Phase Transition and Pattern Formation in a Model of Collective Choice Dynamics

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Phase transition and pattern formation in a model of collective choice dynamics

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We present a model for the emergence of collective decision in a system composed of interacting agents, each of whom are free to choose one of two possible alternatives at every time instant. The choice of each agent is influenced by those of its neighbors, as well as by its personal preference. The agent's preference, in turn, is not fixed, but adjusts to changing circumstances, based on the choices it has made previously (adaptation), as well as whether such choices accorded with those of the majority (learning). We observe a phase transition in the distribution of the collective decision in the presence of learning dynamics. The system gets polarized although individuals may continue to alternate among the choices. This indicates the existence of long-range correlations among the behavior of agents, and is observed in 2-dimensional lattices as spatial patterns in the shape of vortices or spirals. Results of our model corroborate with empirical data on movie popularity, financial markets and voting behavior.

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I. INTRODUCTION

Recently, statistical mechanics has been proposed as a tool to understand various socioeconomic phenomena by viewing them as the outcome of interaction among agents representing individuals, firms or nations (see for example, Ref. [1]). One such phenomenon, that can be viewed as an emergent outcome is the evolution of collective decision in a society where individual agents exercise their free will, partly reflected by their personal preferences, to choose between possible alternatives. However, in a real-life circumstance, an agent's preference is also determined by the information (about the possible alternative choices) it has access to. In a decentralized society, devoid of any central coordinating authority, personal preference differs from agent to agent, making the emergence of collective choice a self-organized coordination problem in a system of heterogeneous entities. An example of such a process is how certain ideas or products earn public approval and become extremely popular, while others are virtually ignored. This is seen frequently in settings as diverse as financial markets, elections, movie popularity, adoption of norms or standards, etc.

The simplest model of collective choice is the one where the agents decide independently of each other and select alternatives at random. In the case of binary choice, where only two options are available to each agent, it is easy to see that the emergence of collective choice is equivalent to a one-dimensional random walk, with the number of steps equal to the number of agents. Therefore, the outcome will be normally distributed, with the most probable outcome being an equal number of agents choosing either alternative. As a result, for most cases, there will be no strong public mandate for either choice. While such unimodal distributions of overall preference is indeed observed in some situations, a large number of real-life examples show the occurrence of bimodal distributions indicative of highly polarized choice behavior among agents. In these cases, a significantly large fraction of the population of agents vote for one alternative over the other, resulting in the most likely outcome being either strong public approval or disapproval. Examples of such bimodality has been observed in the distribution of opening gross for movies released in theaters across the USA [2], the proportion of votes cast for candidates belonging to a certain party in US federal elections [3] and the trading volume imbalance in financial markets [4,5].

The indication of polarization suggests that the agents not only opt for certain choices based on their personal preferences, but are also influenced by other agents belonging to their neighborhood. Also, their personal preferences may themselves change over time as a result of the outcome of their previous choices, e.g., whether or not their choices agreed with that of the majority. This latter effect is an example of global feedback process that we think is crucial in the polarization of public opinion. Here we propose a model of collective choice which takes into account these different effects in the choice behavior of an agent deciding between two alternatives (binary choice) at any given time instant. We observe phase transition in the distribution of the collective decision from unimodal to bimodal under various conditions, such as changing the neighborhood topology from random to regular. However, most strikingly, we observe phase transition to polarized behavior in a regular lattice topology when an agent is capable of learning to adapt its preference according to whether or not its previous choice accorded with that of the majority. One of the striking consequences of this global feedback is that although individuals continue to regularly switch between alternatives, the time for which the collective remains fixed to one alternative is found to diverge. In addition, the long-range correlations among agents manifests itself through striking spatial patterns in the lattice, e.g., vortices or spiral waves.

