Structure and Strategy in Collective Action: Communication and Coordination in Social Networks

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Abstract

This paper considers both structural and strategic influences on collective action. Each person in a group wants to participate only if the total number participating is at least her threshold; people use a social network to communicate their thresholds. Results include: cliques form the common knowledge sufficient and in some sense necessary for collective action; dispersion of "insurgents," people strongly predisposed toward collective action, can be good for collective action but too much dispersion can be bad; classic "bandwagon" models overstate the fragility of collective action.

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Introduction

Although collective action depends on both social structure and individual incentives, these integral aspects have been formalized separately, in the fields of social network theory and game theory. By considering both structure and strategy, this paper engages the classic question of which structures are conducive to collective action and explores how structure and strategy interact.

Here a group of people face a collective action problem in that an individual wants to participate only if joined by others; exactly how many total participants are necessary is given by her "threshold." Social structure is thought of here as a communication network. In each time period, each person tells the people to whom she is connected her own threshold and what she knows about the thresholds of others. As time progresses, communication iterates, and each person knows the thresholds of a larger set of people. Given this knowledge, each person decides whether to participate, considering the rationality and knowledge of others.

The model thus predicts how and among whom collective action emerges and grows, as people know more about each other in time. If it grows quickly, the network, and peoples' thresholds and positions in the network, are considered conducive to collective action. The main motivation here is collective political action, such as protest, revolt, or revolution, but the model of course applies to collective action generally.

The model is based on the "I'll go if you go" mechanism: explicit communication and then individual choices given that communication. I discuss this key assumption and present the model, in "nontechnical" overview and then in complete detail. I explain the model using a simple example and talk about how to compute solutions. I then discuss and defend my modelling choices and assumptions.

In an exploratory spirit, I go through some examples and elementary results. First I show that participation increases in time and network density and decreases in people's thresholds. The increase in time is "clumpy," not smooth, and people who have similar or close social positions and thresholds participate at nearly the same time. Second, I show that cliques, groups in which everyone talks to everyone else, generate the common knowledge sufficient and in some sense necessary for collective action. Thus "strong" links, which form small cliques quickly, are good when thresholds are low, and "weak" links, which form large cliques quickly, are good when thresholds are high. The third topic is centrality. A centralized organization can be especially good at forming common knowledge

and hence acting collectively. The effects of being in a central position vary: peripheral people sometimes participate earlier than central ones, and sometimes collective action is faster when insurgents are peripheral. The fourth topic is dispersion: wide dispersion of insurgents can be good for collective action, but too much dispersion can be bad. Finally, I show that "bandwagon" models, in which each person responds to the number of people already participating, rely on extremely limited communication and hence overstate the fragility of collective action.

In conclusion I discuss generally what structure and strategy can offer each other. Since the examples are the essence of the paper, readers not methodologically inclined might skip to them after overviewing the model.

Communication as the basis for collective action

This paper is built on the assumption that the most basic and common mechanism for collective action is that individuals first communicate with each other about their preferences, and then each individual chooses whether to participate or not. That is, if we see a couple together at the opera, it is possible that they met there by chance, that one followed the other in, or that they meet there out of habit or tradition, but it is safest to assume simply that one invited the other. Even in this simplest of examples, there are aspects of both strategy (I want to go only if he goes with me) and structure (I don't know his telephone number but my friend does).

This "communication mechanism" is easier to use in small groups: it is difficult, but not impossible, for a hundred people to communicate with each other and decide to take a joint action. This difficulty is thought of here as depending on the group's social network: the more contacts people have with each other, for example, the easier communication is.

The main difference between this mechanism and others is that a person's participation depends on what he knows about others; a person's participation does not depend solely on the participation of her neighbors, either through learning, adaptation, or social influence. The physical analogies of contagion, diffusion, or chain reaction are not appropriate; the social network "carries" information, not influence or participation itself. An illustration of this difference is to think of five people, each of whom wants to participate as long as all of the others do. In the model here, after the five people talk together and realize that they

have a common desire, each person participates. The five people participate simultaneously, on the basis of their shared information, not on anyone's previous participation.

The importance of the communication mechanism for collective political action is most clearly shown in the efforts governments take to restrict it, including banning public meetings and public communications such as newspapers, murals, and even graffiti (Sluka 1992, Diehl 1992). The importance of social networks for participation has been demonstrated in many contexts (for example Snow, Zurcher, and Ekland-Olson 1980 and Opp and Gern 1993). Indeed communication is restricted even at personal levels: in the former East Germany, "only about 13 percent of households had a telephone and there were few restaurants or pubs" (Opp and Gern 1993, p. 662); plantation owners in Hawaii around 1900 tried to discourage labor action by conscripting workers who spoke different native languages (Takaki 1983).

A combined structural and strategic analysis of collective action goes back at least to Marx, who saw the proletariat's emergence as a collective actor as resulting from machine production reducing and levelling wages and concentrating workers in factories (Marx and Engels 1848; see also Marx 1986). Interestingly, Marx emphasized improved communication networks resulting from technologies such as the railroad.

The model

Overview

Say there is a group of n people, and each person chooses either to revolt (participate in the collective action) or stay at home (not participate). Each person has an idiosyncratic threshold; a person prefers to revolt only if the number of people who revolt is greater or equal to her threshold. For example, a person with threshold 2 prefers to revolt if he is joined by at least one other; a person with threshold n prefers to revolt only if everyone else does. A person with threshold 1 always prefers to revolt and a person with threshold $n+1$ never prefers to revolt.

Initially, each person knows only his own threshold. After one period of time, a person finds out the thresholds of his neighbors in the social network. After two periods, a person finds out the thresholds of each of his neighbors' neighbors. In general, at time t, a person knows the thresholds of everyone who is within t links away. This communication process

is for simplicity considered as mechanistic and not a matter of individual choice; there is no issue of strategic exaggeration or deception.

At each period of time, each person decides whether to revolt or stay at home given the knowledge she has of other people's thresholds. She thinks strategically, knowing that everyone else is also making a rational choice given their own information. As time progresses, and as each person knows more about other peoples' thresholds, revolt becomes more possible. For simplicity there is no strategic interaction across time; a person's decision depends only on current information and not on any previous actions.

To sum up, the "chain of causality" is as follows. The primitives of the model are the social network and the individual thresholds, which are exogenously given and fixed. The social network determines how communication flows and thus what each person knows at a given time. At a given time, each person, based on his knowledge of other people's thresholds, and his knowledge of what other people know, decides whether to revolt or stay at home; since these decisions are interdependent, we use a game theoretic equilibrium concept to predict people's actions. The model tells us which people revolt first and how revolt grows. If revolt occurs quickly, we consider the social network and individual thresholds conducive to collective action.

Details

The set of communication links among our finite set of $N = \{1, 2, 3, \ldots, n\}$ people is represented by a binary relation $\rightarrow$ on N, where $i \rightarrow j$ means that person i sends information to person j. If we assume that all communication links are reciprocal, then $\rightarrow$ is symmetric ($i \rightarrow j \Rightarrow j \rightarrow i$). Throughout, whether $i \rightarrow i$ or not is immaterial.

If $i_1, i_2, \ldots, i_m \in N$ are distinct and $i_1 \rightarrow i_2 \rightarrow \cdots \rightarrow i_m$, this sequence is called a path of length $m - 1$ from i_1 to i_m. For $j, i \in N$, the distance $d(j, i)$ from j to i is defined to be the length of the shortest path from j to i; if there is no path we say $d(j, i) = \infty$, and if $j = i$, we say $d(j, i) = 0$.

Each person has a threshold $\theta_i \in \{1, 2, 3, \ldots, n + 1\}$. Thresholds are the only source of uncertainty; hence the set of all possible states of the world is $\Theta = \{1, 2, 3, \ldots, n + 1\}^n$. The objective probability distribution $\pi : \Theta \rightarrow [0, 1]$ over Θ is given by $\pi(\theta) = 1/(n + 1)^n$; each state occurs with equal probability.

At time $t = 0$, before any communication has taken place, person i knows only her own threshold. At time $t = 1$, person i knows the thresholds of her immediate neighbors

in the network, those people j such that $j \to i$. Generally, at time t, person i knows the thresholds of all people j who are within distance t of person i, that is, people in the ball $B(i,t) = \{j \in N : d(j,i) \leq t\}$.

Hence if the actual state of the world is $\theta \in \Theta$, then person i knows only that the state of the world is in the set $P_i^t(\theta) = \{(\theta_{B(i,t)}, \phi_{N \setminus B(i,t)}) : \phi_{N \setminus B(i,t)} \in \{1, 2, 3 \ldots, n+1\}^{n - \#B(i,t)}\}$ (throughout we use the notation $\theta_A = (\theta_j)_{j \in A}$). Taken together the sets $\{P_i^t(\theta)\}_{\theta \in \Theta}$ form a partition of Θ, which we call $\mathcal{P}_i^t$ (throughout, individuals are indexed by subscripts and time is indexed by superscripts).

At a given time t, each person i simultaneously chooses an action a_i from the set $\{r, s\}$, where r stands for "revolt" and s stands for "stay at home." Given her own threshold θ_i and everyone's actions $a = (a_1, \ldots, a_n) \in \{r, s\}^n$, person i's utility $u_i(\theta_i, a)$ is given by

$$u_i(\theta_i, a) = \begin{cases} 0 & \text{if } a_i = s \\ 1 & \text{if } a_i = r \text{ and } \#\{j \in N : a_j = r\} \geq \theta_i \\ -z & \text{if } a_i = r \text{ and } \#\{j \in N : a_j = r\} < \theta_i \end{cases}$$

where $-z < -(n+1)^n$. In other words, a person always gets utility 0 by staying at home. If he revolts, he gets utility 1 if the total number of people revolting is at least θ_i, his threshold. If he revolts and not enough people join him, he gets the very large disutility $-z$.

At time t, person i's choice of action is based on her information, represented by the partition $\mathcal{P}_i^t$. Thus we define a strategy for person i at time t to be a function $f_i^t : \Theta \to \{r, s\}$ which is measurable with respect to $\mathcal{P}_i^t$, that is, for all $\theta, \theta' \in \Theta$, if $\theta, \theta' \in P$, where $P \in \mathcal{P}_i^t$, then $f_i^t(\theta) = f_i^t(\theta')$. The idea here is that if θ and θ' are in the same element of the partition $\mathcal{P}_i^t$, then person i at time t cannot distinguish between the two states θ and θ', and hence must take the same action in the two states.

Say that F_i^t is the set of all strategies of person i at time t, and say that $F^t = \times_{i \in N} F_i^t$ is the set of all strategy profiles at time t. If $f^t = (f_1^t, f_2^t, \ldots, f_n^t) \in F^t$, then person i's expected utility is $EU_i(f^t) = \sum_{\theta \in \Theta} \pi(\theta) u_i(\theta_i, f^t(\theta))$, where $f^t(\theta) = (f_1^t(\theta), f_2^t(\theta), \ldots, f_n^t(\theta))$. Note that since expected utility does not depend on t directly, we write EU_i instead of EU_i^t.

Say that $f^t \in F^t$ is a strategy profile at time t. We say that f^t is an equilibrium at time t if for all $i \in N$, and for all $g_i^t \in F_i^t$, $EU_i(f^t) \geq EU_i(g_i^t, f_{N \setminus \{i\}}^t)$. The strategy profile f^t is an equilibrium if no individual can gain by deviating to another strategy $g_i^t \in F_i^t$. Note that since EU_i does not directly change in t, the only things in the definition which change in t are the strategy sets F_i^t.

For a given time t, in general there exist many equilibria. We assume, however, that people play the equilibrium b^t which has the greatest possible revolt: if person i revolts in any equilibrium, she revolts in this "best" equilibrium. This best equilibrium uniquely exists (all proofs are in the appendix).

Result 1. There uniquely exists an equilibrium $b^t \in F^t$ such that if $f^t \in F^t$ is an equilibrium, then $f_i^t(\theta) = r \Rightarrow b_i^t(\theta) = r$.

