels which may reach a 100 or 200 order of magnitude and in land prices per sq meter which may rise to 30 times between the centre and the periphery), proceeding over time from a peripheral spatial expansion as well as from variations in the intensity of land use; 2) the interurban level, or the system of cities, where the size and spacing of towns and cities are organised according to a hierarchical access to services, resulting from a competitive process for capturing and accumulating resources (especially from innovation), inducing a spatially distributed process of growth on very long periods of time, but with heterogeneities leading to a differentiation in city sizes which usually can reach a factor of 1000 within a single country.

We suggest using the term "hierarchical organisation" to describe the existence of two distinct levels in urban systems (the levels of the city and of the system of cities), whereas we reserve the term "hierarchical differentiation" to refer to the macroscopic property of urban systems that leads to a Pareto distribution of city sizes.

3 On autonomy and control

When looking for social or spatial processes that could explain why the configuration of urban systems exhibits rather regular scaling phenomena, it is very difficult to assess where the control of such regularities could come from. Several hypotheses can be made, and one has to be chosen in accordance with the way the scaling effects will be modelled. We shall discuss here only a few.

3.1 Do scaling laws result from random growth processes?

Urban systems involve large numbers (of settlements, and persons), and sometimes their regularities have been conceived to be a consequence of general statistical effects or stochastic processes. For instance, a static explanation of the rank-size rule has been suggested by describing cities as the aggregation of a variety of activities. As each of these industries follow Pareto laws of distribution (for the size of firms), and since a sum of Pareto laws is still a Pareto law, this could explain the shape and regularity of the distribution of city size (Roehner, Wise, 1987). Of course this leaves the problem of how to account for the Pareto distribution of firms within an industry. And it does not explain how cities come to concentrate variable amounts of firms and activities. Moreover, the growth process of firms is not at all the same as the one for cities. Firms can merge or divide, they also can collapse, many are created or die during a short period of time. If we leave aside the rare cases of cities merging (forming conurbations) and of cities dying (this seems to happen very rarely during modern historical times) cities are growing or declining in their population from natural increase, or by migrations. They can grow more or less rapidly, and towns entering the system usually are former villages which overcome the threshold of urban definition.

Better solutions than static explanations can be found by looking at the urban growth process. In the literature two contradictory affirmations are found when justifying the existence of a scaling law like Zipf's distribution by the growth rules of the system. As it has been demonstrated that a random process like the "law of proportional effect" (according to Gibrat, 1931, a randomly distributed exponential growth process) could explain the lognormal distribution, the stochasticity of such effects could be at first sight admitted, and sometimes is claimed (Halloy, 2002). Simple application of "the law of large numbers"? According to Halloy, "possibly the primal feature of complex systems is greed (or more euphemistically, resource attraction) and competition as its secondary outcome. It is this resource attraction and competition which in turns determines the primary interactions between agents, as well as the adaptive nature agents changing rules to outcompete others" (p.2). But a deeper insight into the growth generating process and in the way it is distributed among cities would lead to a less simple explanation. Which social processes can maintain urban growth dP as an average proportion of the initial size of a city and why should the growth rates (dP/P) be randomly distributed at each short time interval, as observed by Robson (1973) for British towns as well as the French ones?

