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Abstract

In this paper, we address the use of adaptive computational modeling techniques in the field of political economy. The introduction considers the advantages of computational methods. The bulk of the paper describes two computational models: a spatial model of electoral competition and a Tiebout model. At the end of the paper, we discuss what the future may hold for these new techniques.

1 Introduction

There has been growing interest in the use of computer experiments in scientific inquiry in recent years. Some advocates believe that computational modeling will lead to breakthroughs and fundamentally alter how we understand both the physical and social worlds (Judd 1995; Holland 1995; Kauffman 1995). Such bold claims are met with varying degrees of skepticism. In

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our view, there are reasons to be optimistic about the use of computational modeling in building theory in the social sciences, particularly in the field of political economy. The potential contributions alone deserve attention: the construction of flexible theoretical models generating rich empirical predictions; the inclusion of dynamic analyses of social systems, including a theory of patterns; and the comparison of institutional structures in complex social environments. Several early, provocative accomplishments have reinforced our optimism and have led us to include these techniques in much of our research.¹ Certainly the grand promises of changes in the nature of science have not yet been fulfilled, and perhaps never will be. Nonetheless, we believe that the current state of the techniques calls for a tempered advocacy.

In political economy, computational techniques are especially valuable because they can complement and extend current theory. “Theory” is often associated with mathematics in the social sciences (especially economics), but it is important to separate the notions of what are useful theories from what are the tools, such as mathematics, that allow us to develop such theories. Useful theories generate accurate and testable predictions about interesting phenomena. Tools have various strengths and weaknesses, but they should be evaluated based on how well they help us construct useful theories. It is possible to use multiple tools.

The interplay between mathematical and computational theoretical investigations promises to be a growing research area (Judd 1995). Though often seen as substitutes, the two approaches overlap on many dimensions. A good example is that both approaches place high value on formally stated assumptions. Also, they often produce similar types of predictions. For example, in our spatial voting models (reported below), we generate predictions about final candidate positions in much the same fashion as mathematical models of electoral competition. Our predictions regarding end states are similar in certain respects to those of the current mathematical theory, even under different conditions than those previously modeled. However, this does not mean that computational and mathematical theory must have identical predictions. In our case, we find several contrasting results, which we then investigate empirically.

As a complement to mathematical theory, computational experiments in general are hardly controversial (Kydlund and Prescott 1996). Few social scientists dispute the usefulness of generating examples and counter-examples,

¹See especially the work of Thomas Schelling (1978) and Robert Axelrod (1986).

testing counterfactuals, and relaxing assumptions. But whether computational modeling alone can solve theoretical problems is another story. Apparent breakthroughs may turn out to be peculiarities of a technique, assumption, or algorithm which initially appears innocuous.² The risk of any one computational model being “a mere example” unfortunately exists. For example, Huberman and Glance (1994) have found that many of the interesting spatial patterns in cellular automata models disappear when updating is asynchronous, and Page (1996) has shown that when the timing of updating is incentive based, the dynamics change dramatically. Unless those using computational techniques set high standards for robustness, theoretical findings will be greeted with skepticism.

Nevertheless, the advantages of a purely computational approach to theory are extensive. Holland and Miller (1991) argue in favor of computational modeling as a good middle ground. Prior to the advent of computational techniques, social science theory relied on one of two methodologies: mathematical or verbal. The strength of mathematical analysis stems from its rigor. However, constructing mathematical models is a slow, challenging pursuit, where assumptions must often be guided by tractability rather than realism. Verbal analysis offers more flexibility and realism at the cost of reduced certainty. James Madison and Karl Marx were unconstrained by the requirements of mathematical consistency imposed upon ideas by 20th Century theorists such as Arrow and Debreu. Accordingly, the scope of analysis and the specificity of explanations in qualitative theory are far greater. An ideal tool for social science inquiry would combine the flexibility of qualitative theory with the rigor of mathematics. But flexibility comes at the cost of rigor. Holland and Miller believe that for computational models this cost is relatively low given that the computer programs guarantee that a logically consistent analysis proceeds from the encoded assumptions. They also state that the attendant gain in flexibility is large, as computer programs can encode a wide-range of behaviors. Alternative behavioral and environmental assumptions can be included quickly and at low cost in a computational model. Assumptions need not be dictated by the abilities of the researcher to prove formal claims.