Given the relation $\rightarrow$ and thresholds θ, the set of people who revolt at time t is $R^t(\rightarrow, \theta) = \{i \in N : b_i^t(\theta) = r\}$. We show later (Result 2) that $R^0(\rightarrow, \theta) \subset R^1(\rightarrow, \theta) \subset R^2(\rightarrow, \theta) \subset \cdots$, and hence the limit of this sequence $R^\infty(\rightarrow, \theta)$, the set of people who eventually revolt, is well defined. Revolt begins at time $t_{begin}(\rightarrow, \theta) = \min\{t \geq 0 : R^t \neq \emptyset\}$, and everyone revolts starting at time $t_{full}(\rightarrow, \theta) = \min\{t \geq 0 : R^t = N\}$ (if $R^t(\rightarrow, \theta)$ is always empty, we say $t_{begin}(\rightarrow, \theta) = \infty$; if $R^t(\rightarrow, \theta)$ never reaches N, we say $t_{full}(\rightarrow, \theta) = \infty$).

Example

Say $N = \{1, 2\}$; thus $\Theta = \{11, 12, 13, 21, 22, 23, 31, 32, 33\}$ is the set of states of the world (we write 21 instead of $(2, 1)$ for convenience). For now, let the actual thresholds θ_1, θ_2 remain unspecified. Say the relation $\rightarrow$ is given by $1 \rightarrow 2,\ 2 \rightarrow 1$.

At time $t = 0$, each person knows only her own threshold: we have $\mathcal{P}_1^0 = \{\{11, 12, 13\}, \{21, 22, 23\}, \{31, 32, 33\}\}$ and $\mathcal{P}_2^0 = \{\{11, 21, 31\}, \{12, 22, 32\}, \{13, 23, 33\}\}$, shown in Figure 1. Given $\theta = 21$, the element of $\mathcal{P}_1^0$ which contains θ is $P_1^0(21) = \{21, 22, 23\}$. Thus when the state of the world is 21, person 1 knows only that the state of the world is in the set $\{21, 22, 23\}$, that is, only that his own threshold is 2. In other words, person 1 cannot distinguish between the states of the world in $\{21, 22, 23\}$.

Person 1's strategy is a function $f_1^0 : \Theta \rightarrow \{r, s\}$, a choice of action in every state of the world. Since he cannot distinguish between states in $\{21, 22, 23\}$, his action must be the same in all of them, that is, $f_1^0(21) = f_1^0(22) = f_1^0(23)$; this is necessary for f_1^0 to be measurable with respect to $\mathcal{P}_1^0$.

The best equilibrium b^0 is shown in Figure 1. Consider person 1. If his threshold is 1, that is in states 11, 12,and 13, he doesn't mind revolting all by himself. If his threshold is 3, then he never revolts. If his threshold is 2, then he would like to revolt if person 2 revolts, for example in state 21. But he must take the same action in all three states 21, 22, and

23. If he revolts, there is the possibility that he will revolt alone, since person 2's threshold could be 3; hence he plays it safe and stays at home.

At time $t = 1$, since $1 \to 2$ and $2 \to 1$, persons 1 and 2 know each other's threshold. So we have $\mathcal{P}_1^1 = \mathcal{P}_2^1 = \{\{11\}, \{12\}, \{13\}, \{21\}, \{22\}, \{23\}, \{31\}, \{32\}, \{33\}\}$, shown in Figure 1. Person 1 and person 2 can now distinguish between all states of the world, and hence their strategies $f_1^1 : \Theta \to \{r, s\}$ and $f_2^1 : \Theta \to \{r, s\}$ have no restriction.

Figure 1 shows the best equilibrium b^1. Person 1 now knows person 2's threshold and hence his action can be conditioned on it: revolt in states 21 and 22 and stay at home in state 23. Similarly, person 2 revolts in states 12 and 22. Note that if both people stayed at home in state 22, this would also be an equilibrium, but it would not be the best one.

Figure 1. Partitions $\mathcal{P}_1, \mathcal{P}_2$ and the best equilibrium strategies b_1, b_2 for $t = 0, 1$

Figure 2 shows the results of the model when $\theta_1 = 1, 2, 3$ and $\theta_2 = 2$. Since $\rightarrow$ is symmetric, we can represent $\rightarrow$ as an undirected graph, where each person is represented as a point; since $1 \rightarrow 2$ and $2 \rightarrow 1$, a line is drawn connecting the two. We write a person's threshold θ_i next to the point corresponding to person i. If at time t person i revolts, that is, $b_i^t(\theta) = r$, we make person i's point a large dot; the sequence of graphs as time progresses conveniently shows both the model's primitives and results. Figure 2 also includes the "summary statistics" t_{begin}, t_{full}, and R^∞.

Figure 2. Results for $\theta_1 = 1, 2, 3$ and $\theta_2 = 2$, where $1 \rightarrow 2,\ 2 \rightarrow 1$

Not surprisingly, the lower person 1's threshold, the more favorable conditions are for revolt. Clearly $\theta_1 = 3$ is the worst of the three cases; the cases of $\theta_1 = 1$ and $\theta_1 = 2$ require the same amount of time for everyone to revolt, but one might say that $\theta_1 = 1$ is better because someone revolts earlier.

How to compute the best equilibrium

To compute the best equilibrium, say thresholds are given by θ. We first simplify things by making the state space $\{\theta_1, n+1\} \times \{\theta_2, n+1\} \times \cdots \times \{\theta_n, n+1\}$ instead of the much larger Θ. We can do this because you either know a person's threshold exactly or not at all; because revolt is so risky, if you don't know it, you assume the worst.

The assumption which guarantees the existence of a best equilibrium is that other people revolting less always makes you want to revolt the same amount or less, never more. So when we iteratively apply the best response correspondence, we get a "decreasing" sequence; since the set of strategy profiles is finite, the sequence must eventually reach a fixed point, which is by definition an equilibrium (see the proof of Result 5). Computing the best equilibrium is just this iterative process; a computer program, based on published routines (Skiena 1990), is available from the author.

Discussion

First I should make clear that my usage of the term "collective action" differs from the more common usage associated with the free rider problem and the Prisoners' Dilemma (as in Olson 1971), in which an individual never prefers to participate in the collective action, regardless of who else is participating. In this paper we have a "coordination problem": since each person wants to participate if enough others do, the problem is not individual motivation but getting everyone to move from nonparticipation to participation.

Why base our model on a coordination problem instead of a free rider problem? First, communication cannot affect a free rider problem: if I never want to participate regardless of what others do, more information about others will not make me change. Structural issues can be interestingly addressed in the repeated Prisoners' Dilemma context (for example Kandori 1992 and Fearon and Laitin 1996), where communication is about who defected and not about individuals' desires. Second, some argue that the coordination problem model is more appropriate factually (for example Calvert 1992, Chong 1991, Goldstone 1994, Karklins and Petersen 1993); one justification might be that once popular action makes its demise

inevitable, even the staunchest supporter of the old regime would join in revolt, perhaps for fear of being ostracized or punished by the ascendant regime. Third, even if participation is made more attractive than free riding, thereby "solving" the free rider problem (see Lichbach 1995 and Moore 1995 for recent surveys), still probably no one would want to participate all by herself. So when solving a free rider problem, there is also an accompanying coordination problem.

In the coordination problem here, there is no disagreement over what to coordinate on: a person chooses to revolt or stay home, not which banner to stand under. More often there are competing goals: socialism versus social democracy, or less consequentially VHS versus Beta (Katz and Shapiro 1986); the social network might then influence what the group coordinates on. Here we look at the more basic problem of how a group goes from nonparticipation to participation.

Here each person cares only about the total number, not the positions, of fellow participants; for simplicity we assume that the coordination problem has no locality. The opposite "extreme" common in adaptive or evolutionary models is to assume complete locality, each person caring about the actions of only local others (Blume 1993, Ellison 1993, Young 1996a, 1996b; see also Brock and Durlauf 1995). Most collective actions fall between the two extremes: I care about what computer system my colleagues use, but I also care about each system's total market share, which determines how much software is available; I care most about which side of the road to drive on in my city, but sometimes I drive in other cities or countries. Political action certainly has an important "mass" component: my three friends and I will not start a revolt alone even if we all want to; even if my friends are not interested, I might join in a demonstration of ten thousand people. In any case, here all locality is through communication and hence knowledge; in a more complicated model, one could explore how locality of knowledge and locality in the "underlying game" interact.

How exactly does a person's utility depend on the number of total participants? In other words, where does a person's threshold come from? A person's threshold is best thought of as a "reduced form" parameter of a possibly complicated idiosyncratic decision process. For example, say I participate if the expected benefits to me of a new government outweigh the expected costs of being punished. If the probability that my participation makes a difference increases and the probability of being caught by the police decreases with the total number of participants, then I will participate if many others do and I will not

if only a few others do; my threshold is the number of people necessary to "tip me over" from staying at home to participating (Schelling 1978). Say I want to join in a protest not instrumentally but simply out of personal integrity, but the cost of actually protesting is too high unless enough people join in (Kuran 1991); here also I participate only if the total number of protesters is at least a certain level. Say I want to participate but will follow what the majority does; my threshold would then be $n/2$.

So using thresholds does not depend on whether people are "narrowly" rational, where benefits or costs come from, or if they change smoothly or discontinuously. Of course, certain things are ruled out: if I especially value being one of the first to protest, for example, then I might join in only if the number of participants is small rather than large (Granovetter and Soong 1983).

The model does assume that the penalty for revolting when not enough people do so is very large; when $n = 10$, the penalty is greater than 25 billion. This is for modelling convenience; we make a person choose to revolt only if she knows with absolute certainty that the total number revolting will be at least her threshold. We do this by making even a very small probability of not having enough comrades a sufficiently large deterrent. Another simplifying assumption is that staying at home here always guarantees the safe utility of zero.

What about our network model of communication? One objection might be that people should be able to communicate not only thresholds, but also the very fact of participation (see Arthur and Lane 1993, Banerjee 1992, Bikhchandani, Hirshleifer, and Welch 1992). It turns out that adding this does not change anything: for example, say you revolt today and I am your neighbor; if you tell me about your revolt, I find out tomorrow. But tomorrow I will also know everything you know about people's thresholds today, and hence I will be able to deduce your participation.

Here either two people are linked or they are not. More realistically, I randomly bump into and talk with some people more often than others. Communication links might also be "noisy," from misunderstandings or imperfections in technology. This might make communication more local, since reliability suffers in each iteration (Chwe 1995a; see also Burt 1980, p. 88).

There is also uncertainty in the network itself: I usually know which of my friends know each other, but sometimes I make mistakes. Since the network here is fixed and well

known, each person at any given time knows exactly the extent of other people's knowledge. More reasonable might be to assume that each person knows the structure only locally. If we assume that I know the social structure only among the local people whose thresholds I know, it turns out that nothing changes. Intuitively this is because if I don't know your threshold, it may as well be $n+1$ and you never revolt; hence what I know about what you know doesn't matter. So in this sense at least the model is consonant with local but growing knowledge of the network.

Another source of uncertainty might arise from people's actions themselves. For example, large groups especially are susceptible to "trembles": although everyone plans to meet at the square at sunrise tomorrow, someone might oversleep. In the model, the only uncertainty is over thresholds; all of these other uncertainties are assumed away for simplicity.

Here we take a strategic approach, but perhaps we do not take it far enough: here strategy involves only whether to revolt or not. But people also communicate strategically, including exaggeration, misrepresentation, and outright lying. Incorporating this, however, is not straightforward: existing strategic communication models focus on the simplest case of one sender and one receiver and have only begun to look at a wider social context (Farrell and Gibbons 1989 and Matthews and Postlewaite 1995 for example).