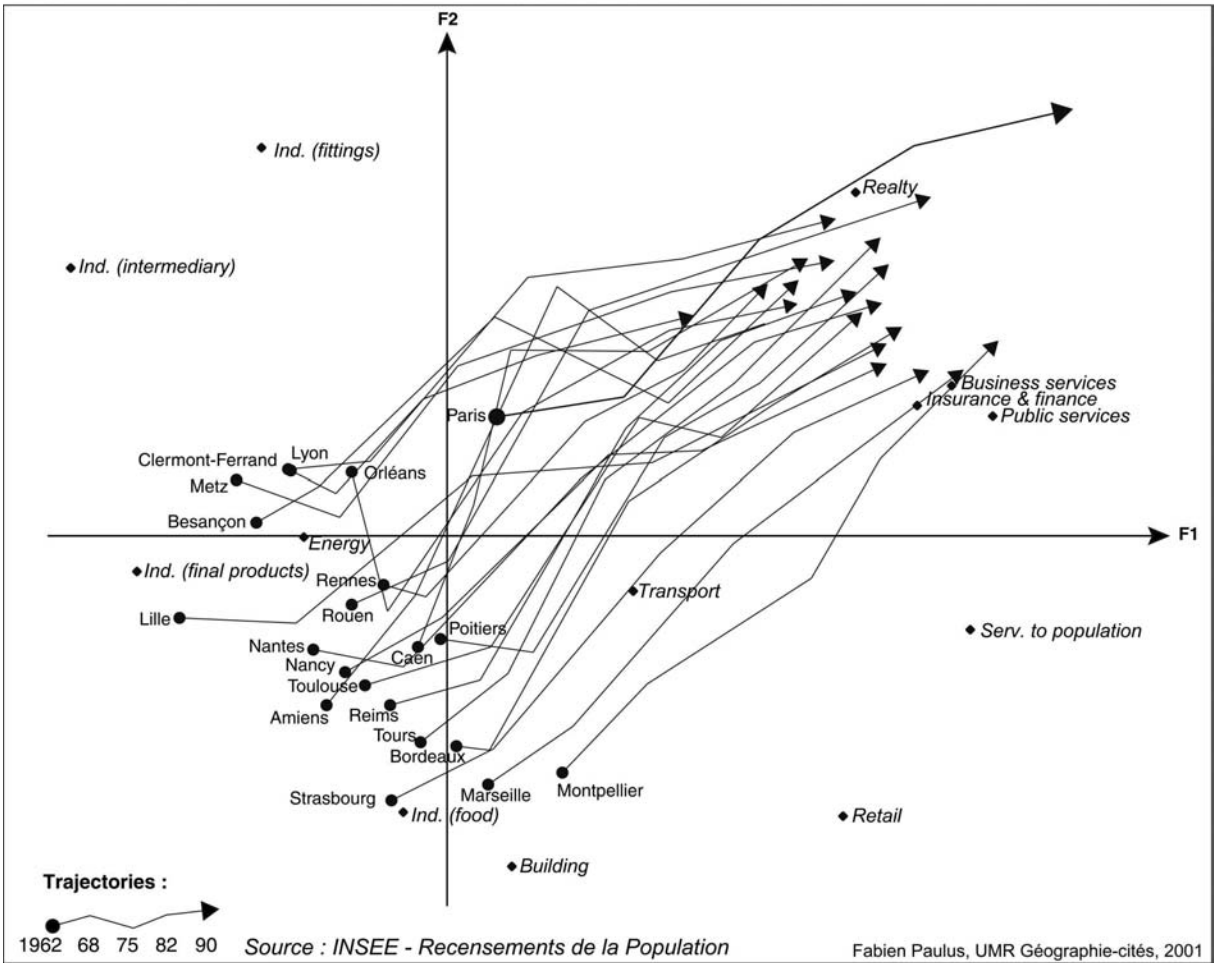
When observing carefully how cities transform over long periods of time, it can be shown that the urban changes (new techniques, but also new economic activities, professions, physical infrastructures, as well as fashions, cultural practices and so on) are adopted in each very quickly, compared to the general speed of change. But cities are adapting to the changes (that they contribute to create as well) by small discontinuous adjustments: the deviations from the mean change are generally not found in the same cities at two successive time intervals. On the whole, urban qualitative changes (and corresponding quantitative growth) diffuse themselves rapidly within the entire urban system and therefore do not alter the initial structure of the system; the relative situations (in terms of size or economic specialisation for instance) remain the same (Pumain, Saint-Julien, 1978). This can be illustrated by the evolution of the position of cities projected on a graph defined by a multivariate analysis of their economic profiles at different dates (figure 5).

This is made possible because every town or city within the system is “trying” to do the same as its neighbour. Or, put in another way, they are in competition for the same thing (to attract population and activities, to capture investment or the benefits of innovations). In some sense, they “behave” as if they were “greedy” (Halloy, 2002). So even behind a process that could apparently be reproduced by a purely stochastic model, there may be a substantive explanation in social terms.

If so, is this process totally free, or is it controlled, constraint, or regulated?

Figure 5 Co-evolution of economic profiles of French agglomerations (1962-1999)

Figure 5 : Trajectories of main cities in the space of economic activities, 1968 - 1990



Source: Paulus F. in Pumain D. (2002)

3.2 Are scaling laws the product of any optimisation principle?

Cities are neither firms nor simple institutions. They don't have any general aim or function to fulfil and, even if subsets of interested actors or some specialised institutions can be identified for representing an activity sector or a partial group of citizens, there is no responsible omnipotent body for supporting the general city development in all its dimensions. This could have been the case in historical time for instance in Europe when cities were governed by a prince or a bishop having full authority over the population and territory. Actually such cities should be considered as "states" and in fact are identified as "city-states" by historians. Their rivalries and the events in their development could be related to well identified "actors" who took decisions according to their representation of what their city should become. On the contrary we have shown, when discussing the difficulty of providing reliable urban statistics, the usual mismatch between consistent physical or functional urban entities and the administrative limits on which they are built. Today the real political power that controls a city's development has been delegated to upper levels of governance of territories or economic institutions and is no longer unified at the scale of a single city.