In the literature on collective choice, various strategies have been proposed to model the problem of group decision-making from the perspective of the individual agent's choice. For instance, in Bikhchandani et al [6], uniform social behavior of all agents in a local neighborhood (localized conformity) was observed in a model where agents choose sequentially by using the choice information of the preceding agent. Though previous work based on positive pay-off externalities [7] suggested the formation of stable coalitions, this model argues that the convergence to such a coalition will be fragile in the sense of being susceptible to rapid changes in the group behavior. However, the individuals do not have the ability to adapt based upon either their previous preferences or their information about the collective decision. Brock and Durlauf [8] have incorporated private as well as social influence on individual decision-making process in a model of theory choice (e.g., between two competing scientific theories). Social influence is modeled by having the agent place greater weight on theories which others have already accepted (conformity effect). They approach the problem through mean-field theory where there is no distinction between local (i.e., neighborhood) and global effects. Hence, social influence through conformity effect requires perfect rationality in the sense of agents having access to unlimited information which is clearly unrealistic. In Axelrod's model of dissemination of culture [9,10], individual agents are randomly matched according to similarity of preferences that promote exchange, which in turn leads to more similarity. This local convergence through the interplay between different cultural features leads to global polarization and suggests plausible explanation for state formation, transnational integration and social cleavages. However, this global convergence is purely an outcome of local polarization, where individual agents can no longer alter their choice once they are part of the majority group. Another strategy is to model the problem of collective choice as a cascade in a network of agents [11], where the decision of an individual agent depends not only on its own state and the states of its neighbors, but also upon the number of its neighbors. The interaction between the distribution of private thresholds and the degree of connectivity among agents sometimes leads to significantly large cascades and has been suggested as a possible explanation for cultural fads and technological innovations. Galam [12] has proposed a mechanism for group polarization by incorporating social pressure as an external magnetic field in a random field Ising model representing a collection of agents having individual biases. While our model also reduces to the random field Ising model in a limit, we do not introduce an external field, but rather induce transitions by coupling the individual spins to the magnetization.

Another approach towards modeling collective choice based on social percolation focuses on the adaptive nature of decision making by an individual agent. Weisbuch and Stauffer [13] propose a model where agents, having heterogeneous beliefs, are influenced by the choice behavior of their immediate neighbors, and the beliefs are, in turn, adjusted according to the choices that the agents make. The belief adaptation is such that similar choices in successive periods decrease the probability of making the same choice in the subsequent period. Increasing the rate of adaptation destroys synchrony between agents and produces smaller connected domains of similar choice behavior, with the collective decision showing small fluctuations about the average. This dynamics is incapable of exhibiting social polarization, unlike the models discussed in the preceding paragraph, because of the uncorrelated volatility in the individual agent's choice behavior.

II. THE MODEL

Here we present a general model of collective decision that, in contrast to those discussed above, shows how polarization in the presence of individual choice volatility can be achieved with an adaptation and learning dynamics of the personal preference. In this model, the choice of individual agents are not only affected by those of their neighbors, but, in addition, their preference is modified by their previous choice as well as information about how successful their previous choice behavior was in coordinating with that of the majority. An example of such limited global information obtained by an agent are the results of opinion polls disseminated through the mass media. Our model is defined as follows. Consider a population of N agents, each of which can be in one of two choice states $S = \pm 1$ (e.g., to buy or not to buy, to vote Party A or Party B, etc.). In addition, each agent has an individual preference, θ, that is chosen from a uniform random distribution initially. At each time step, every agent considers the average choice of its neighbors at the previous instant, and if this exceeds its personal preference makes the same choice; otherwise, it makes the opposite choice. Then, for the i-th agent, the choice dynamics is described by:

$$S_i^{t+1} = \mathrm{sign}(\Sigma_{j \in \mathcal{N}} J_{ij} S_j^t - \theta_i^t), \tag{1}$$

where $\mathcal{N}$ is the set of neighbors of agent i ($i = 1, \ldots, N$), and sign $(z) = +1$, if $z > 0$, and $= -1$, otherwise. The coupling coefficient among agents, J_{ij}, is assumed to be a constant ($= 1$) for simplicity and normalized by z ($= |\mathcal{N}|$), the number of neighbors. In a lattice, $\mathcal{N}$ is the set of spatial nearest neighbors and z is the coordination number, while in the mean field approximation, $\mathcal{N}$ is the set of all other agents in the system and $z = N - 1$.

The individual preference, θ evolves over time as:

$$\begin{aligned} \theta_i^{t+1} &= \theta_i^t + \mu S_i^{t+1} + \lambda S_i^t, \text{ if } S_i^t \neq M^t, \\ &= \theta_i^t + \mu S_i^{t+1}, \text{otherwise}, \end{aligned} \tag{2}$$

where $M^t = (1/N)\Sigma_j S_j^t$ is the collective decision of the entire community at time t. Adjustment to previous choice is governed by the adaptation rate μ in the second term on the right-hand side of Eq. (2), while the third term, governed by the learning rate λ, represents the correction when the individual choice does not agree with that of the majority at the previous instant. The desirability of a particular choice is assumed to be related to the fraction of the community choosing it; hence, at any given time, every agent is trying to coordinate its choice with that of the majority. Note that, for $\mu = 0, \lambda = 0$, the model reduces to the well-known zero-temperature, random field ising model (RFIM) [14].