How the network is formed is also at least partly a strategic issue. We assume that the network is unchanging, but of course people intentionally make new acquaintances; even maintaining the existing network can be costly (Boorman 1975, Hendricks, Piccione, and Tan 1995). To say what a person gains by changing the network, however, one should be able to say what happens in the static cases of before and after (for example Aumann and Myerson 1985 and Jackson and Wolinsky 1995). A strategic model of network formation would include activists building organizations and creating links and governments repressing communication, destroying and restricting links.

By considering a sequence of static games, our model also proscribes any strategy involving time. For example, no one ever revolts earlier than they "should," anticipating that their courage will inspire others. Again, we leave this out for simplicity.

How then should time be interpreted? If taken literally, then the length of a period might be less than an hour, the time it takes for me to call and talk with each of my friends; whether revolt takes two or ten periods then doesn't seem important. But a time period is perhaps the time it takes for me and a friend to come to talk seriously, probably face to

face, about something as serious as our feelings about a potential revolution. With action seemingly far off, a time period could be weeks or months. If action is imminent, or in a period of crisis, a time period could be much shorter.

Time can also be considered more abstractly as an index of communication iterations and hence locality of knowledge. If full revolt takes two periods, for example, then we can say that revolt only requires one level of hearsay, that it is enough for each person to know only about her friends' friends. If it takes ten periods, then communication between "distant" people is necessary.

Finally, some "technical" aspects of the model should be discussed. Modelling knowledge as a partition of the set of states of the world and defining a strategy as a function measurable with respect to that partition goes back to Aumann's (1974) definition of correlated equilibrium (see also Mailath, Samuelson, and Shaked 1996). This setup uses "ex ante" expected utility; more natural perhaps would be to define a person's expected utility given his own threshold, since it is fixed and he always knows it. But this would not change anything, given our partitions and prior (Brandenburger and Dekel 1987). We use ex ante instead of "interim" expected utility for convenience.

Assuming complete rationality is common in game theory, but not in theories of social structure, which more often make adaptive, bounded rationality, or behavioral assumptions. There is no reason for this in principle, although the complexities of modelling complete rationality in a structural context might be a reason in practice. This paper shows that some of these complexities can be manageably handled, resulting in some structural observations not otherwise discernible (see also Morris 1996). Some coordinations, such as driving on the right (Young 1996a) and especially language (Blume, Kim, and Sobel 1993, Kim and Sobel 1995, Warneryd 1993), result from adaptation, but some are achieved through explicit communication. Sweden's switch from driving on the left to the right on a single day would be inconceivable without mass media ("They've done it" 1967); corporations try to make brand names into "household words" not gradually but by saturation advertising. Whether adaptation or explicit coordination, or complete or bounded rationality, better applies to a given real-world setting is an empirical as well as theoretical question. In any case, how to move "beyond" rationality (Macy 1995) should depend at least partly on what results from rationality.

Finally, by using an equilibrium concept we implicitly assume that people know each others' strategies (Aumann and Brandenburger 1995; see also Osborne and Rubinstein 1994); peoples' decisions are interdependent, not isolated. But we do not go so far as assuming explicit binding contracts (Marwell and Oliver 1993, p. 32); here a person participates because she wants to, not because dropping out makes others drop out.

By selecting the best equilibrium, it seems we are biasing our prediction in favor of revolt; presumably, how people get from "bad" to "good" equilibria is the entire issue, assumed away when we assume the best equilibrium. We could say that the best equilibrium at least places an upper bound on the amount of revolt; for example, t_{begin} is the earliest possible time at which anyone will revolt.

But if we make no assumption other than equilibrium, we would have to say that a coordination problem cannot be solved by communication: no matter how perfectly we communicate, there is still an equilibrium in which everyone ignores the communication and does not participate. Since people do in fact communicate purposefully to coordinate their actions, it is reasonable to assume outright that communication works (see also Johnson 1993). How well it works depends on the amount, kind, and structure of communication and the underlying game, but this is what the model is about.

How participation grows

Monotonicity

We first show that revolt increases in time and that denser networks and lower thresholds are more conducive to revolt. If $\rightarrow$ and $\rightarrow'$ are two relations on N, say that $\rightarrow$ is contained in $\rightarrow'$, written $\rightarrow \subset \rightarrow'$, if $i \rightarrow j \Rightarrow i \rightarrow' j$ for all $i, j \in N$. In other words, $\rightarrow \subset \rightarrow'$ means that $\rightarrow'$ has every link that $\rightarrow$ has. We say $\theta' \leq \theta$ if $\theta'_i \leq \theta_i$ for all $i \in N$: everyone's threshold in θ' is less than or equal to their threshold in θ.

Result 2. Say $t \leq t'$, $\rightarrow \subset \rightarrow'$ and $\theta' \leq \theta$. Then $R^t(\rightarrow, \theta) \subset R^{t'}(\rightarrow', \theta')$.

That density is good for collective action might be considered obvious (Marwell, Oliver, and Prahl 1988), but there are reasons to think otherwise: if nonparticipation can spread through the network, then more links might be bad (Gould 1993, Macy 1991). In our model, if you don't know someone's threshold, you essentially assume that the person never

revolts; hence more links and thus more information always helps. If people could discover that others' thresholds are lower than expected, then more links would not necessarily help.

One convenient way of thinking about how revolt increases in time is that "time makes the network denser." Given $\rightarrow$ and $t \geq 0$, define $\rightarrow^t$, the tth power of $\rightarrow$, as $i \rightarrow^t j$ if there is a path of length less than or equal to t from i to j (Buckley and Harary 1990). Figure 3 shows an example in which $\rightarrow$ is symmetric.

Figure 3. Powers of $\rightarrow$

Since being within distance t in the network $\rightarrow$ is the same as being within distance 1 in the network $\rightarrow^t$, we immediately have the following.

Result 3. $R^t(\rightarrow, \theta) = R^1(\rightarrow^t, \theta)$.

Say $d = \max\{d(i,j) : i,j \in N,\ d(i,j) < \infty\}$ is the length of the longest path of $\rightarrow$; if $d(i,j) < \infty$ always, then d is called the diameter of $\rightarrow$ (Buckley and Harary 1990). From Result 3 we get $R^\infty(\rightarrow, \theta) = R^d(\rightarrow, \theta)$; after time d, nothing changes.

Clumpiness: correlation in time and space

Since revolt here is based on explicit communication, it typically does not grow gradually. When a person revolts, the total number revolting must be at least that person's threshold; no one ever revolts without enough people joining in.

Result 4. $i \in R^t(\rightarrow, \theta) \Rightarrow \#R^t(\rightarrow, \theta) \geq \theta_i$.

This result is simple but sometimes useful: for example, if the group has three people with threshold 3 and three people with threshold 6, the threshold 6 people will all revolt at the same time (or never revolt), regardless of the network structure. The only case in which anyone revolts alone is if that person has threshold 1.

As for spatial correlation, we say two people i and j are structurally equivalent if $i \rightarrow j \Leftrightarrow j \rightarrow i$ and for all $k \in N \setminus \{i,j\}$, $k \rightarrow i \Leftrightarrow k \rightarrow j$ and $i \rightarrow k \Leftrightarrow j \rightarrow k$ (this definition is adapted from Burt 1976). In other words, i and j are structurally equivalent if they have exactly the same relationships with "others" and are either reciprocally connected or not connected at all; it is easy to see that this is an equivalence relation. We then have the following.

Result 5. If i and j are structurally equivalent and $\theta_i \leq \theta_j$, then $j \in R^t(\rightarrow, \theta) \Rightarrow i \in R^t(\rightarrow, \theta)$.

Hence if two people have the same threshold and are structurally equivalent, they either both revolt at the same time or both never revolt. Two people can be structurally equivalent even though they are "far apart" and never communicate with each other. If their

thresholds are the same, they revolt at the same time because they receive the same information about others and face the same decision; they coordinate not reciprocally but through "third parties."

Close people are not usually structurally equivalent, but they do have roughly similar information as time progresses: if you are my neighbor, I know everything you know, just one period later. Thus we get the following.

Result 6. If there is a path from j to i and $\theta_i \leq \theta_j$, then $j \in R^t(\rightarrow, \theta) \Rightarrow i \in R^{t+d(j,i)}(\rightarrow, \theta)$.

For example, if two people have the same threshold and are reciprocally connected (distance 1 apart), if one of them revolts at t, then the other revolts at either $t-1$, t, or $t+1$. When two people have the same threshold, the closer they are, the tighter the temporal correlation.

These results are interesting in relation to Coleman, Katz, and Menzel's (1966) study of how each doctor in a social structure starts to prescribe a newly introduced drug. Since a doctor might be reluctant to be the only one adopting the new drug, this might be thought of as a coordination problem. Burt's (1987) analysis of this data looks at correlation of adoption times, and finds a lot among doctors who are structurally equivalent and little among doctors who are close together in the structure. For people with similar thresholds, our results suggest greater correlation among people who are structurally equivalent, but also some correlation among people who are close.

A simple "logistic" model of social contagion, in which the fraction of nonadopters who become adopters tomorrow is proportional to the number of adopters today, results in a very gradual increase of adopters early on (Braun 1995; more generally, Allen 1982a). Since the data do not show this gradual increase, Burt concludes that purely personal choice, more than any social interaction, best explains adoption. But this conclusion depends completely on the simple logistic model. A model such as the one here can include both individual preferences (thresholds) and network interactions; rapid, "clumpy," adoption early on might be evidence not of the absence of social interaction but of a coordination problem.

Common knowledge, cliques, and strong links

Pluralistic ignorance and common knowledge

The entire point of this model is how the social network enables a group to know itself, to achieve a "collective self-understanding" (Gould 1995, p. 149). This topic has come up before, notably in the concept of "pluralistic ignorance" from social psychology (Allport 1924, Katz and Allport 1931, O'Gorman 1986) and "common knowledge," brought into game theory from philosophy (Lewis 1969, Aumann 1976). Consider the following example.

Say that we have four people, each with threshold 3; that is, each person will revolt as long as two others revolt also. Figure 4 shows two social structures, the "square" and the "kite."

Figure 4. Two structures: square and kite

Consider first the square. Before communication takes place, each person knows only his own threshold and hence does not revolt. After one period of time, each person knows that his two neighbors also have threshold 3. After two periods, each person knows the thresholds of his neighbors and his neighbors' neighbors, that is, everyone.

After the first period, each person knows that there are three people with threshold 3: himself and his two neighbors. That is, each person knows that "objective conditions" make revolt possible. But say I'm considering whether to revolt. What do I know about, say, my neighbor to the right? I know that he has threshold 3. I am his neighbor, and hence I know that he knows I have threshold 3. But I do not know anything about his other neighbor ("across" from me), and hence I cannot count on him revolting. Hence I do not revolt. So after one period of time, even though everyone knows that revolt is possible, no one in fact revolts.

In the kite after the first period, each individual in the "triangle" knows similarly that his two neighbors have threshold 3. But here, each individual knows that his two neighbors know the thresholds of each other. Among the three of us in the triangle, the fact that there are three people with threshold 3 is not only known by each person; each person knows that each other person knows this fact. Thus the three members of the "triangle" revolt after only one period. Figure 5 compares the square and kite.

Figure 5. The square and kite compared

In both square and kite, it takes two periods of time for everyone to revolt; in the kite, however, some people revolt earlier and in this sense the kite is more conducive to collective action. Note that this difference cannot be accounted for by macroscopic characteristics such as the total number of links (four in both cases), or even by finer measures such as the number of links each person has (in the kite, two of the early revolters have only two neighbors, as in the square). The difference between the square and kite is truly a structural difference. Note also that the assumption of strategic thinking is also critical; in a simple

diffusion or adaptive model in which I revolt if enough of my neighbors have low thresholds, there would be revolt in the square after only one period.

Pluralistic ignorance refers to a situation in which people hold very incorrect beliefs about the beliefs of others. To take one of many examples, in a 1972 survey, 15 percent of white Americans favored racial segregation, but 72 percent believed that a majority of the whites in their area favored segregation; in particular, whites who favored strict segregation were more likely to believe that a majority of whites favored segregation (O'Gorman 1979; see also Shamir 1993).