Our own recent work (Kollman, Miller, Page 1995b) on federal systems of government exemplifies the flexibility of computational approaches. We began to develop a theory of using states as policy laboratories by highlighting

²Of course, mathematical theories run similar risks.

four basic characteristics of the problem: the difficulty of the policy function, the relative abilities of states and the federal government to search for innovative policies, the heterogeneity of preferences across states, and whether states could instantaneously adopt new policies or whether states could adopt policies gradually.³ None of these characteristics fits easily into a dynamic programming framework, the standard mathematical approach to modeling search for innovation (Dearden, et al. (1990)). Yet, in a computational model, we were able to include these features. We found unexpected interaction effects between the difficulty of policy functions and whether policies could be implemented by states instantaneously.

The second area in which computational modeling has advantages concerns dynamics. Many social phenomena are dynamic in nature. The best way to understand some social systems is to watch them unfold rather than to compare end states or equilibria (which may or may not exist). An array of computational techniques have been developed to explore and capture dynamical phenomena. In many instances, mathematical techniques are often strained, and computational techniques serve as neither complement nor substitute: they are the only alternative. Returning to our voting model example, mathematical models typically ignore the process of candidate position-taking over time, instead focusing on the final positions (Downs 1957). Our computational techniques enable us to make predictions about paths taken by candidates toward equilibria.

One way to gauge the contribution of computational models to theory in political economy is to compare them with rational choice and game theory models, methodologies which have been applied for a longer period of time. Though initial findings evoked interest, game theory and rational choice modeling survived a long period of indifference by many social scientists. Yet, despite recent attacks (Green and Shapiro 1994), the rational choice/game theoretic approach has to be considered a big success. It has allowed researchers to generate theoretical insights, to guide empirical investigations, and to assist in the evaluation of potential political institutions. Computational modeling has also generated provocative insights (Axelrod 1986; Axelrod and Bennet 1993; Kollman, Miller, and Page 1992) yet presently endures a mild response. Whether it will make similar or even larger contributions to social science than rational choice modeling, or whether it will collapse

³By difficult here, we mean a static, hard to solve problem. See Page (1995) for a more complete discussion of the difference between difficult and complex.

under its own weight in the manner of catastrophe theory, remains to be seen. The point is that methodological movements take time. One should not expect the discipline to embrace these new techniques, or to agree upon their contributions at such an early stage.

In the next section, we describe some of our work in computational political economy focusing on two models: a spatial model of electoral competition and a Tiebout model. We conclude with a discussion of the future of computational modeling in political economy. The description of the two models demonstrates some of the advantages of computational modeling. In the spatial electoral model, we see how computational methods can extend previous mathematical results and develop new testable hypotheses of party position-taking in two-party elections. We find strong support for these hypotheses in data from American presidential elections.⁴

In the Tiebout model, we demonstrate how computational models can lead to new insights, in this case an unexpected change in institutional performance when conditions change. When citizens can relocate to other jurisdictions, unstable electoral institutions, which perform poorly in single jurisdiction models, may outperform more stable institutions. This counter-intuitive finding can be explained by borrowing an idea from computational physics: annealing. In an annealing algorithm, the level of noise is decreased, or “cooled,” over time to help a complex dynamical system settle into a good equilibrium. In the Tiebout model, instability together with citizen relocations function as noise. The level of noise cools over time because the districts become more homogeneous, reducing the institutional instability and the incentives to relocate.

2 A Computational Agenda

In our own computational modeling research, we try to balance the generation of new theoretical insights with an attempt to understand better the methodology itself. The research described below on spatial voting models extends and challenges well-known theoretical models and leads to new, testable hypotheses. In developing these and other computational models, the most pressing methodological issues have concerned the modeling

⁴For a detailed overview of political methodology which includes a discussion of path dependency see Jackson (1996).

of adaptive, non-optimizing behavior. We shall discuss our approach and the motivations which led to it in the following subsection.

2.0.1 Modeling Adaptive Parties

The spatial voting models and the Tiebout model described below incorporate what we call *adaptive political parties* in place of the fully rational parties used in nearly all mathematical theories of political competition. Modeling parties as adaptive captures the many limitations on parties when they attempt to change platforms to appeal to voters. Standard rational choice models assume parties can maneuver virtually without limit in an issue space (Downs 1957; Kramer 1977; Enelow and Hinich 1984). In contrast, adaptive parties are limited informationally, computationally and spatially. We assume that adaptive parties' information about voters' preferences comes entirely from polling data. Parties do not know individual voters' utility functions. Nor do adaptive parties respond to their information optimally. They do not begin with Bayesian priors from which they update. Instead, they rely on heuristics, or rules of thumb, to navigate around the issue space in search of votes. Finally, adaptive parties are restricted in how far they can move in the policy space.