At another scale, namely systems of towns and cities, there is neither any decisional body that would take decisions for keeping the size of cities within the general model of city sizes. Of course towns and cities are connected by a multitude of links corresponding to a variety of social networks. A few networks can be identified that would be interested in lobbying for the sake of cities that belong to them. But no single institution, nor even a few competing ones, can be taken for being directly responsible for the persistence of the general model of city size distribution. As mentioned above, if cities keep relatively constant shares in general economic, demographic and cultural or technical developments, it is probably by a deconcentrated process of many decisions taken at a micro level (to invest in such activity, to migrate to a city, to adopt such innovation…), that drives the transformation of each town or city, in an incremental way, towards the general direction of change.

The economic theory could provide explanations by demonstrating that this structure of the urban system corresponds to an optimisation. However, the urban economic theory mainly conceive cities as places where agglomeration economies can be produced for attracting firms (Derycke et al., 1996; Huriot, Thisse, 1999). City size is theorised as resulting from the compromise between agglomeration economies and congestion costs (both quantities are very rarely empirically measured or estimated). Usually such a theory leads to the conception of an optimal city size (Bairoch, 1978). The economic models which try to produce as an output a rank size distribution (Cordoba, 2003, Gabaix and Ioannides, 2003, Fujita, 2000) remain theoretical and have not been empirically tested. Economic theory cannot yet explain why there should be a Zipf's law, and it is highly significant that in their last book on spatial and urban economy, Fujita, Krugman and Venables (1999) devote a whole chapter (chapter 12) to that question, entitled: "an empirical digression: the size of cities"! They quote a suggestion by Krugman to develop an analogue between cities and river networks, forgetting to recall that such an analogy was already suggested by Chorley and Haggett in their book on geographical models in 1967. Moreover, Krugman suggests that the "percolation theory" of physics could be used for modelling urban hierarchies as the result of a diffusion process. However, it is well known that diffusion processes in urban systems are very often "hierarchical", in the sense that the adoption of innovation "jumps" from a large city to a very distant large one before going to the closest (Pred, 1977) and that it cannot be simulated by a passive and contiguous diffusion process.

3.3 Alternative approaches for models of urban growth

Playing with power laws and trying to generate them from agent based models or multi-agents systems that define rules of interaction at a mico-level is not specific to the urban sciences. A large variety of examples can be found, from stock market and wealth distribution in a population (Solomon, Richmond, 2001), to the size of firms in a nation state (Axtell, 2001) or length of words in languages (Cancho, Solé, 2002). However, hypothesis and results should not simply be transferred from one discipline to the others, because the selection of relevant variables and processes is very specific, if one wants to represent even in a simplified way the empirical knowledge associated with each field.

S.E. Page (1988) proposes an agent-based model for simulating the emergence of cities, from very simple assumptions about agent's location behaviour, depending on a preference for agglomeration and an average distance to other agents. However, in this model an agent's utility is defined that refers to the distribution of agents on the lattice, it is not very plausible that real agents could attain such an information. Axtell and Florida (2000) provide a more detailed microeconomic multi-agent model of endogeneous firm formation, allowing agents to move between firms and between clusters of firms (then assimilated to cities). Under the hypothesis of increasing returns from clustering at the level of the firm, they simulate a size distribution with constant returns (average growth rate) at the aggregate level. A stationary macro-structure is generated from a non equilibrium microeconomics. However appealing because it conciliates two apparently contradictory but observed processes (search for increasing returns at the individual level, no decisive increasing returns at the aggregate level), this model has not yet reached the stage of validation from empirical observations.

Anderson et al. (2003) use an algorithm building « scale-free » networks. This corresponds to a class of growing networks whose node degrees are power law distributed. In their model, the nodes of the network represent pieces of land which become over time more and more connected by edges representing exchanges of goods and services (actually the result of this trade is simulated by a trade benefit or financial investment directed from one node to another). The model proceeds by adding new links between already developed nodes, with a probability which is proportional to the relative size of the node in the total of nodes, and by selecting new nodes. The mean probability of developing existing nodes is significantly higher than the one attached to the development of new nodes. Spatial rules are added for specifying this selection process, according to hypothesis about a distance-decay interaction model.