This concept did not come out of thinking about collective action; in fact, early accounts and most later work see it as a distortion more at the individual than the social level (O'Gorman 1986, Mullen and Hu 1988): a person reduces dissonance by thinking that her own view is the majority view, for example. Recently it has been applied to the former Soviet Union and eastern European states, the idea being that dissatisfaction was widespread but that few people knew how widespread it was. These accounts focus on limited communication due to criminal penalties for self-expression, a government-controlled press, and in the spirit of our model, a lack of social ties (Coser 1990, Kuran 1991). "The reduction of pluralistic ignorance," due to modern communication technology and increased foreign contacts, "has led. . . to a political wave of tremendous power" (Coser 1990, p. 182).

There are no mistaken beliefs in our model, but people can be unaware of how much dissatisfaction there is, in other words, other people's thresholds. In both square and kite, before communication starts, each person only knows her own threshold and no revolt takes place.

What our model makes absolutely clear is that overcoming pluralistic ignorance is not necessarily enough. In the square after one period of communication, everyone knows that three people have threshold 3, that there is enough discontent to make revolt possible; pluralistic ignorance is no longer a problem. But revolt still does not take place, because I do not know that you know that there is enough discontent. Overcoming pluralistic ignorance is thus necessary but not sufficient. What is required is "common knowledge": everyone has to know that there is sufficient discontent, everyone has to know that everyone knows, everyone has to know that everyone knows that everyone knows, and so on; this is achieved in the square after two time periods. In the kite, pluralistic ignorance is eliminated and common knowledge created (among three people) simultaneously, after one period.

The importance of common knowledge and "metaknowledge" generally for collective action has been explored in a variety of strategic contexts (Chwe 1995b, Morris, Rob, and Shin 1995, Rubinstein 1989, Shin 1996). Our model shows how the social network determines metaknowledge and hence revolt (see also Halpern and Moses 1990, Morris 1996, Parikh and Krasucki 1990). This simple example shows that generating the common knowledge sufficient for revolt is sometimes harder than merely eliminating pluralistic ignorance.

Cliques and common knowledge

A clique is a set of people in which each person talks to every other: $Q \subset N$ such that $i \to j$ for all $i, j \in Q$ such that $i \neq j$. This concept goes back to the beginnings of social network theory and is basic to defining and detecting social subgroups (Homans 1951, Luce and Perry 1949, Luce 1950; see Scott 1991). In our model, a clique is exactly what is necessary to form common knowledge, a basic concept in game theory.

Say Q is a clique of $\to^t$ and $\#Q \geq \theta_j$ for all $j \in Q$; that is, the size of the clique is greater than the threshold of each person in the clique. We call such a clique a "revolting clique" because everyone in such a clique revolts.

Result 7. Say Q is a clique of $\to^t$ and $\#Q \geq \theta_j$ for all $j \in Q$. Then $Q \subset R^t(\to, \theta)$.

In a revolting clique, thresholds are low enough to make everyone want to revolt given that everyone else revolts, and this fact is common knowledge among members of the clique; hence they all revolt. The thresholds and social structure of people outside do not matter; a revolting clique has enough members to support its own revolt and forms the necessary "local" common knowledge. So being in a revolting clique is sufficient for participation.

By the way, Q being a clique of $\to^t$ is equivalent to Q being a "t-clique" of $\to$ (Luce 1950; see Scott 1991, p. 118). Also, the usual definition of clique requires that it be maximal, but we drop this for simplicity; the results here hold under the usual definition also.

An accompanying "necessary" result is that if a person revolts, then he knows about, but is not necessarily a member of, a revolting clique.

Result 8. If $i \in R^t(\to, \theta)$, then there exists a clique $Q \subset R^t(\to, \theta) \cap B(i, t)$ of $\to^t$ such that $\#Q \geq \theta_j$ for all $j \in Q$.

Again, if person i revolts, then he knows about a revolting clique $Q \subset B(i,t)$, but is not necessarily a member of it. Revolting cliques are in this weak sense necessary for revolt. Result 8 can have strong conclusions: for example, if everyone's threshold is 7 or higher, then there can be no revolt until a clique of size 7 forms (that is, $\rightarrow^t$ has a clique of size 7).

To illustrate these results, consider five people on a "line"; Figure 6 shows the results together with the powers of $\rightarrow$.

Figure 6. Results together with $\rightarrow^t$

Before any communication, at $t = 0$, each person is in a clique of size 1, and since no one has threshold 1, by Result 8 no one revolts. At $t = 1$, each person is in a clique of size 2; the two people with threshold 2 form a revolting clique and we know they revolt by Result 7. At $t = 2$, the two people with threshold 3 are in a clique of size 3, but it is not a revolting clique since it includes the person with threshold 5. Finally, at $t = 3$, the two people with threshold 3 join the nearer threshold 2 person to make a revolting clique of size 3. The person with threshold 5 revolts even though he is not in a clique of size 5; by Result 8, he at least knows about a revolting clique, such as the threshold 2 pair. Here the threshold 5 person revolts because he knows that everyone else is revolting "by themselves"; no one relies on him but he relies on everyone else.

If everyone has the same threshold x, then being in a revolting clique, a clique of size at least x, is necessary and sufficient for revolt.

Result 9. Say $\theta = (x, x, \ldots, x)$. Then $i \in R^t(\rightarrow, \theta)$ if and only if there exists a clique Q of $\rightarrow^t$ such that $\#Q \geq x$ and $i \in Q$.

Making thresholds uniform suppresses "strategic" heterogeneity and highlights purely "structural" issues; our model then has a simple graph-theoretic interpretation. In Figure 7, we have a four-person clique with a two-person "tail," and everyone has a threshold of 4.

Figure 7. Results together with $\rightarrow^t$ (all thresholds are 4)

The four-person clique revolts at $t = 1$ since their clique is already formed. At $t = 2$, the closer member of the tail is in a clique of size 5 and thus revolts; the person at the end of the tail is only in a three-person clique. Finally at $t = 3$, a six-person clique includes everyone and everyone revolts.

We can define common knowledge precisely given the knowledge partitions $\mathcal{P}_i^t$ (Aumann 1976; see also Geanakoplos 1992 and Osborne and Rubinstein 1994). Here we derive knowledge partitions from a social structure; a clique is then simply a graphical representation of "local" common knowledge.

In models by Macy (1991) and Gould (1993), cliques are bad because nonparticipants in cliques reinforce each other; sparse networks, in which a participant can have greater influence on her nonparticipant neighbors, are better. Again, if people could discover that others' thresholds are lower than expected, we might get a similar result: cliques would make it harder to sustain a "collective fiction" that revolt has enough support. Cliques have a neutral or negative effect in a model by Marwell and Oliver (1993), but their quite different definition is based on the absence, not the presence, of links.

Intuitively, groups which have a sense of "community" are more likely to collectively act. This could be because in a community, everyone knows each other; the resulting clique creates the common knowledge sufficient for collective action.

Strong and weak links

The distinction between "strong" and "weak" links is basic to social network theory (Granovetter 1973). Roughly speaking, a strong link joins close friends and a weak link joins acquaintances. A simple empirical finding (Rapoport and Horvath 1961; see White 1992) is that strong links tend to traverse a society "slowly": start with an arbitrary person, find two of her close friends, then find two close friends of each of these two people, and continue in this manner. As you iterate, the group increases slowly because often no one new is added: the close friends of my close friends tend to be my close friends also. If instead you successively add two acquaintances, the group grows quickly: the acquaintances of my acquaintances tend not to be my acquaintances. Because of the empirical robustness of this finding, the terms "weak" and "strong" are used to indicate structural properties of relationships, as opposed to how good a friendship a given relationship is. Weak links traverse a society "quickly": a demonstration suggests that any two people in the United

States can be connected by as few as six weak links (Milgram 1992; see also Kochen 1989). Weak links tend to scatter widely, while strong links tend to be involuted.

To connect a large society, then, weak links are more important than strong links; weak links are more important for spreading information (Granovetter 1995; see also Montgomery 1991). For collective action, however, the importance of strong versus weak links is unclear. Data from volunteers in the 1964 Mississippi Freedom Summer, for example, show that the presence of a strong link to another potential participant correlates strongly and positively with participation while the presence of a weak link has no correlation (McAdam 1986, McAdam and Paulsen 1993; see also Fernandez and McAdam 1988).

Here I show that strong links are better for participation when thresholds are low and weak links are better when thresholds are high. This is because the involutedness of strong links, their tendency to form small cliques, is exactly what is needed to form common knowledge at a local level; if my friend's friend is my friend also, then common knowledge among the three of us is formed quickly. If thresholds are high, common knowledge must be formed among a large group of people; then weak links are better simply because they speed up communication.

Say we have thirteen people and consider the two network structures in Figure 8.

Figure 8. Circulant graphs: strong and weak links

Both structures are symmetric and in both each person is connected to four others; these are the "circulant graphs" $C_{13}(1,2)$ and $C_{13}(1,5)$ (Buckley and Harary 1990). The only difference between the two structures is that the first has strong links while the second has weak links: in the first, it can take as many as three steps to get from one person to another, while in the second it never takes more than two. Correspondingly, in the first, some of your friends' friends are your friends, while in the second, none of them are.

Figure 9 shows that when $\theta = (3, 3, \ldots, 3)$, the strong link network is better, and when $\theta = (7, 7, \ldots, 7)$, the weak link network is better (for clarity, a graph is drawn only when the set of people revolting changes).

Figure 9. When $\theta = 3$, strong links are better; when $\theta = 7$, weak links are better

When $\theta = (3, 3, \ldots, 3)$, strong links allow collective action quickly: there is a 3 person clique and hence common knowledge among three people is formed in only one unit of time. In the weak link network, after one period each person knows the thresholds of four other people, but none of these four people know the threshold of each other. When $\theta = (7, 7, \ldots, 7)$, the weak link network is better: common knowledge among seven people (in fact all thirteen people) is formed in two time periods. In the strong link network, communication spreads more slowly and hence three periods are necessary. Figure 10 compares t_{full}, the time necessary for full collective action, for the two structures as θ goes from $(1, 1, \ldots, 1)$ to $(13, 13, \ldots, 13)$.

Figure 10. Comparison of strong and weak links for $\theta = (1, 1, \ldots, 1), \ldots, (13, 13, \ldots, 13)$

That strong as opposed to weak links are important in recruiting for Freedom Summer is interpreted by McAdam (1986, p. 80) as the links having different functions: "although weak links may be more effective as diffusion channels, strong ties embody greater potential for influencing behavior." This is of course reasonable, but our model suggests it is not necessary. In Freedom Summer, a person might participate if a few friends also participate, which is like having a low threshold (strictly speaking, in our model only the number of fellow participants matters, not whether they are close friends). If you and I are potential participants connected by a strong link, your friends are likely to be my friends, and participation among our group of friends would be common knowledge among us. If you and I are connected by a weak link, your friends and my friends don't know each other, and hence there is no common knowledge group to which we both belong.

In other words, the idea that weak links are better for communication relies on the assumption that communication is about knowledge only and not metaknowledge. Strong links are better for forming common knowledge at a local level, and when thresholds are low, local mobilization is all that is necessary (see also Marwell, Oliver, and Prahl 1988, p. 532). Indeed, informal evidence suggests that the importance of weak links is in coordinating scattered "movement people" who are already participating, rather than influencing individual participation decisions (Freeman 1973, Jackson et al 1960).

In an expanded analysis, McAdam and Paulsen (1993, p. 658) find that organizations such as religious groups and civil rights groups gave individuals "a highly salient identity and strong social support for activism based on that identity." Interestingly, when organizational affiliation and the presence of a strong tie are included in the same regression, the strong positive effect of strong ties disappears. But organizational affiliation and strong ties may simply be indicators of the same underlying "variable": belonging to a group among which wanting to participate is common knowledge.