III. RESULTS

Random neighbor and mean field model. For mathematical convenience, we choose the z neighbors of an agent at random from the $N-1$ other agents in the system. We also assume this randomness to be "annealed", i.e., the next time the same agent interacts with z other agents, they are chosen at random anew. Thus, by ignoring spatial correlations, a mean field approximation is achieved.

For $z = N-1$, i.e., when every agent has the information about the entire system, it is easy to see that, in the absence of learning ($\lambda = 0$), the collective decision M follows the evolution equation rule:

$$M^{t+1} = \text{sign}[(1-\mu)M^t - \mu\Sigma_{\tau=1}^{t-1} M^\tau].$$

For $0 < \mu < 1$, the system alternates between the ordered states $M = \pm 1$ with a period $\sim 4/\mu$. The residence time at any one state ($\sim 2/\mu$) diverges with decreasing μ, and for $\mu = 0$, the system remains fixed at one of the ordered states corresponding to $M = \pm 1$, as expected from RFIM results. At $\mu = 1$, the system remains in the disordered state, so that $M = 0$. Therefore, we see a transition from a bimodal distribution of the collective decision, M, with peaks at non-zero values, to an unimodal distribution of M centered about 0, at $\mu_c = 1$. When we introduce learning, so that $\lambda > 0$, the agents try to coordinate with each other and at the limit $\lambda \to \infty$ it is easy to see that $S_i = \text{sign}(M)$ for all i, so that all the agents make identical choice. In the simulations, we note that the bimodal distribution is recovered for $\mu = 1$ when $\lambda \geq 1$.

For finite values of z, the population is no longer "well-mixed" and the mean-field approximation becomes less accurate the lower z is. For $z << N$, the critical value of μ at which the transition from a bimodal to a unimodal distribution occurs in the absence of learning, $\mu_c < 1$. For example, $\mu_c = 0$ for $z = 2$, while it is 3/4 for $z = 4$. As z increases μ_c quickly converges to the mean-field value, $\mu_c = 1$. On introducing learning ($\lambda > 0$) for $\mu > \mu_c$, we again notice a transition to an ordered state, with more and more agents coordinating their choice.

Lattice. To implement the model when the neighbors are spatially related, we consider d-dimensional lattices ($d = 1, 2, 3$) and study the dynamics numerically. We report results obtained in systems with absorbing boundary conditions; using periodic boundary conditions leads to minor changes but the overall qualitative results remain the same. It is worth noting that the adaptation term disrupts the ordering expected from results of the RFIM for $d = 3$, so that for any non-zero μ the system is in a disordered state when $\lambda = 0$.

In the absence of learning ($\lambda = 0$), starting from a initial random distribution of choices and personal preferences, we observe only very small clusters of similar choice behavior (Fig. 1) and the average choice M fluctuates around 0. In other words, at any given time on an average equal number of agents have opposite choice preferences. In fact, the most stable state under this condition is the antiferromagnetic ground state, with neighboring agents in the lattice making opposite choices. This manifests itself as a checkerboard pattern in simulations carried out in a two-dimensional square lattice (Fig. 1, bottom). Introduction of learning in the model ($\lambda > 0$) gives rise to significant clustering as well as a non-zero value for the average choice M. We find that the probability distribution of M (Fig. 2) evolves from a single peak at 0, to a bimodal distribution as λ increases from 0. This is similar to second-order phase transition in systems undergoing qualitative changes at a critical threshold. The collective decision M switches periodically from a positive value to a negative value having an average residence time which diverges with λ and with N (Fig. 3). For instance, when λ is very high relative to μ, we see that the collective choice gets locked into one of two states (depending on the initial condition), corresponding to the majority preferring either one or the other choice. This is reminiscent of lock-in in certain economic systems subject to positive feedback [15]. The special case of $\mu = 0, \lambda > 0$ also results in a lock-in of the collective decision, with the time required to get to this state diverging as $\lambda \to 0$. For $\mu > \lambda > 0$, large clusters of agents with identical choice are observed to form and dissipate throughout the lattice (Fig. 4). After sufficiently long times, we observe the emergence of structured patterns having the symmetry of the underlying lattice (Fig. 5), with the behavior of agents belonging to a particular structure being highly correlated. Note that these patterns are dynamic, being essentially concentric waves that emerge at the center and travel to the boundary of the region, which continually expands until it meets another such pattern. Where two patterns meet their progress is arrested and their common boundary resembles a dislocation line. In the asymptotic limit, several such patterns fill up the entire system. Ordered patterns have previously been observed in spatial prisoner's dilemma model [16]. However, in the present case, the patterns indicate the growth of clusters with strictly correlated choice behavior. The central site in these clusters act as the "opinion leader" for the entire group. This can be seen as analogous to the formation of "cultural groups" with shared preferences [9]. It is of interest to note that distributing λ from a random distribution among the agents disrupt the symmetry of the patterns, but we still observe patterns of correlated choice behavior. It is the global feedback ($\lambda \neq 0$) which determines the formation of large connected regions of agents having similar choice behavior. This is reflected in the order parameter, $< |M| >$, where $<>$ indicates time averaging. Fig. 6 shows the order parameter increasing with λ in both one and two dimensional lattices, signifying the transition from a disordered state to an ordered state, where neighboring agents have coordinated their choices. The transition curves are independent of the system size N.