The model is calibrated in order to fit an impressive empirical data set about land values in Sweden (almost 3 million observations). The paper demonstrates the ability of the model to reproduce the global statistical distribution (frequency of land squares with according to land price) and its main parameter (Pareto exponent of 2.1). The authors assume a linear relationship between the value per unit of urban land and the size of urban population, so their model could be used as a starting point for fitting population data as well.

The paper is not clear bout the scale of application of the model: whereas referring at first to Zipf's law, which is a model of the interurban distribution of city sizes, it represent "systems of specialised trading activities" that "can be resolved to any resolution down to individual transactions", whereas the explanation of the model in "an urban economy context" seems to refer mainly to intra-urban land values formation (for instance, looking at different processes at the perimeter of urban areas and predicting the emergence of urban sub-centres). In any case, the model predicts a single and unified statistical distribution of land values at a country scale, making no distinction between the intra-urban gradient of land prices and the interurban distribution of land values. The model produces only a sharp break between rural and urban land values. To be coherent with the existing state of knowledge, the authors should have tested the variations of land prices inside the nodes (between centres and peripheries) as well as between the aggregated nodes. It could happen that the rather high level of inequalities they find between land prices is more linked with intra-urban inequalities than to interurban. Actually, when I looked at the average housing, offices and land prices per urban area in France, I discovered that prices are surprisingly similar from one city to the next (low variance) and the correlation with city size (as measured by population figures) is rather low for the entire distribution (even if a very large city like Paris has the highest prices in the country). Meanwhile, the inequalities between the prices per hectare inside the same city may reach a factor 10 and more, at the block level.

The simulation by network building is however an interesting way of specifying a growth process in an urban system. Models of distributed growth are at the basis of the generation of power law distributions. This was shown as early as 1922 by Willis and Yule in a deterministic model explaining the linear relation between the logarithms of the numbers of genus and species; such a model was applied for cities by Steindl who demonstrated again in

1965 that two hypothesis are sufficient for generating a rank-size distribution: all towns and cities have the same growth rate, and there is a constant ratio over time between this urban population growth rate and the rate of appearance of new towns in the urban system. Before that, other authors willing to take into account the numerous fluctuations of observed urban growth rates between cities and over time had invented stochastic processes including similar principles: H. Simon (1955), transposing a model first elaborated by Yule (1924), shows how a Pareto-like distribution, which is characteristic of contagious processes, can be generated if the probability for a migrant to reach a city of size i is proportional to the total number of people already living in cities of size i and if the individual probability of migrating to a new town remains constant over time; R. Gibrat (1931) demonstrates that the "law of proportional effect" as a stochastic model is sufficient for explaining the emergence of a lognormal distribution of city sizes.

Our first SIMPOP model (Bura et al., 1996) generates rank-size distribution from a multi-agents system simulating a spatially competitive urban growth. We made the spatial process of urban growth more explicit by including the centre-periphery effects in a less detailed but more powerful simulation model (Page et al., 2001). In these models, we did not look at the intra-urban distribution of population, obviously for reasons of limitation of computer memory capacities, but also because we think that intra-urban and inter-urban distributions have different forms, which may be explained by different morphogenetic processes. Anderson and his colleagues may be right in considering the urban land values as a scale-free distribution for a whole country, but they seem to neglect an essential feature of such a distribution, its multi-fractality. As observed in the case of spatial distributions of population or of built-up areas, there are at least two different gradients, one inside intra-urban aggregates and the other for inter-urban nodes. We would like to develop a theory that generates both, if possible within a single model. The urban space has not the same properties within and between cities. The land use is somehow more intense inside cities, generating a better accessibility to a larger diversity of activities, together with a high level of mobility, but the transportation speed for connecting intra-urban places is rather slow. The space between cities is more diluted, accessibility and frequency of use are lower, but the speed of connection is much higher.

The solution could be to include a model of growth with similar characteristics as the already existing models that can generate skewed distribution of size aggregates, but at two different scales according to the intensity of activities allowed by speed of transportation, and with evolving speed and range over historical time. The fractal dimension of the generated aggregates could be used for validation of the model.