Centrality

Organizations as optimal networks

Since revolt increases with network density, the best possible network would be the "complete graph," a clique of everyone, in which each person is connected to every other. Communication would then be complete in one period; an example would be a public meeting

in which everyone expresses her opinion to everyone else. The worst possible network would be the "empty graph," which has no connections at all.

Short of these extremes, say if the total number of links are constrained, which networks are best and worst for collective action? Say that links are symmetric and that we have $n-1$ links, the minimum necessary to connect n people. Which network is best depends on people's thresholds. But speaking very roughly, the diameter of the network is the time it takes for the complete graph to form, and hence the upper bound on the full revolt time t_{full}; we can thus think of it as a worst-case measure of the difficulty of collective action.

The best network would then be a "star," shown in Figure 11 for $n = 10$. Since this network has diameter 2, complete communication is achieved in two periods; even adding more links would not decrease the diameter, unless you add enough to make the complete graph. The worst network which still keeps the group connected would be a "line," which has diameter 9.

Figure 11. The minimum and maximum diameter connected networks with $n - 1$ symmetric links ($n = 10$)

Anyhow, the star, the "best" network short of the complete graph, looks something like an organization; communication flows fastest when everyone reports to the same person, who then reports back to everyone. More "decentralized" networks such as the line do not perform well.

One might object that the star puts too much burden on the central person; we could then add the constraint that each person can be connected to at most three others. The optimal network when $n = 10$ is the "tree" shown in Figure 12.

Figure 12. The minimum diameter network with $n - 1$ symmetric links when each person is connected to at most three others ($n = 10$)

Even though collective action in our model has no inherent need for centralization or a command structure, the "best" networks, even considering information processing limitations, have a tree structure reminiscent of hierarchy or organization. That trees can reduce communication time is hardly a new idea in the context of organizational decision making (see for example Radner 1992). In the collective action context, Marwell, Oliver, and Prahl (1988) also stress centralization: in their model, people all linked to the same person, as in the star, are more likely to participate; they conclude that organizations are important for collective action because they are centralized. Gould (1993) finds stars optimal because a star not only allows a central participant to influence the most nonparticipants but also isolates the nonparticipants from each other. Here, stars are best because they "bring people together," form cliques, as quickly as possible.

Organizations such as parties, unions, and citizens' groups are obviously very important for collective action; a communication structure perspective is only part of the story, but it does allow great flexibility in thinking about what a (not necessarily "formal") organization is. One interpretation of the star would be people expressing their preferences by voting, with the results centrally counted and then publicly announced. Another might be a reporter or pollster who interviews people and then publishes the results; a newspaper's common readership might thus be considered an organization and hence collective actor (Anderson 1991).

In spite of the growing ease of electronic communication, Gould (1995, p. 205) predicts the continued primacy of collective action based on physical locality (see also Hedström 1994) because physical, not "virtual," neighbors of my neighbors are necessarily my neighbors; strong links provide collective identity, in our terms common knowledge. But many self-described Internet "communities" (Rai 1995, Slatalla 1996) are mailing lists or "bulletin boards," in which each person sends messages to a central person or computer which then retransmits the message to the entire group.

Does revolt start in the center?

We might expect revolt to start in areas of high communicative density, such as cities or central locations generally. But consider the two networks in Figure 13, and say all people have threshold 3. In the first network, the center is a clique of three people; in the second, the center is a single person (for various definitions of centrality, see Buckley and Harary 1990 and Burt 1980). In both networks, people in the center have the most links to others and also receive information about the entire group fastest.

Figure 13. Sometimes the center revolts
first, sometimes last (all thresholds are 3)

In the first network, the center revolts first, but in the second network, the center revolts last. This is because in the second network, none of the people the center knows about after one period know each other.

In general, although the center might have many interactions, they might be scattered throughout a large population; out in the periphery, where interactions are more "local" and my friends all know each other, cliques might be more likely. People in the periphery do not find out about the rest of the periphery as quickly as the center does, but if their thresholds are low, they don't need to; they just need to form common knowledge among themselves. Between 1853 and 1870, the city center of Paris was rebuilt, pushing working class people

out and resulting in neighborhood-based "communities" in the periphery, forming the basis for participation in the Paris Commune in 1871 (Gould 1995).

Do insurgents in the center help?

Centrality also comes up in the question of whether having people with low thresholds, "insurgents," in the center encourages or discourages revolt. When insurgents are in the center, more people find out about them faster. But the center also receives information faster; since people with high thresholds, "conservatives," need to know about a lot of people before participating, it might be good if they are in the center.

In Figure 14, we have three conservatives with threshold 3 and four insurgents with threshold 1 (for clarity, insurgents' thresholds are not shown). Here it is best if each conservative is "surrounded" by two insurgents; this requires that one insurgent be in the center, to be close to all three conservatives. This distribution is optimal; any other takes two periods for full revolt. Figure 14 shows that although it is not good for a conservative to be in the center, it is not good for him to be at the far periphery either. Also, when a conservative is in the center, it can delay not only his own participation but the participation of other conservatives.

Figure 14. Best to have insurgent in
the center (thresholds not shown are 1)

Say we have the same network and instead have six insurgents with threshold 1 and a single very conservative person with threshold 7 (Figure 15). Since the insurgents all have threshold 1, full revolt depends only on how quickly the conservative learns about everyone else; having the conservative in the center is optimal because the center receives information quickly. So the impact of centrality is not obvious; depending on the situation, the different roles of the center take on different importance.

Figure 15. Best to have conservative in the center (thresholds not shown are 1)

The optimal dispersion of insurgents

The question of optimal distribution of insurgents also concerns dispersion: here we illustrate the intuitive idea that having insurgents widely dispersed is good, but too much dispersion can be bad. Say that there are six insurgents with threshold 2 and six conservatives with threshold 6, and the social network is a circle, shown in Figure 16 (for clarity, only the insurgents' thresholds are shown).

Figure 16. The optimal dispersion of

insurgents (thresholds not shown are 6)

In the first case, the insurgents are extremely concentrated: since insurgents have threshold 2, any two of them revolt once they discover each other, and hence all the insurgents revolt after one period. It takes five periods, however, to achieve full revolt. Here the insurgents are ghettoized: since they are in one corner of the population, it takes a long time for others to find out about them. In the second case, the insurgents are split up into two groups, and now it takes only four periods for full revolt. The third case, in which pairs of insurgents are evenly distributed, is even better, requiring only three periods.

But when insurgents are maximally dispersed, with each conservative surrounded by insurgents, collective action is greatly hindered, requiring five periods. Since insurgents are separated from each other, it takes two periods for them to revolt. But revolt is slow not so much because the insurgents are separated but because the conservatives are separated. After three periods, a conservative knows that there are four people with threshold 2, but he requires at least one other fellow participant. So he has to rely on the conservatives two steps away, and it takes five time periods for him to make sure that they will revolt.

This example naturally applies to "vanguard" party organization (Lenin 1969) but also to marketing (Iacobucci and Hopkins 1992): if people are more likely to use a new software program if their friends use it, discount coupons, which lower thresholds, should neither be widely individually scattered nor concentrated all in one region, but should have many local concentrations, perhaps in various "niche markets."

"Spontaneous" coordination and limited communication

One of the earliest and simplest explicit models of a coordination process was introduced by Schelling (1978) and further discussed by Granovetter (1978): first, only people with low thresholds participate, but their participation makes people with slightly higher thresholds want to participate. As the number participating grows, people join in successively, in a "snowball" or "bandwagon" effect. I call this the "simple model."

Here I represent the simple model as a special case; our model might thus be considered its extension to arbitrary networks. Doing this shows that the simple model implicitly assumes that communication possibilities are extremely restricted, indeed to a maximal nontrivial degree. One compelling result of the simple model is that collective action depends

crucially on the thresholds of people "early" in its growth. I show that this fragility depends heavily on the assumption of extremely restricted communication.

In a discrete version of the simple model, we have n people and the number of people with thresholds less than or equal to m is given by the function $g(m)$. Say m_0 people participate at first; if we assume that we start off with no participation, we have $m_0 = 0$. In the next period, if a person decides to participate, he knows that the total number of participants will be at least $m_0 + 1$; hence everyone with a threshold of $m_0 + 1$ or lower will join in. Now there are $m_1 = g(m_0 + 1)$ people participating. Similarly in the second period, $m_2 = g(m_1 + 1)$ people participate. Hence we have the sequence $m_0, m_1, m_2, \ldots$, where $m_l = g(m_{l-1} + 1)$, and this nondecreasing sequence bounded above by n converges to some equilibrium m^*, where $m^* = g(m^* + 1)$. We can represent the simple model with our model and get the same result.

Result 10. Say $\theta_1 \leq \theta_2 \leq \cdots \leq \theta_n$, and $\rightarrow$ is defined as $1 \rightarrow 2 \rightarrow \cdots \rightarrow n$. Define $g(m) = \#\{i \in N : \theta_i \leq m\}$ and say m^* is the limit of the sequence $m_0, m_1, m_2, \ldots$, where $m_0 = 0$ and $m_l = g(m_{l-1} + 1)$. Then $R^\infty(\rightarrow, \theta) = \{1, 2, \ldots, m^*\}$.

In the simple model, a person knows only how many have already participated, while in our model, a person knows others' thresholds and deduces their participation; they end up with the same conclusion.

The communication network which adapts our model to the simple model is very sparse; in fact it could not be made any more sparse without disconnecting the group. Communication is ample in that a person eventually learns about everyone with a lower threshold (Braun 1995), but is restricted in that a person never learns about anyone with a higher threshold; communication is in only one direction, never reciprocal.

The effects of this sparse network are clear in an example. Say we have ten people with thresholds 1,2,3,4,5,6,7,8,9,10: first the threshold 1 person revolts, then the threshold 2 person joins in, and so on, snowballing all the way to total participation. Now change the first person's threshold from 1 to 2, so thresholds are 2,2,3,4,5,6,7,8,9,10. The equilibrium now is $m^* = 0$; no one participates. The first person no longer wants to jump in all by himself, and hence the second person doesn't follow. This result, that participation is vulnerable to small perturbations of people "early" in the bandwagon, is a central claim of Granovetter (1978).

But if the first person could talk to the second person and discover that he also had threshold 2, then the two would jump in together and the bandwagon would go all the way, just as before. That is, when $1 \to 2 \to 3 \to \cdots \to 10$, we get $R^{\infty}(\to, \theta) = \emptyset$; if we add $2 \to 1$, we get $R^{\infty}(\to, \theta) = N$.

In other words, the breakdown of collective action when going from 1,2,3,4,5,6,7,8,9,10 to 2,2,3,4,5,6,7,8,9,10 depends entirely on the assumption that communication is extremely limited. A more extreme example is when thresholds are 2,2,2,2,2,2,2,2,2,2; here the simple model predicts that there will be no collective action, even though compared to 1,2,3,4,5,6,7,8,9,10 almost everyone has a lower threshold. Here if any two people could communicate reciprocally, then at least some collective action would occur.

One might defend the simple model as applying to a spontaneous, unorganized process, explicit communication being unnecessary or unavailable. Communication is not necessarily automatic or easy; this is the premise of our model also. But it is hard to believe that a conversation or even eye contact between two people at least could not arise spontaneously. In any case, the simple model does not approximate the "general" case; when coordination takes place under the simple model, it takes place under communicative conditions so limited that no pair of individuals can reciprocally communicate. In the simple model, a person's choice depends only on the number already participating, not the possibly many eager people just waiting to jump in: there is no revolt unless someone is willing to take on the repressor utterly alone.

Collective action still can depend on slight changes in thresholds: going from 1,2,3,4,5,6,7,8,9,10 to 2,3,4,5,6,7,8,9,10,11, collective action totally collapses, regardless of how rich communication is. But it is not as fragile as the simple model suggests.

Structure and strategy combined

I hope that this paper offers an example of what structure and strategy can offer each other. What about generally?