The depiction of party behavior as adaptive and incremental has several justifications. Here we consider two. First, parties, like other organizations, need to maintain credibility and keep diverse supporters aboard, so wild policy swings or rapid changes are unlikely (Stokes 1963; Dahl and Lindblom 1953; Bendor and Hammond 1992). Second, uncertainty over whether a policy change will lead to an increase or decrease in vote totals—the uncertainty might be a product of sampling or measurement error, or just the difficulty of the decision—makes parties tentative in changing their issue positions. Granted, if a party interprets polls as suggesting that a particular shift in a policy position will lead to a larger percentage of the vote, then the party will probably adopt the change. However, because parties do not know actual voters' preferences, when they change positions toward the median voter on one issue with the rest of the platform unchanged, they cannot be certain this will lead to an increase in vote totals. This uncertainty stems not so much from the fact that parties do not know the median position, but rather from the imperfect information and multidimensional nature of the issue space.

Precisely how to model formally the behavior of adaptive organizations is not straightforward. Undoubtedly the biggest criticism of computational

models is that findings are sensitive to particular heuristics and parameters. While optimal choices are often unique, there are typically many choices that lead to improvement. For example, a political party may be adapting in an environment in which many platforms generate higher vote totals, though typically only one will yield a maximal number of votes. This potential multitude of good choices creates the possibility of the search rule determining the predictions a given adaptive model will generate. Findings from a particular rule of thumb, such as a steepest or first ascent method, may not extend to other rules of thumb.

To guard against non-robust findings, we consider several decision heuristics and consider a wide range of parameters in our research. If we find similar results under all rules and many parameter values, we can be more secure that we are reporting general phenomena and not anomalies. Besides ensuring robustness, our search heuristics also satisfy two criteria. The first is that the rules are crudely accurate as descriptions of the behavior of actual parties both in the information they acquire and how they process it. More specifically, the amount, type, and quality of information should correspond to that which parties may actually obtain. To assume that parties rely on polling data is reasonable. To assume that they take gradients of utility functions is not. In addition, the parties should process their information in a realistic fashion: trying new policies to gather more information and updating their beliefs about voters. The second criterion is that the rules be widely used search algorithms with known strengths and weaknesses. In addition to having a class of problems for which it performs well, every search heuristic has an Achilles heel. Comparing performance across algorithms, we can try to discover which algorithms perform well in which environments. Also, if we amend an algorithm, such as appending an operator onto a genetic algorithm, we may introduce behavior which contaminates our findings.

2.0.2 Three Heuristics

In our research, we have relied for the most part on three algorithms to model adaptive party behavior: a hill-climbing algorithm, a random search algorithm, and a genetic algorithm. A hill-climbing algorithm is a sequential search algorithm. A single platform is chosen randomly in the first election as the party's status quo point. In subsequent elections, the party's platform from the previous election becomes the status quo. The algorithm proceeds in two steps. A platform is chosen in a neighborhood of the status quo and

a poll is taken comparing this platform against the opponent. If it receives more votes than the status quo, it becomes the new status quo. Otherwise the status quo remains. This process continues for a fixed number of iterations and then an election is held. A great deal is known about hill climbing algorithms. For example, they perform well on functions with low levels of difficulty and poorly on very difficult functions (Page 1995). Metaphorically, we can interpret the hill-climbing algorithm as a party which selects a candidate and then alters the candidate's platforms during the course of the campaign in response to polls and focus groups.

A random search algorithm differs from a hill-climbing algorithm in two ways. First, rather than pick one random neighboring platform, many are chosen. The new status quo is simply the the best from the group of randomly chosen platforms and the old status quo. The random algorithm is only run for one generation. Random algorithms perform moderately well on functions of varying degrees of difficulty. They outperform hill-climbing on difficult functions but are less effective on easy functions. They are also not deceived easily. An algorithm is deceived if, during the course of search, it is systematically led away from good regions of the domain. A random search algorithm represents a party which chooses a candidate from a collection of volunteers. Once the candidate is selected, her platform is assumed to be immutable.