3.4 The outcome of a constrained evolutionary process?

Rather than conceiving, as economists do, that at any time there is an optimisation, or an equilibrium, our observations would lead us to consider that urban systems do evolve far from equilibrium. (For instance, when we adjusted the evolution of inter-urban migration flows in France (between 1954 and 1990, Haag and al., 1992) trough a dynamic model, the resulting distribution of urban population among cities was far from the equilibrium that had been computed after the dynamics of migrations). The general structure of urban systems, including scaling effects, could be the result of social evolutionary processes : as in biological sciences, one can identify effects of mutation, competition, cooperation and selection, but in this case the evolution is also partly driven by a cognitive activity of inventing technical and social artefacts. Improvement of accessibility to more and more sophisticated activities, in order to reduce life uncertainties (ecological and social), could be considered as the main constraint operating on the evolution of the system, at both scales of analysis. But the action of this organising principle on the spatial structure of the urban systems is almost always indirect : especially at the level of the system of cities, there is no conscious will neither no responsible institution for organising and adapting the system to ensure this increasing power of accessibility. The global structure and its more or less continuous adaptation are emerging from the interurban competition.

In such an interpretation, the accessibility constraint would be the systemic ordering principle of the spatial structure of urban systems. It could be a kind of collective "rationale", distinct from the actual individual intentions of urban actors, which could be, on the whole, to make a better living by adding value to the urban "heritage" (global value of urban estates, production capital of firms, human capital of resident population, development potential of activities and urban capacities, symbolic value inducing urban attractivity, and so on). This very general aim (?), at the level of urban actors, produces, when aggregated, the apparent "greediness" at the level of each city, which explain their competitive incremental adaptation to change and the process of creation and capture of innovation that characterises their interactive dynamics within the systems of cities. It is this historical competitive process which explains the persistence of a continuum of towns and city sizes, representative of individual stories of towns which succeeded or failed to adapt at each step of the evolution. The city size (and the correlated variety and complexity of the activity portfolio and social and cultural sophistication) is the cumulated product of such an history.

The largest metropolises usually have very long stories of successful adaptation. For instance, Paris and London in Europe, even if they were state capitals, have both benefited from the industrial revolution of the XIXth century – probably because they also were since a long time the largest city in their country, and then the more likely to be open to any kind of innovation. The industrial revolution can be seen as a major perturbation for the European urban system because specialised cities, sometimes very large, as Birmingham, or Manchester, or cities of the Ruhr area, were boosted from a status of small towns to the upper ranks of the urban hierarchy. The industrial revolution had another important effect by reinforcing the inequalities in city size all over Europe (mainly through the effects of rapid transportation means). But despite this, it has to be recalled that the urban hierarchy in Europe was not totally upset by the industrial revolution: the correlation between the rank of cities in the middle of eighteen century, before it started, and their rank in 1950, long after its end, is very high (coefficient around 0.8!). The capacity of this system of cities to absorb big shocks was already attested in XVth century by its recovery, making it similar both in cities' size and ranking, one century after the Black Death had divided the population by a factor two. This is another indication of the number and strength of the links that have for centuries tightly knotted the network of European towns and cities, making their co-evolution a fully competitive process much before the continent was equipped with direct and rapid transport connexions between all of them. The major transformation that happened between the XVIth and XVII century, transferring the core of the system from the Mediterranean coast to the Nordsea (de Vries, 1984, after Braudel) was a very slow process. The development of functionally specialised cities is usually linked with different economic cycles which have favoured the growth of particular places : before the industrial revolution, there are references to similar "generations" of cities, that were once driven by textile industry, trade with the colonies, or since the end of nineteenth century by mass tourism. A recent version of this specialisation process would occur for instance in the cities of high tech, or finance activities. However successful during the boom of the specialising activities, such places may have difficulties to continue their further adaptation to new cycles of products, or ways of producing, unless they continuously invent new kinds of innovation. Some of them may share the destiny of the smallest towns, that were once successful but not for long –but that can become again adapted to some innovation and are susceptible to grow again, even if the probability that they would ever challenge the largest metropolis is very small.

At least three time scales have then to be considered for describing the main processes of this evolution of urban systems : the short time process of innovation and competition, as it may be seen at the actor level, the mean time –usually a few decades- process of specialisation as related to economic cycles at the level of each town and city, and the long time process of the emergence and slow transformation of the urban hierarchy – in general, several centuries- at the level of the whole system of cities.