Much work on the influence of social structure on collective action can be criticized for not being specific about what collective action is. For example, in Gould's (1993) model of social influence through fairness norms, a person's participation level, say in helping clean up the street, is basically the average of the participation levels of his neighbors. But social influence could also work through conformity; this model would then apply to completely

independent decisions, such as cleaning up one's own front yard. Adaptive models (such as Macy 1991) typically think of social structure as each person's payoff depending only on the actions of his neighbors: collective action problems are then essentially local, and hence the application to large scale mobilizations, political revolt as opposed to cleaning up the street, is unclear. Game theory is good at making these distinctions: between independent and collective decisions and between local and large scale collective action problems.

In game theory, social structure has come up in several contexts; for example, models of trade patterns (Kalai, Postlewaite, and Roberts 1978), bargaining (Myerson 1977), and coalition formation (Aumann and Myerson 1985, Kirman, Oddou, and Weber 1986) all model social structure with a network or graph. Coalition formation generally (for example Hart and Kurz 1983) in some sense has to do with endogenous social structures. Recently, social structure has been considered in adaptive contexts (Blume 1993, Ellison 1993, Young 1996a, 1996b). In general equilibrium theory, local interactions were considered early on (for example Föllmer 1974 and Allen 1982b). But social structure still is not a prominent concern of game theory or economic theory generally.

Game theory has traditionally answered the question "Can a group of people, if each individual wants to, collectively act?" in only two ways: yes or no. Cooperative theory assumes outright that coalitions can form and collectively act. Without more complex constructions (for example, Harsanyi and Selten 1988) noncooperative theory predicts Pareto dominated Nash equilibria. The much more interesting way to answer this question is "it depends"; a social network is one thing it could depend on. For collective political action at least, much empirical and theoretical work has shown that social structure is crucial. What social structure can offer, then, is a greater "real world" contextualization and richness, which could give cooperative theory better individual-level foundations and give noncooperative theory another explanatory factor in equilibrium selection.

Appendix

When there is no risk of confusion, we leave out the t superscripts for convenience. The following lemma is useful.

Lemma. Say $f \in F$ and $\theta \in \Theta$. Then $\text{argmax}_{a\in\{r,s\}} \sum_{\phi\in P_i(\theta)} \pi(\phi)u_i(\theta_i, a, f_{-i}(\phi))$ is always a singleton, which we call $BR_i(f)(\theta)$, the best response of person i to f in state θ. Also, $BR_i(f)(\theta) = r$ if and only if $\#\{j \in N\setminus\{i\} : f_j(\phi) = r\} \geq \theta_i - 1$ for all $\phi \in P_i(\theta)$.

Proof. Since π is positive and constant, we maximize the sum $\sum_{\phi\in P_i(\theta)} u_i(\theta_i, a, f_{-i}(\phi))$. If $a = s$, then $u_i(\theta_i, a, f_{-i}(\phi))$ is always 0, and hence the sum is 0. If $a = r$, then $u_i(\theta_i, a, f_{-i}(\phi))$ is either 1 or $-z$. We know $\#P_i(\theta) \leq \#\Theta = (n+1)^n$; by assumption $-z < -(n+1)^n$. Hence the sum is strictly positive, and $a = r$ is the argmax, if and only if $u_i(\theta_i, a, f_{-i}(\phi)) = 1$ for all $\phi \in P_i(\theta)$, that is, $\#\{j \in N\setminus\{i\} : f_j(\phi) = r\} \geq \theta_i - 1$ for all $\phi \in P_i(\theta)$. Otherwise, the sum is strictly negative and $a = s$ is the argmax. $\square$

Result 1. There uniquely exists an equilibrium $b^t \in F^t$ such that if $f^t \in F^t$ is an equilibrium, then $f_i^t(\theta) = r \Rightarrow b_i^t(\theta) = r$.

Proof. This follows directly from the Knaster-Tarski fixed point theorem (Tarski 1955; see also Davey and Priestley 1990): an order-preserving function on a lattice has a maximal fixed point. We set up the lattice structure and show that the best response function is order-preserving; a fixed point of the best response function is an equilibrium, and the maximal fixed point is the best equilibrium. We go through the Knaster-Tarski proof because we need it later.

Define the binary relation $\leq$ on the set of strategy profiles F: we say $f \leq g$ if $f_i(\theta) = r \Rightarrow g_i(\theta) = r$ for all $i \in N$ and all $\theta \in \Theta$. It is easy to see that $\leq$ is transitive ($e \leq f,\ f \leq g \Rightarrow e \leq g$) and antireflexive ($f \leq g,\ g \leq f \Rightarrow f = g$).

Given $E \subset F$, $E \neq \emptyset$, define $(\vee_{f\in E} f)_i : \Theta \to \{r, s\}$ as:

$$(\vee_{f\in E} f)_i(\theta) = \begin{cases} r & \text{if } \exists f \in E \text{ such that } f_i(\theta) = r \\ s & \text{otherwise.} \end{cases}$$

First show $(\vee_{f\in E} f)_i$ is measurable with respect to $\mathcal{P}_i$, and hence $\vee_{f\in E} f \in F$. Let $\theta, \theta' \in P_i \in \mathcal{P}_i$. If $(\vee_{f\in E} f)_i(\theta) = r$, then $\exists f \in E$ such that $f_i(\theta) = r$. Since f_i is measurable with respect to $\mathcal{P}_i$, we know $f_i(\theta') = r$, and hence $(\vee_{f\in E} f)_i(\theta') = r$.

Given $f \in F$, we defined $BR_i(f)(\theta)$ in the Lemma. If $\theta, \theta' \in P_i \in \mathcal{P}_i$, then $\theta_i = \theta'_i$ and $P_i(\theta) = P_i(\theta')$ and thus $BR_i(f)(\theta) = BR_i(f)(\theta')$. Hence $BR_i(f)$ is measurable with respect to $\mathcal{P}_i$ and thus $BR(f) \in F$.

Show that $BR : F \to F$ is order-preserving, that is, $f \leq g \Rightarrow BR(f) \leq BR(g)$. Say $f \leq g$. Say $BR_i(f)(\theta) = r$ and show $BR_i(g)(\theta) = r$. From the Lemma, $\#\{j \in N \setminus \{i\} : f_j(\phi) = r\} \geq \theta_i - 1$ for all $\phi \in P_i(\theta)$. But since $f \leq g$, we know $f_j(\phi) = r \Rightarrow g_j(\phi) = r$ and thus $\#\{j \in N \setminus \{i\} : g_j(\phi) = r\} \geq \theta_i - 1$ for all $\phi \in P_i(\theta)$. Hence by the Lemma, $BR_i(g)(\theta) = r$.

Let $E \subset F$ be defined by $E = \{f \in F : f \leq BR(f)\}$. We know $E \neq \emptyset$ because if we define $\tilde{f} \in F$ as $\tilde{f}_i(\theta) = s$ for all $i \in N$ and $\theta \in \Theta$, we get $\tilde{f} \leq BR(\tilde{f})$ trivially. Let $b = \vee_{f \in E} f \in F$. It is easy to see that $f \leq b$ for all $f \in E$. Since BR is order-preserving, we know $BR(f) \leq BR(b)$ for all $f \in E$. It is easy to see that therefore $\vee_{f \in E} BR(f) \leq BR(b)$. Since $f \leq BR(f)$ for all $f \in E$, it is also easy to see that $\vee_{f \in E} f \leq \vee_{f \in E} BR(f)$. Since $\leq$ is transitive, we have $\vee_{f \in E} f \leq BR(b)$. But $\vee_{f \in E} f = b$, and thus $b \leq BR(b)$. Since $b \leq BR(b)$ and BR is order-preserving, we have $BR(b) \leq BR(BR(b))$. Hence $BR(b) \in E$ and thus $BR(b) \leq b$. Since $b \leq BR(b)$ and $BR(b) \leq b$, we have $b = BR(b)$.

It is easy to see that $f \in F$ is an equilibrium if and only if $f = BR(f)$. Hence b is an equilibrium. Let f be any equilibrium: since $f = BR(f)$, we have $f \leq BR(f)$, and so $f \in E$. Hence $f \leq b$, that is, $f_i(\theta) = r \Rightarrow b_i(\theta) = r$, and b is the best equilibrium. $\square$

Result 2. Say $t \leq t'$, $\to \subset \to'$ and $\theta' \leq \theta$. Then $R^t(\to, \theta) \subset R^{t'}(\to', \theta')$.

Proof. Step 1 shows that revolt decreases in thresholds; step 2 shows that revolt increases in density; we bring everything together with the concluding step.

Step 1. Say $b \in F$ is the best equilibrium. Define $c_i : \Theta \to \{r, s\}$ as

$$c_i(\theta) = \begin{cases} r & \text{if } \exists \bar{\theta} \geq \theta \text{ such that } b_i(\bar{\theta}) = r \\ s & \text{otherwise} \end{cases}$$

where $\bar{\theta} \geq \theta$ means $\bar{\theta}_i \geq \theta_i$ for all $i \in N$. We immediately have fact 1, $b_i(\theta) = r \Rightarrow c_i(\theta) = r$, and fact 2, if $\theta' \leq \theta$, then $c_i(\theta) = r \Rightarrow c_i(\theta') = r$.

First show that c_i is measurable with respect to $\mathcal{P}_i$, and hence $c \in F$. Let $\theta, \theta' \in P_i \in \mathcal{P}_i$. By the definition of $\mathcal{P}_i$, we know $\theta_{B(i,t)} = \theta'_{B(i,t)}$. Say $c_i(\theta) = r$; hence $\exists \bar{\theta} \geq \theta$ such that $b_i(\bar{\theta}) = r$. Let $\bar{\theta}' = (\bar{\theta}_{B(i,t)}, \theta'_{N \setminus B(i,t)})$. Since $\bar{\theta}_{B(i,t)} \geq \theta_{B(i,t)} = \theta'_{B(i,t)}$, we have

$\bar{\theta}' \geq \theta$. Since $\bar{\theta}, \bar{\theta}' \in \bar{P}_i$ for some $\bar{P}_i \in \mathcal{P}_i$ and b_i is measurable with respect to $\mathcal{P}_i$, we have $b_i(\bar{\theta}') = b_i(\bar{\theta}) = r$, and hence $c_i(\theta') = r$.

Now show $c \leq BR(c)$, where the relation $\leq$ on F is defined in the proof of Result 1 and $BR : F \to F$ is defined in the Lemma. Let $c_i(\theta) = r$ and show $BR_i(c)(\theta) = r$. Since $c_i(\theta) = r$, there exists $\bar{\theta} \geq \theta$ such that $b_i(\bar{\theta}) = r$. Since b is an equilibrium, $BR_i(\bar{\theta}) = b_i(\bar{\theta}) = r$, and thus by the Lemma, $\#\{j \in N \setminus \{i\} : b_j(\phi) = r\} \geq \bar{\theta}_i - 1$ for all $\phi \in P_i(\bar{\theta})$. Since $\bar{\theta} \geq \theta$, we have $\#\{j \in N \setminus \{i\} : b_j(\phi) = r\} \geq \theta_i - 1$ for all $\phi \in P_i(\bar{\theta})$. By fact 1 above, we have $\#\{j \in N \setminus \{i\} : c_j(\phi) = r\} \geq \theta_i - 1$ for all $\phi \in P_i(\bar{\theta})$. Since $\phi \in P_i(\bar{\theta})$ can be written as $\phi = (\bar{\theta}_{B(i,t)}, \phi_{N \setminus B(i,t)})$, we have $\#\{j \in N \setminus \{i\} : c_j(\bar{\theta}_{B(i,t)}, \phi_{N \setminus B(i,t)}) = r\} \geq \theta_i - 1$ for all $\phi_{N \setminus B(i,t)} \in \{1, 2, \ldots, n+1\}^{n - \#B(i,t)}$. But since $(\theta_{B(i,t)}, \phi_{N \setminus B(i,t)}) \leq (\bar{\theta}_{B(i,t)}, \phi_{N \setminus B(i,t)})$, by fact 2 we have $\#\{j \in N \setminus \{i\} : c_j(\theta_{B(i,t)}, \phi_{N \setminus B(i,t)}) = r\} \geq \theta_i - 1$ for all $\phi_{N \setminus B(i,t)} \in \{1, 2, \ldots, n+1\}^{n - \#B(i,t)}$, which is the same thing as saying $\#\{j \in N \setminus \{i\} : c_j(\phi) = r\} \geq \theta_i - 1$ for all $\phi \in P_i(\theta)$. Then by the Lemma, $BR_i(c)(\theta) = r$.