To get a better idea about the distribution of the most popular events (corresponding to the highest values of M) we have looked at the rank ordered plot of M, i.e., the curve obtained by putting the highest value of M in position 1, the second highest value of M in position 2, and so on. The rank distribution behavior of M shows that with $\lambda = 0$, the distribution varies smoothly over a large range, while for $\lambda > 0$, the largest values are close to each other, and then shows a sudden decrease. In other words, events are either met with significant popular approval or disapproval, when agents have limited information about the global response to an event (or alternatively, are susceptible to the media). Random distribution of λ among the agents results in only small changes to the curve. However, certain distributions of λ are seen to elevate the highest values of M beyond the trend of the curve, which reproduces an empirically observed feature in many popularity distributions that has sometimes been referred to as the "king effect" [17,18].

IV. DISCUSSION

Our model seems to provide an explanation for the observed bimodality in a large number of social or economic phenomena, e.g., in the initial reception of movies, as shown in the distribution of the opening gross of movies released in theaters across the USA during the period 1997-2003 [2]. Bimodality in this context implies that movies either achieve significant success or are dismal box-office failures initially. We have considered the opening, rather than the total, gross for our analysis because the former characterizes the uncertainty faced by the moviegoer (agent) whether to see a newly released movie, when there is very little information available about its quality. Based on the model presented here, we conclude that, in such a situation the moviegoers' choice depends not only on their neighbors' choice about this movie, but also on how well previous action based on such neighborhood information agreed with media reports and reviews of movies indicating the overall or community choice. Hence, the case of $\lambda > 0$, indicating the reliance of an individual agent on the aggregate information, imposes correlation among agent choice across the community which leads to bimodality in the opening gross distribution.

Based on a study of the rank distribution of movie earnings according to their ratings [19], we further speculate that movies made for children (rated G) have a significantly different popularity mechanism than those made for older audiences (PG, PG-13 and R). The former show striking similarity with the rank distribution curve obtained for $\lambda = 0$, while the latter are closer to the curves corresponding to $\lambda > 0$. This agrees with the intuitive notion that children are more likely to base their choices about movies (or other products, such as toys) on the choice of their friends or classmates, while adults are more likely to be swayed by reports in mass media about the popular appeal of a movie.

Our model may also provide justification for the two-phase behavior observed in the financial markets wherein volume imbalance clearly shows a bimodal distribution beyond a critical threshold of local noise intensity [4]. In contrast to many current models, we have not assumed a priori existence of contrarian and trend-follower strategies among the agents [20]. Rather such behavior emerges naturally from the micro-dynamics of agents' choice behavior. Further, we have not considered external information shocks, so that all observed fluctuations in market activity is endogenous. This is supported by recent empirical studies which have failed to observe any significant correlation between market movements and exogenous economic variables like investment climate [21].