Conclusion

"Scaling laws typically reflect, and often reveal, the general principles underlying the structure of a physical problem" (West, 2002). Does this assumption hold for social systems?

In the case of urban systems, scaling effects could be studied by relating either cities population size, their occupied surface, density of activities, speed of transports, income levels and/or accessibility in transportation systems... Perhaps the easiest and most fruitful direction would be to consider the question of urban spatial expansion at two scales of analysis, trying to understand two contrasting processes: at the level of the city, slower growth in surface than in population for some time after industrial revolution, increasing urban densities, then rapid urban sprawl with a larger increase in surface than in population during the last decades, inside a general model of spatial distribution characterised by a rather steep but recently decreasing density gradient from the centre to the periphery ; and growth in size and number of cities but with increasing size inequalities (and local concentrations) at the scale of the interurban processes (national or regional territories). In both cases, the controlling parameters could be the means of transportation, with varying speeds over time, and their typical spatial range that is different when adapting to the daily urban system or to the connection within networks of cities. But perhaps other social (economic) or physical processes have to be included to provide a consistent model at both scales.

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Figure 3c

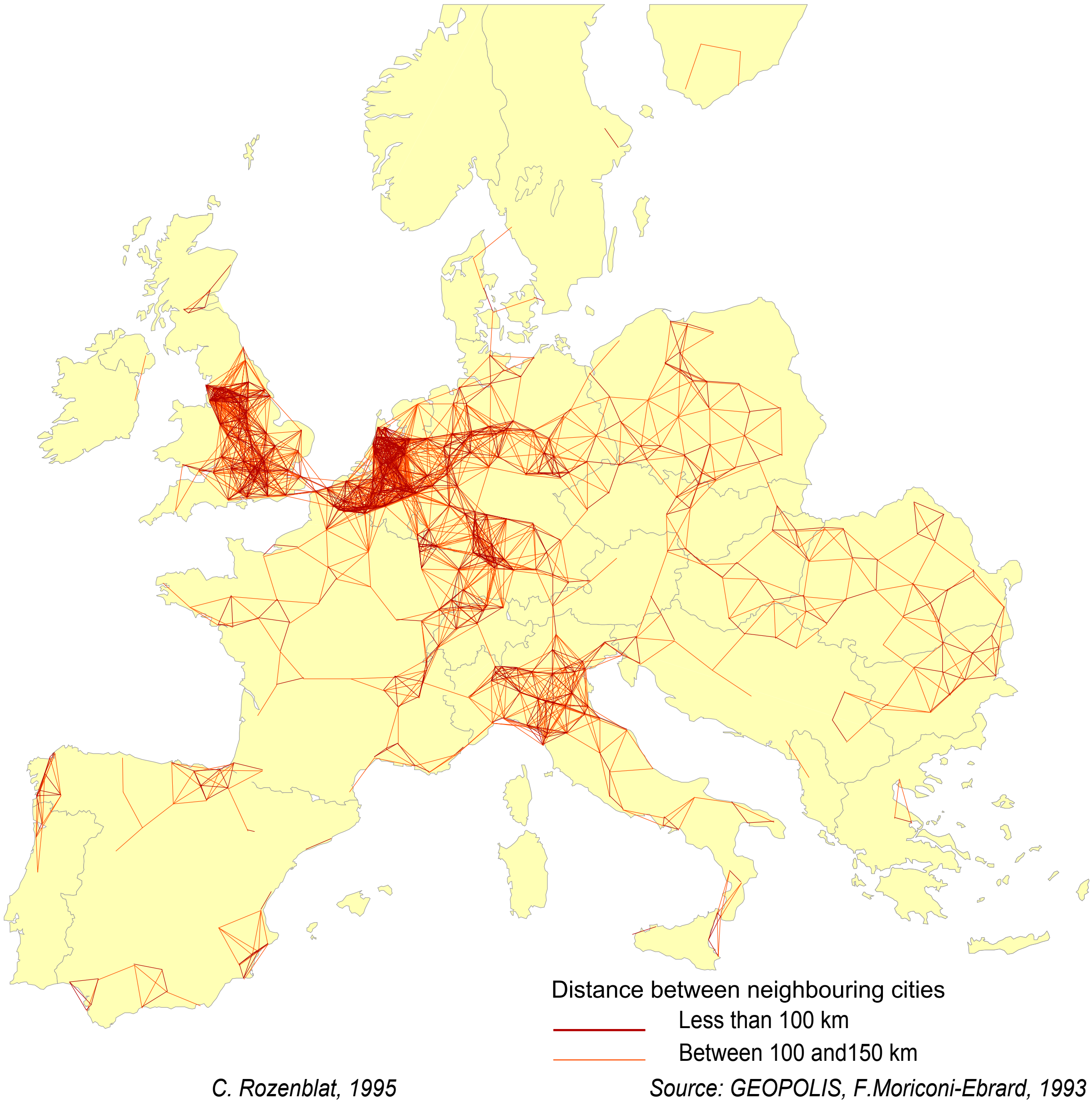

Distance between neighbouring cities
Less than 100 km
Between 100 and150 km
C. Rozenblat, 1995
Source: GEOPOLIS, F.Moriconi-Ebrard, 1993