In the proof of Result 1, we defined $E = \{f \in F : f \leq BR(f)\}$ and showed $f \leq b$ for all $f \in E$. Since $c \leq BR(c)$, we have $c \leq b$. But $b \leq c$ from fact 1 and thus $b = c$. From fact 2, we know $\theta' \leq \theta \Rightarrow R^t(\to, \theta) \subset R^t(\to, \theta')$.

Step 2. Say $\to \subset \to'$. We defined F to be the set of strategy profiles given $\to$; say F' is the set of strategy profiles given $\to'$, and note that the relation $\leq$ is well-defined over F'. Similarly, say $B'(i,t)$ is the ball given $\to'$ and $\mathcal{P}'_i$ is person i's partition given $\to'$. Since $\to \subset \to'$, we have $B(i,t) \subset B'(i,t)$ and thus $P_i(\theta) \supset P'_i(\theta)$ for all $\theta \in \Theta$; the partition $\mathcal{P}'_i$ given $\to'$ is "finer" than the partition $\mathcal{P}_i$ given $\to$. Hence if f_i is measurable with respect to $\mathcal{P}_i$, it is also measurable with respect to $\mathcal{P}'_i$. Hence $F \subset F'$.

In the Lemma, we defined the best reply function $BR : F \to F$ given $\to$; say that $BR' : F' \to F'$ is the best reply function given $\to'$. Show that for all $f \in F$, $BR(f) \leq BR'(f)$: say $BR_i(f)(\theta) = r$ and show $BR'_i(f)(\theta) = r$. Since $BR_i(f)(\theta) = r$, by the Lemma, $\#\{j \in N \setminus \{i\} : f_j(\phi) = r\} \geq \theta_i - 1$ for all $\phi \in P_i(\theta)$. But $P_i(\theta) \supset P'_i(\theta)$: hence $\#\{j \in N \setminus \{i\} : f_j(\phi) = r\} \geq \theta_i - 1$ for all $\phi \in P'_i(\theta)$, and thus by the Lemma $BR'_i(f)(\theta) = r$.

We thus have $BR(b) \leq BR'(b)$. Since b is an equilibrium given $\to$, we know $b = BR(b)$ and thus $b \leq BR'(b)$. If we define $E' = \{f \in F' : f \leq BR'(f)\}$ as in the proof of Result 1, we have $b' = \vee_{f \in E'} f$. But since $b \in E'$, we have $b \leq b'$, which means $b_i(\theta) = r \Rightarrow b'_i(\theta) = r$, or in other words, $R^t(\to, \theta) \subset R^t(\to', \theta)$.

Concluding step. Say $t \leq t'$, $\rightarrow \subset \rightarrow'$ and $\theta' \leq \theta$. Since $t \leq t'$ and $\rightarrow \subset \rightarrow'$, it is easy to see that $\rightarrow^t \subset (\rightarrow')^{t'}$. By step 2, $R^1(\rightarrow^t, \theta) \subset R^1((\rightarrow')^{t'}, \theta)$. Since $\theta' \leq \theta$, by step 1, $R^1((\rightarrow')^{t'}, \theta) \subset R^1((\rightarrow')^{t'}, \theta')$. Hence from Result 3, $R^t(\rightarrow, \theta) = R^1(\rightarrow^t, \theta) \subset R^1((\rightarrow')^{t'}, \theta') = R^{t'}(\rightarrow', \theta')$. $\square$

Result 3. $R^t(\rightarrow, \theta) = R^1(\rightarrow^t, \theta)$.

Proof. $B(i,t)$ defined with $\rightarrow$ is equal to $B(i,1)$ defined with $\rightarrow^t$. $\square$

Result 4. $i \in R^t(\rightarrow, \theta) \Rightarrow \#R^t(\rightarrow, \theta) \geq \theta_i$.

Proof. Say $b_i(\theta) = r$; since b is an equilibrium, we have $BR_i(b)(\theta) = b_i(\theta) = r$, and hence by the Lemma, $\#\{j \in N \setminus \{i\} : b_j(\phi) = r\} \geq \theta_i - 1$ for all $\phi \in P_i(\theta)$. Since $\theta \in P_i(\theta)$ and $b_i(\theta) = r$, we have $\#\{j \in N : b_j(\theta) = r\} = \#R^t(\rightarrow, \theta) \geq \theta_i$. $\square$

Result 5. If i and j are structurally equivalent and $\theta_i \leq \theta_j$, then $j \in R^t(\rightarrow, \theta) \Rightarrow i \in R^t(\rightarrow, \theta)$.

Proof. If i and j are structurally equivalent in $\rightarrow$, it is easy to see that i and j are structurally equivalent in $\rightarrow^t$. Hence by Result 3 we can assume that $t = 1$. If $i = j$, we are done trivially, so assume $i \neq j$ and let $M = N \setminus \{i, j\}$. Since i and j are structurally equivalent, we either have case 1, $i \rightarrow j$ and $j \rightarrow i$, or case 2, $i \not\rightarrow j$ and $j \not\rightarrow i$.

Case 1. Say $i \rightarrow j$ and $j \rightarrow i$. Then $B(i,1) = B(j,1)$ and hence $P_i(\theta) = P_j(\theta)$. Say $b_j(\theta) = r$. Using the "indicator" notation $I(\text{true}) = 1$ and $I(\text{false}) = 0$, by the Lemma we know $\#\{k \in M : b_k(\phi) = r\} + I(b_i(\phi) = r) \geq \theta_j - 1$ for all $\phi \in P_j(\theta)$. Hence $\#\{k \in M : b_k(\phi) = r\} \geq \theta_j - 2$ for all $\phi \in P_j(\theta)$. Since $b_j(\theta) = r$, by measurability of b_j, we know $b_j(\phi) = r$ for all $\phi \in P_j(\theta)$. So $\#\{k \in M : b_k(\phi) = r\} + I(b_j(\phi) = r) \geq \theta_j - 1$ for all $\phi \in P_j(\theta)$. But since $P_i(\theta) = P_j(\theta)$ and $\theta_j \geq \theta_i$, we know $\#\{k \in M : b_k(\phi) = r\} + I(b_j(\phi) = r) \geq \theta_i - 1$ for all $\phi \in P_i(\theta)$, which by the Lemma means $b_i(\theta) = r$.

Case 2. Say $i \not\rightarrow j$ and $j \not\rightarrow i$. Since i and j are structurally equivalent, there exists $A \subset M$ such that $B(i) = A \cup \{i\}$ and $B(j) = A \cup \{j\}$.

Say that $f \in F$ satisfies two properties: (1) $f_j(\theta) = r \Rightarrow f_i(\theta) = r$ and (2) if $k \in M$ and $\sigma \in \{1, 2, \ldots, n+1\}^{n-\#A-2}$, then $f_k(\theta_A, \sigma, n+1, \theta_j) = r \Rightarrow f_k(\theta_A, \sigma, \theta_i, \omega_j) = r$ for all $\omega_j \in \{1, 2, \ldots, n+1\}$. Show that $BR(f)$ satisfies these two properties also.

To show (1), say that $BR_j(\theta) = r$. By the Lemma, and since $B(j) = A \cup \{j\}$, we know $\#\{k \in M : f_k(\theta_A, \sigma, \omega_i, \theta_j) = r\} + I(f_i(\theta_A, \sigma, \omega_i, \theta_j) = r) \geq \theta_j - 1$ for all $\omega_i \in \{1, 2, \ldots, n+1\}$ and for all $\sigma \in \{1, 2, \ldots, n+1\}^{n-\#A-2}$. Since $f_i(\theta_A, \sigma, \omega_i, \theta_j) = s$ when $\omega_i = n+1$, we know $\#\{k \in M : f_k(\theta_A, \sigma, n+1, \theta_j) = r\} \geq \theta_j - 1$ for all $\sigma \in \{1, 2, \ldots, n+1\}^{n-\#A-2}$. Since f satisfies (2), we know $\#\{k \in M : f_k(\theta_A, \sigma, \theta_i, \omega_j) = r\} \geq \theta_j - 1$ for all $\sigma \in \{1, 2, \ldots, n+1\}^{n-\#A-2}$ and for all $\omega_j \in \{1, 2, \ldots, n+1\}$. Since $B(i) = A \cup \{i\}$ and $\theta_j - 1 \geq \theta_i - 1$, by the Lemma we have $BR_i(\theta) = r$.

To show (2), say $k \in M$, $\sigma \in \{1, 2, \ldots, n+1\}^{n-\#A-2}$, and $BR_k(f)(\theta_A, \sigma, n+1, \theta_j) = r$. Since i and j are structurally equivalent, either $B(k, 1) \subset M$ or $i, j \in B(k, 1)$. If $B(k, 1) \subset M$, then $(\theta_A, \sigma, n+1, \theta_j), (\theta_A, \sigma, \theta_i, \omega_j) \in P_k \in \mathcal{P}_k$, and thus from measurability of $BR_k(f)$, we are done. Say $i, j \in B(k, 1)$ and let $\phi \in P_k(\theta_A, \sigma, n+1, \theta_j)$. By the Lemma, $\#\{l \in M \setminus \{k\} : f_l(\phi) = r\} + I(f_i(\phi) = r) + I(f_j(\phi) = r) \geq \theta_k - 1$. Since $\phi_i = n+1$, we know $I(f_i(\phi) = r) = 0$. Since $B(j) = A \cup \{j\}$, we know $\phi \in P_j(\theta)$ and thus $f_j(\phi) = f_j(\theta) = r$ by measurability; hence $I(f_j(\phi) = r) = 1$. So $\#\{l \in M \setminus \{k\} : f_l(\phi) = r\} \geq \theta_k - 2$. Now let $\phi' \in P_k(\theta_A, \sigma, \theta_i, \omega_j)$. Since f satisfies (2), we know $\#\{l \in M \setminus \{k\} : f_l(\phi') = r\} \geq \theta_k - 2$. Since $f_j(\theta) = r$ and f satisfies (1), we know $f_i(\theta) = r$; since θ and $(\theta_A, \sigma, \theta_i, \omega_j)$ are in the same element of $\mathcal{P}_i$, we know $I(f_i(\phi') = r) = 1$. Hence $\#\{l \in M \setminus \{k\} : f_l(\phi') = r\} + I(f_i(\phi') = r) \geq \theta_k - 1$; by the Lemma, $BR_k(f)(\theta_A, \sigma, \theta_i, \omega_j) = r$.

If we define $\bar{f} \in F$ as $\bar{f}_i(\phi) = r$ for all $\phi \in \Theta$, clearly $\bar{f}$ satisfies (1) and (2). We showed above that $BR(\bar{f})$ must satisfy (1) and (2) also. Since $\bar{f} \geq BR(\bar{f})$, and BR is order-preserving, we have $BR(\bar{f}) \geq BR(BR(\bar{f}))$; iterating, we have the sequence $\bar{f} \geq BR(\bar{f}) \geq BR(BR(\bar{f})) \geq \cdots$, in which each element satisfies (1) and (2). Since the set F is finite, there must be m such that $BR^m(\bar{f}) = BR^{m+1}(\bar{f})$, and hence $BR^m(\bar{f})$ is an equilibrium. To see that it is the best, say f is an equilibrium: since $f \leq \bar{f}$, we have $f = BR^m(f) \leq BR^m(\bar{f})$ (this is another proof of the existence of b). Hence b satisfies (1). $\square$

Result 6. If there is a path from j to i and $\theta_i \leq \theta_j$, then $j \in R^t(\rightarrow, \theta) \Rightarrow i \in R^{t+d(j,i)}(\rightarrow, \theta)$.