Our model also throws light on the long-standing debate on vanishing marginals, i.e., decline in the marginal seats in the federal US congressional elections from 1956-1972 [3]. This has been ascribed to incumbency effect, leading to a wave of new electoral laws from campaign finance regulation to term limits. However, other studies have shown that although similar incumbency advantage is present in the state congressional elections in USA, there is no evidence of vanishing marginals in those elections [22]. This has been sought to be explained by introducing the concept of bandwagon effect, where the information about the majority opinion, widely dispersed in the community, causes people to alter their opinion [23]. This is equivalent in our model to having $\lambda > 0$. As seen in the results of our model, the distribution of the two-party vote will show an unimodal pattern for state elections (where local issues are often more important than the role of the media, and hence $\lambda = 0$) and a bimodal distribution for federal elections, where voters are more reliant on media coverage for individual-level voting cues (hence $\lambda > 0$).

Similar behavior possibly underlies the emergence of cooperative behavior in societies. As shown in our model, each agent can switch regularly between cooperation and defection. However, society as a whole can get trapped in a non-cooperative mode (or cooperative mode) if the global feedback is high enough. One can also tailor marketing strategies to different segments of the population depending on the role that global feedback plays in their decisions. Products whose consumers have $\lambda = 0$ can be better disseminated through distributing free samples in neighborhoods; while for $\lambda > 0$, a mass media campaign blitz will be more effective.

V. CONCLUSION

In summary, we have presented here a model of the emergence of collective decision through interactions between agents who are influenced by their personal preferences which change over time through processes akin to adaptation and learning. We find that introducing these effects produce a two-phase behavior, marked by a unimodal distribution and a bimodal distribution of the collective decision, respectively. There are multiple mechanisms for the phase transition to occur: (i) keeping the adaptation and learning rate fixed but switching from an initially regular neighborhood structure (lattice) to a random structure (mean-field) one sees a transition from unimodal to bimodal behavior; (ii) in the lattice, by increasing the learning rate λ (keeping μ fixed) one sees a transition from unimodal to bimodal behavior; and (iii) in the case where agents have randomly chosen neighbors, by increasing the adaptation rate μ beyond a critical value (keeping λ fixed) one sees a transition from bimodal to unimodal behavior.

The principal interesting observation seems to be that while individual agents constantly adapt their beliefs and hence alter their choices, the collective choice may remain polarized in either one of the two choices. This has been observed, for example, in voter behavior, where preferences have been observed to change at the individual level which is not reflected in the collective level so that the same party remain in power for extended periods. Even with randomly distributed λ (according to uniform, exponential or log-normal distributions) we see qualitatively similar results.

A possible extension of the model involve introducing stochasticity in the deterministic dynamics. In real life, the information an agent obtains about the choice behavior of others in the community is not completely reliable, as media often report news according to their own bias. This can be incorporated in the model by making the updating rule Eq. (1) probabilistic. The degree of randomness can be parameterized by a "temperature", which represents the degree of reliability an agent attaches to the information available to it. Preliminary results indicate that higher temperature produces unimodal distribution of the collective decision. Another possible extension is enabling the agents to choose from m alternatives where $m > 2$. This will allow testing the predictions of our model with many more real-life instances of collective decision formation.