Figure 3b

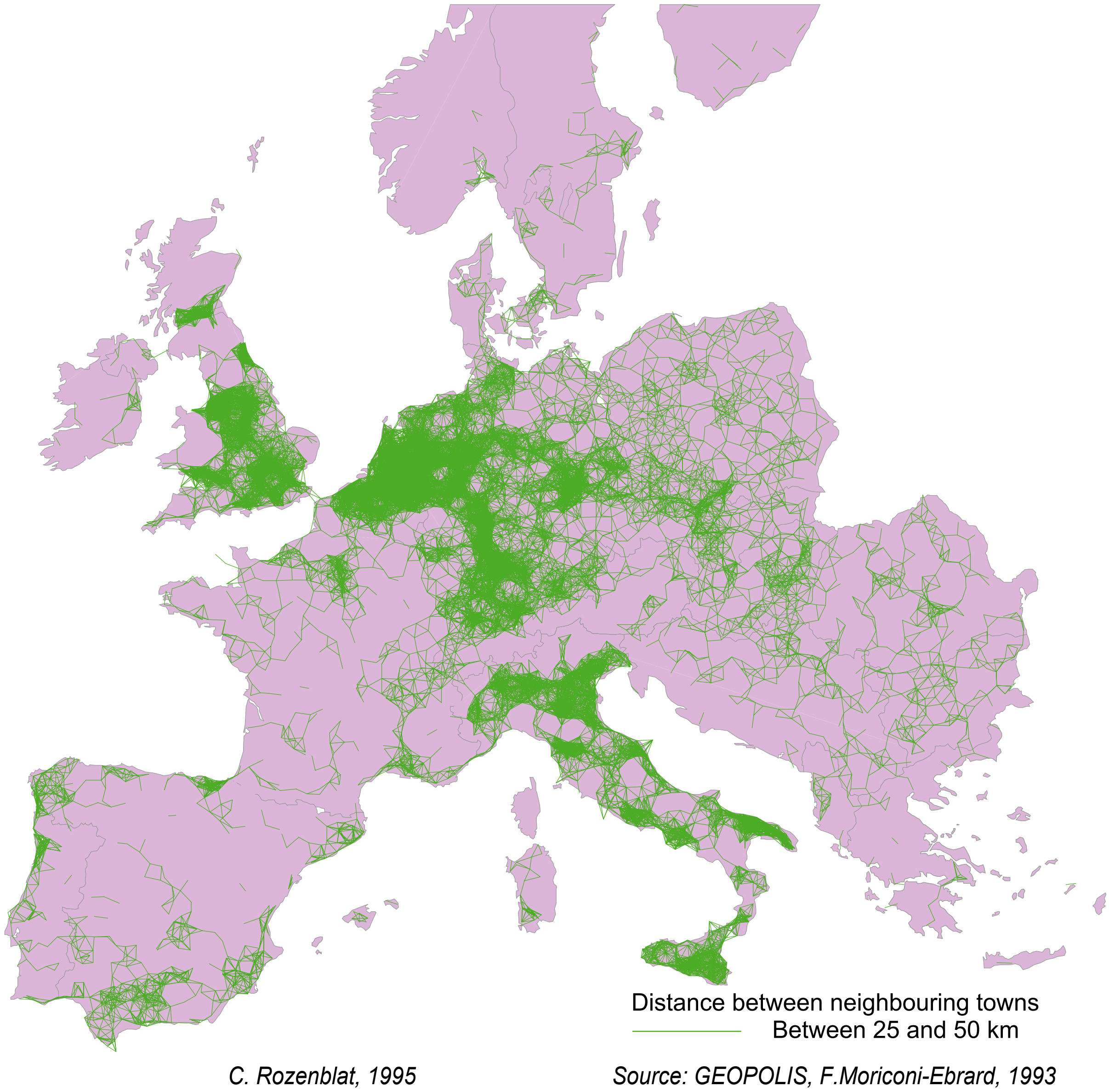
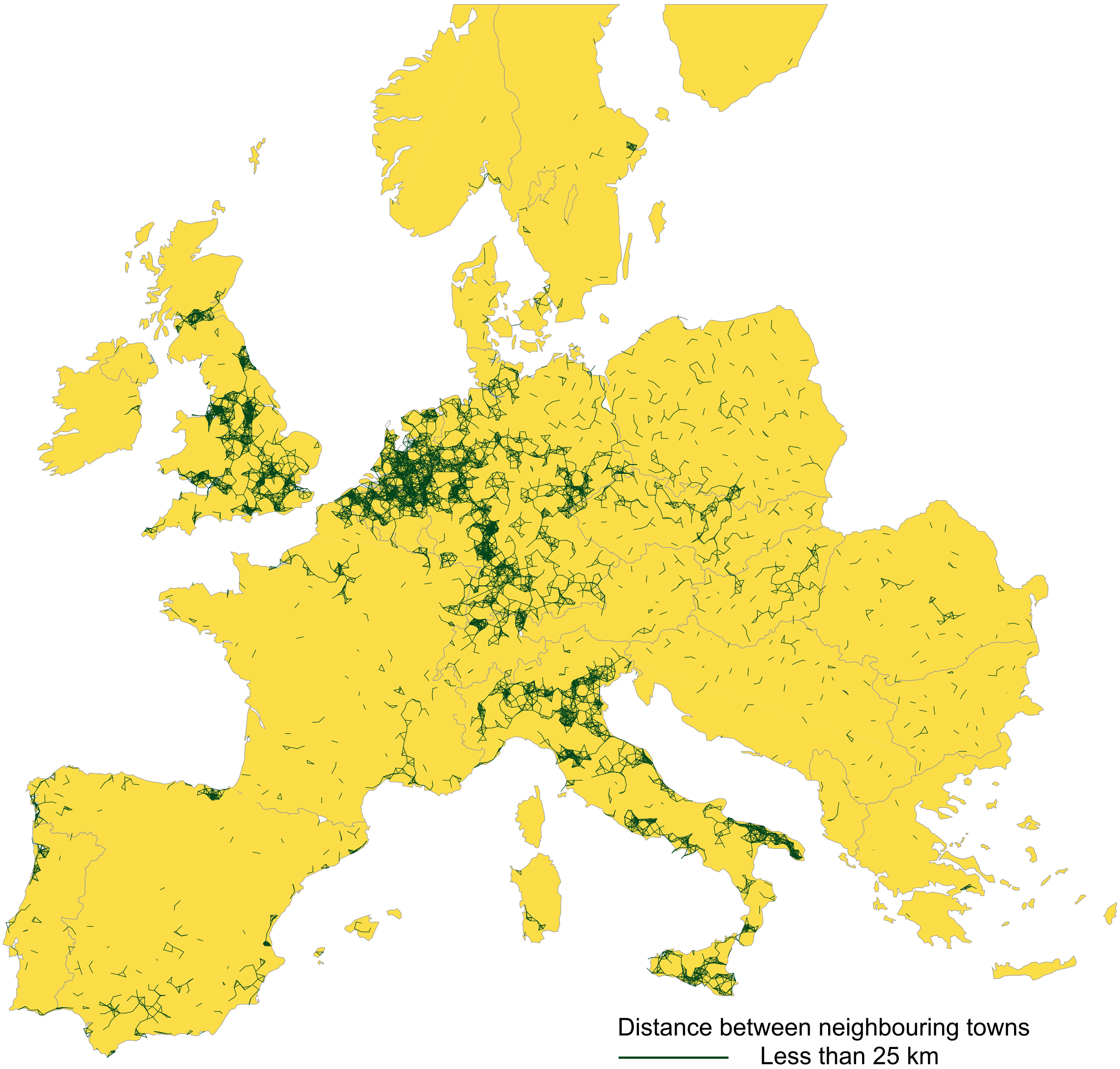

Distance between neighbouring towns
Between 25 and 50 km
C. Rozenblat, 1995
Source: GEOPOLIS, F.Moriconi-Ebrard, 1993

Figure 3a



C.Rozenblat, 1995

Source, GEOPOLIS, F.Moriconi-Ebrard, 1993