Proof. If $i = j$, we are done because of Result 2. So let $i \neq j$ and let $M = N \setminus \{i, j\}$. Say $b_j^t(\theta) = r$. Using the "indicator" notation $I(\text{true}) = 1$ and $I(\text{false}) = 0$, by the Lemma we know $\#\{k \in M : b_k^t(\phi) = r\} + I(b_i^t(\phi) = r) \geq \theta_j - 1$ for all $\phi \in P_j^t(\theta)$. Hence $\#\{k \in M : b_k^t(\phi) = r\} \geq \theta_j - 2$ for all $\phi \in P_j^t(\theta)$.

Since $b_j^t(\theta) = r$, by measurability of b_j^t, we know $b_j^t(\phi) = r$ for all $\phi \in P_j^t(\theta)$. So $\#\{k \in M : b_k^t(\phi) = r\} + I(b_j^t(\phi) = r) \geq \theta_j - 1$ for all $\phi \in P_j^t(\theta)$.

Since there is a path from j to i of length $d(j, i)$, it is easy to see that $B(j, t) \subset B(i, t + d(j, i))$. Hence $P_i^{t+d(j,i)}(\theta) \subset P_j^t(\theta)$. We also know by Result 2 that $b_k^t(\phi) = r \Rightarrow b_k^{t+d(j,i)}(\phi) = r$. Hence $\#\{k \in M : b_k^{t+d(j,i)}(\phi) = r\} + I(b_j^{t+d(j,i)}(\phi) = r) \geq \theta_j - 1 \geq \theta_i - 1$ for all $\phi \in P_i^{t+d(j,i)}(\theta)$, and hence by the Lemma, $b_i^{t+d(j,i)}(\theta) = r$. $\square$

Result 7. Say Q is a clique of $\rightarrow^t$ and $\#Q \geq \theta_j$ for all $j \in Q$. Then $Q \subset R^t(\rightarrow, \theta)$.

Proof. Define $f_j : \Theta \rightarrow \{r, s\}$ as

$$f_j(\phi) = \begin{cases} r & \text{if } j \in Q \text{ and } \phi_Q = \theta_Q \\ s & \text{otherwise.} \end{cases}$$

Show f_j is measurable with respect to $\mathcal{P}_j$, and hence $f \in F$. If $j \in N \setminus Q$, f_j is constant and we are done. Say $j \in Q$. Let $\phi, \phi' \in P_j \in \mathcal{P}_j$; hence $\phi_{B(j,t)} = \phi'_{B(j,t)}$. Since Q is a clique of $\rightarrow^t$, we have $Q \subset B(j, t)$ and thus $\phi_Q = \phi'_Q$.

Show $f \leq BR(f)$. Say $f_j(\phi) = r$; from the definition $j \in Q$ and $\phi_Q = \theta_Q$ (hence $\phi_j = \theta_j$). For all $\omega \in P_j(\phi)$, $\omega_{B(j,t)} = \phi_{B(j,t)}$. Since Q is a clique of $\rightarrow^t$ and $j \in Q$, we know $Q \subset B(j, t)$ and hence for all $\omega \in P_j(\phi)$, $\omega_Q = \phi_Q = \theta_Q$. Hence for all $\omega \in P_j(\phi)$, $f_k(\omega) = r$ for $k \in Q$. Hence $\#\{k \in N \setminus \{j\} : f_k(\omega) = r\} \geq \#Q - 1$ for all $\omega \in P_j(\phi)$. Since $\phi_j = \theta_j \leq \#Q$, by the Lemma we know $BR_j(\phi) = r$. Hence $f \leq BR(f)$, and therefore by the proof of Result 1, $f \leq b$. Since $f_j(\theta) = r$ for $j \in Q$, we know $Q \subset R^t(\rightarrow, \theta)$. $\square$

Result 8. If $i \in R^t(\rightarrow, \theta)$, then there exists a clique $Q \subset R^t(\rightarrow, \theta) \cap B(i, t)$ of $\rightarrow^t$ such that $\#Q \geq \theta_j$ for all $j \in Q$.

Proof. Since $\rightarrow$ and t are constant throughout, for convenience write $R(\theta)$ instead of $R^t(\rightarrow, \theta)$, and $B(i)$ instead of $B(i, t)$.

Given $A \subset N$ and $\theta \in \Theta$, define $\theta[A] = (\theta_A, (n+1)_{N \setminus A}) \in \Theta$. Because $\theta[B(j)] \in P_j(\theta)$, since b_j is measurable, we have fact 1, $j \in R(\theta) \Rightarrow j \in R(\theta[B(j)])$. Because a person with threshold $n+1$ never revolts, we have fact 2, $R(\theta[A]) \subset A$.

Say $i \in R(\theta)$. Define a nested sequence $A_1 \supset A_2 \supset \cdots$ of subsets of N recursively. Let $A_1 = B(i)$. Given A_l, define A_{l+1}: if $A_l \subset B(j)$ for all $j \in R(\theta[A_l])$, then let $A_{l+1} = A_l$; if there exists $j \in R(\theta[A_l])$ such that $A_l \not\subset B(j)$, then let $A_{l+1} = A_l \cap B(j)$.

Show $R(\theta[A_l]) \neq \emptyset$ for all $l \geq 1$ by induction. Since $i \in R(\theta)$, by fact 1, $i \in R(\theta[B(i)]) = R(\theta[A_1])$, and so $R(\theta[A_1]) \neq \emptyset$. Say $R(\theta[A_l]) \neq \emptyset$ and show $R(\theta[A_{l+1}]) \neq \emptyset$. If $A_{l+1} = A_l$, we are done. Say there exists $j \in R(\theta[A_l])$ such that $A_l \not\subset B(j)$. Since $j \in R(\theta[A_l])$, by fact 1, we have $j \in R(\theta[A_l \cap B(j)]) = R(\theta[A_{l+1}])$; hence $R(\theta[A_{l+1}]) \neq \emptyset$.

Since $A_1 \supset A_2 \supset \cdots$ are subsets of the finite set N, there exists $A_l \neq \emptyset$ such that $A_l = A_{l+1}$. Hence if we let $Q = R(\theta[A_l])$, we know $A_l \subset B(j)$ for all $j \in Q$. But by fact 2, $Q \subset A_l$, so $Q \subset B(j)$ for all $j \in Q$. Hence Q is a clique of $\rightarrow^t$.

Since $\theta[A_l] \geq \theta$, by Result 2, $Q = R(\theta[A_l]) \subset R(\theta)$. We also know $Q \subset A_l \subset A_1 = B(i)$. We showed above that $Q = R(\theta[A_l]) \neq \emptyset$, so let $j \in Q = R(\theta[A_l])$. By Result 4, $\#Q = \#R(\theta[A_l]) \geq \theta_j$. $\square$

Result 9. Say $\theta = (x, x, \ldots, x)$. Then $i \in R^t(\rightarrow, \theta)$ if and only if there exists a clique Q of $\rightarrow^t$ such that $\#Q \geq x$ and $i \in Q$.

Proof. ($\Rightarrow$) Again for convenience write $R(\theta)$ instead of $R^t(\rightarrow, \theta)$ and $B(i)$ instead of $B(i,t)$. Say $i \in R(\theta)$. In the proof of Result 8, we defined A_l and showed that $Q = R(\theta[A_l])$ is a clique of $\rightarrow^t$, where $\#Q \geq x$. Since $A_l \subset A_1 = B(i)$, for all $\phi \in P_i(\theta[A_l])$, we know $\phi \leq \theta[A_l]$. Hence by Result 2, for all $\phi \in P_i(\theta[A_l])$, $R(\phi) \supset R(\theta[A_l]) = Q$ and thus $\#R(\phi) \geq \#Q \geq x$. Hence for all $\phi \in P_i(\theta[A_l])$, $\#\{j \in N \setminus \{i\} : b_j(\phi) = r\} \geq x - 1$; thus by the Lemma, $b_i(\theta[A_l]) = r$, that is, $i \in R(\theta[A_l]) = Q$.

($\Leftarrow$) This follows directly from Result 7. $\square$

Result 10. Say $\theta_1 \leq \theta_2 \leq \cdots \leq \theta_n$, and $\rightarrow$ is defined as $1 \rightarrow 2 \rightarrow \cdots \rightarrow n$. Define $g(m) = \#\{i \in N : \theta_i \leq m\}$ and say m^* is the limit of the sequence $m_0, m_1, m_2, \ldots$, where $m_0 = 0$ and $m_l = g(m_{l-1} + 1)$. Then $R^\infty(\rightarrow, \theta) = \{1, 2, \ldots, m^*\}$.

Proof. For convenience, write R^t instead of $R^t(\rightarrow, \theta)$. Note that since $\theta_1 \leq \theta_2 \leq \cdots \leq \theta_n$, if $i \leq g(m)$ then $\theta_i \leq m$ and if $i \geq g(m) + 1$ then $\theta_i \geq m + 1$. Since the length of the longest path is $n - 1$, by Result 3 we have $R^\infty = R^{n-1}$. Clearly $B(i, n-1) = \{1, 2, \ldots, i\}$ for all $i \in N$; hence if $j \leq i$, then $B(j, n-1) \subset B(i, n-1)$ and thus $P_j(\theta) \supset P_i(\theta)$ (leave off $n - 1$ superscripts for convenience).

Show $i \leq m_l \Rightarrow i \in R^{n-1}$ by induction on l. It is trivially true for $l = 0$ since $m_0 = 0$. Assume it is true for $i \leq m_{l-1}$ and show it is true for $i \in \{m_{l-1} + 1, \ldots, m_l\}$. Since for all $j \leq m_{l-1}$, $b_j(\theta) = r$, from measurability, we know for all $j \leq m_{l-1}$, $b_j(\phi) = r$ for all $\phi \in P_j(\theta)$. From earlier we know that for $i \geq m_{l-1}+1$, $P_j(\theta) \supset P_i(\theta)$. Hence for $i \geq m_{l-1}+1$, $\#\{j \in N \setminus \{i\} : b_j(\phi) = r\} \geq m_{l-1}$ for all $\phi \in P_i(\theta)$. So by the Lemma, $b_i(\theta) = r$ for all $i \geq m_{l-1} + 1$ which have thresholds $\theta_i \leq m_{l-1} + 1$, that is, for $i \leq m_l = g(m_{l-1} + 1)$.

Since $m^* = m_{l^*}$ for some l^*, we know $i \leq m^* \Rightarrow i \in R^{n-1}$. Now show by induction that $i \geq m^* + 1 \Rightarrow i \notin R^{n-1}$. Say $i = m^* + 1$; hence $\theta_i \geq m^* + 2$. Since $B(i, n-1) = \{1, 2, \ldots, i\}$, we have $\phi = (\theta_{\{1,2,\ldots,i\}}, (n+1)_{\{i+1,\ldots,n\}}) \in P_i(\theta)$. Since $f_j(\phi) = s$ for $j \geq i + 1$, we know $\#\{j \in N \setminus \{i\} : b_j(\phi) = r\} = i - 1 = m^*$. But $\theta_i - 1 \geq m^* + 1$, and thus by the Lemma, $b_i(\theta) = s$.

Now let $i \geq m^* + 2$ and say $j \notin R^{n-1}$ for $j = m^* + 1, \ldots, i - 1$. Clearly $\theta_i \geq m^* + 2$. Again define $\phi = (\theta_{\{1,2,\ldots,i\}}, (n+1)_{\{i+1,\ldots,n\}}) \in P_i(\theta)$. Since $b_j(\theta) = s$ for $j = i + 1, \ldots, n$, and we assumed $b_j(\theta) = s$ for $j = m^* + 1, \ldots, i - 1$, we have $\#\{j \in N \setminus \{i\} : b_j(\phi) = r\} = i - 1 = m^*$. But again $\theta_i - 1 \geq m^* + 1$, and thus by the Lemma, $b_i(\theta) = s$. $\square$

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