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FIGURES

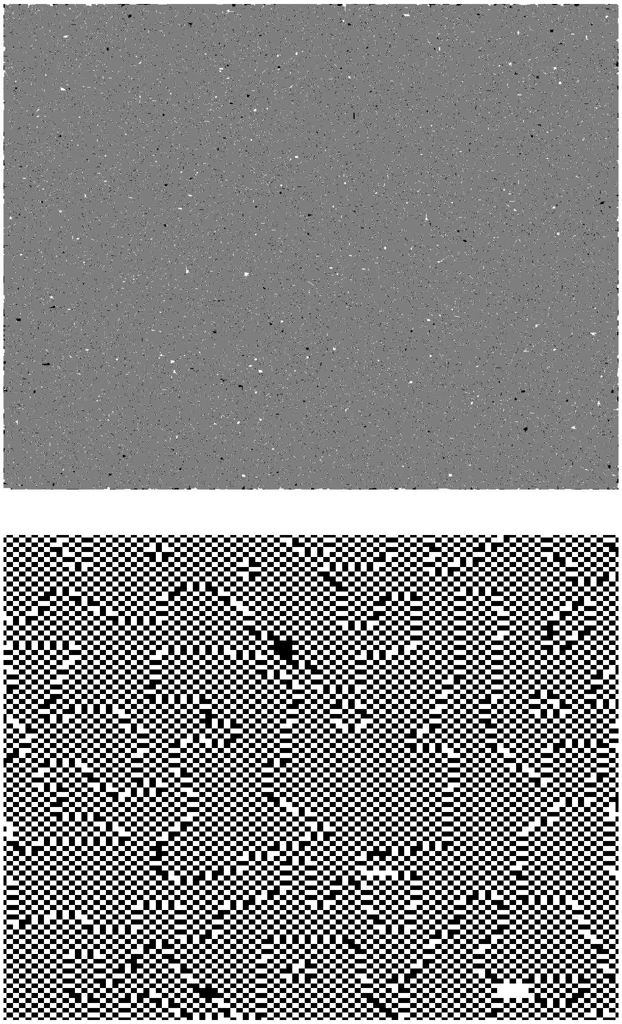

FIG. 1. The spatial pattern of choice (S) in the absence of learning ($\lambda = 0$) in a two-dimensional square lattice of 1000×1000 agents after 500 iterations starting from a random configuration (top). The adaptation rate is $\mu = 0.1$. (Bottom) A magnified view of the central 100×100 region showing the absence of long-range correlation among the agents.

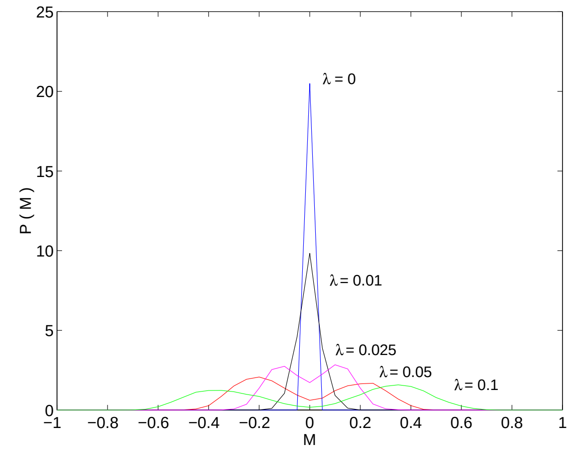

FIG. 2. The probability distribution of the collective decision M in a two-dimensional square lattice of 100×100 agents. The adaptation rate $\mu = 0.1$, and the learning rate λ is increased from 0 to 0.1 to show the transition from unimodal to bimodal behavior. The system was simulated for 5×10^4 iterations to obtain the distribution.

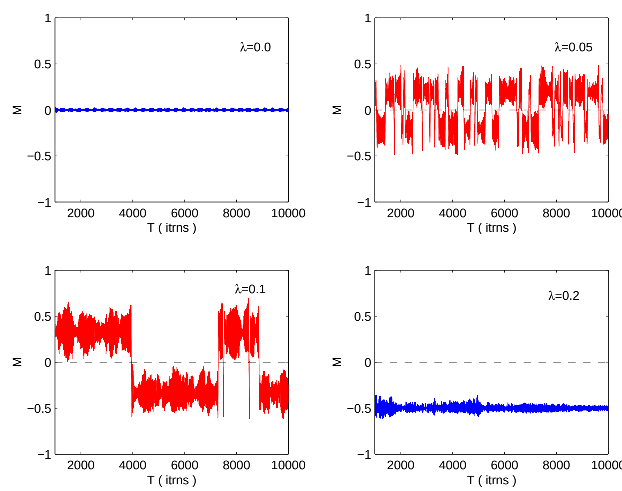

FIG. 3. Time series of the collective decision M in a two-dimensional square lattice of 100×100 agents. The adaptation rate $\mu = 0.1$, and the learning rate λ is increased from 0 to 0.2 to show the divergence of the residence time of the system in polarized configurations.

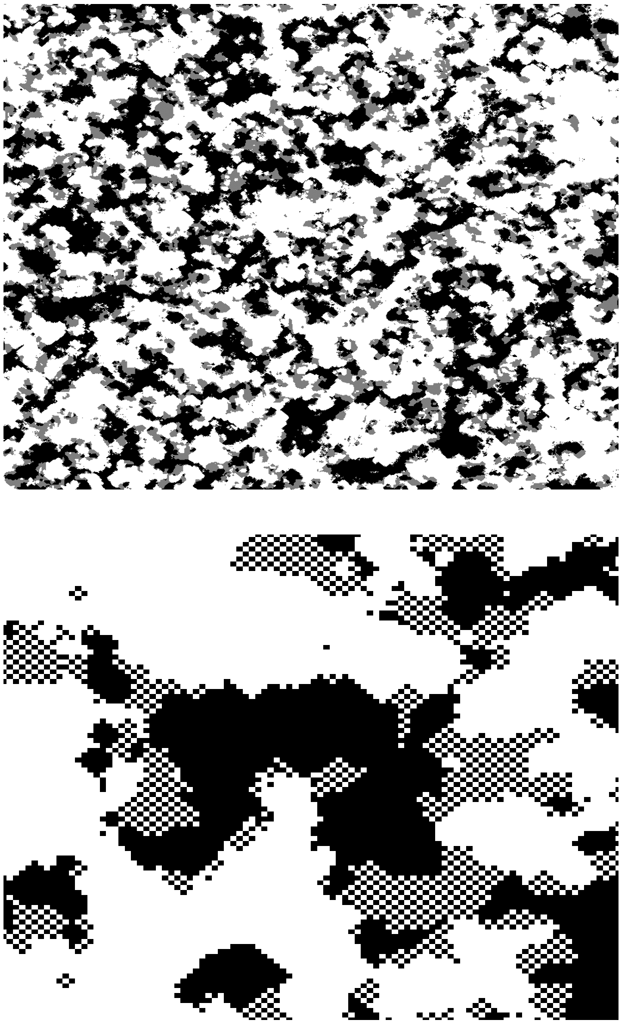

FIG. 4. The spatial pattern of choice (S) with learning ($\lambda = 0.05$) in a two-dimensional square lattice of 1000×1000 agents after 500 iterations starting from a random configuration (top). The adaptation rate is $\mu = 0.1$. A majority of agents are in the choice state $S = +1$. (Bottom) A magnified view of the central 100×100 region showing the coarsening of regions having agents aligned in the same choice state.

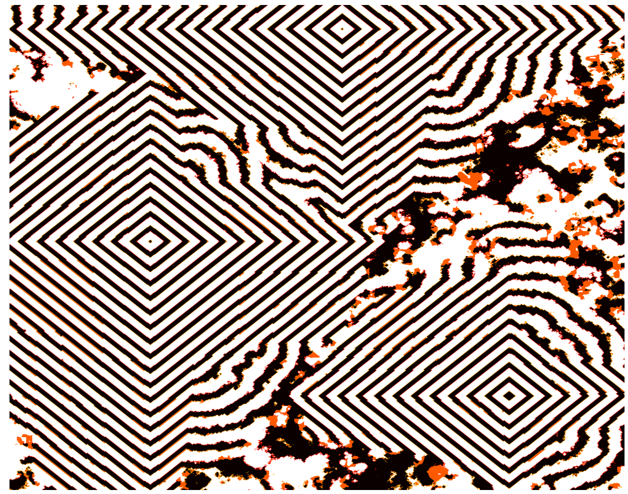

FIG. 5. The spatial pattern of choice (S) in a two-dimensional square lattice of 1000×1000 agents ($\mu = 0.1$, $\lambda = 0.05$) after 5×10^4 iterations. Note the presence of three regions of tightly correlated choice behavior among agents belonging to that region. The symmetric shape of these regions reflect the underlying square symmetry of the lattice.

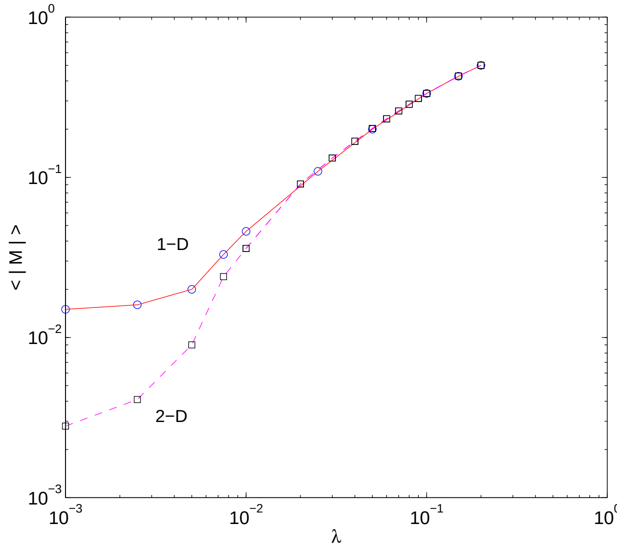

FIG. 6. The order parameter $< |M| >$ for one- and two-dimensional lattices. The adaptation rate is $\mu = 0.1$, while λ is increased gradually to show the transition to an ordered state. Note that for higher values of μ the two curves are virtually identical. There is very little system size-dependence of the curves.