A genetic algorithm (GA) is a population-based search algorithm (Holland 1975). A GA begins with a population of platforms all near the party's status quo. A GA proceeds in two steps: *reproduction* and *modification*. Each step captures a characteristic of biological evolution. In the reproduction step, more fit platforms—those that attract more votes—are more likely to be reproduced than less fit platforms. In the modification stage, platforms exchange blocks of positions on issues (crossover), and randomly change some positions on individual issues (mutation). Each application of reproduction and modification is called a generation. A GA is run for several generations and then the best platform in the final generation becomes the party's new platform. GAs perform well for many types of functions, usually far better than random search. Hill-climbing can outperform a GA on simple functions. Unlike random search though, a GA can be deceived. GAs perform poorly on some functions. Metaphorically, the GA represents competition and deliberation within the party. Candidates compete with the better surviving (reproduction), they share ideas (crossover), and they experiment with novel policy changes (mutation).

In each of the models we describe, our GA parties outperformed on average the other two types of parties. The differences in performance, however, were not large. The results suggest that the vote function, while not separable across issues, has sufficient regularity, that is moves towards the median tend to be good, to enable simple search algorithms to perform well. In the abbreviated presentations which follow, we restrict attention to the hill-climbing algorithm. Qualitatively similar results hold for the other two types. For a comparison of algorithms we refer readers to the original papers.

2.1 Spatial Elections

The spatial election model is a building block of formal political theory. In a series of four papers, we examine the spatial model from a computational perspective. Our research in this area has combined mathematical and computational theories with empirical testing. In our first paper (KMP 1992), we construct a computational model of adaptive parties in two-party elections. We find that parties tend towards moderate positions but that they do not converge to a single platform. In a second paper (KMP 1994a), we vary voters' preferences and discover correlations between voter characteristics and the separation or divergence of parties in the issue space. A mathematical paper (KMP 1994b) demonstrates why these correlations occur. Finally, in KMP 1995a, we test our results empirically and find support for our claims about party separation.

We begin by describing a spatial model of electoral competition. In the model we consider, each voter attaches both a strength and an ideal position to each of n issues. Voter j 's strength on issue i , $s_{ji} \in [0, 1]$, measures the issue's relative importance to the voter. For example, a voter considers an issue of strength zero irrelevant. The ideal position of voter j on issue i , $x_{ji} \in \mathfrak{R}$, denotes the voter's preferred position on the issue. The utility to voter j from a party's platform, $y \in \mathfrak{R}^n$, equals the negative of the squared weighted Euclidean distance between the vector of j 's ideal positions and the party's platform weighted by the voter's strengths:

$$u_j(y) = - \sum_{i=1}^n s_{ji} \cdot (x_{ji} - y_i)^2.$$

Voter j computes the personal utility from each party's platform and casts a ballot for the party whose platform yields the higher utility. Parties care

only about winning elections, and they compete for votes by adapting their platforms.⁵ That is, they “move,” or adapt, about the multidimensional issue space over the course of campaigns. Each election campaign begins with two parties, one of which is the incumbent. The incumbent’s platform remains fixed during the campaign while the challenger party adapts its platform.⁶

The challenger party uses information about its current popularity and applies decision rules to change its platform. Polling information during the campaign comes in the form of mock elections, indicating the percentage of votes the party would receive if an election was held at the time of the poll. A party can try a platform change and observe whether the change will lead to an increase or decrease in vote percentages. An important feature is that the polls provide noisy, or imperfect, signals of the popularity of proposed platform alterations. At the completion of the campaign, the challenger party selects a platform and the two parties stand for election with the winning party becoming the new fixed incumbent (at the winning platform) and the losing party becoming the challenger.

The movements of the parties in the issue space over the course of several elections can be monitored, thus giving a picture of the trajectory of party policies over time. An intuitive way to conceive of the process being modeled is as parties adapting on an *electoral landscape*. Each possible platform is perceived as a physical location and its corresponding vote total against the incumbent’s platform is perceived as an elevation.⁷

2.1.1 A Basic Computational Spatial Model

In KMP 1992, we analyze the behavior of adaptive parties in a two-party computational model. Our main findings are threefold. First, as previously mentioned, parties tend to converge to similar platforms that yield high aggregate utility. This appears robust to wide ranges of parameter values and methods of adaptive search. The fact that these platforms have high aggregate utility suggests that parties reliant on adaptive search rules processing limited information tend to adopt moderate platforms. They do not wander to the extremes of the issue space, even though such an outcome is mathematically possible. Instead, predictable, moderate, consensual (though distinct)

⁵In KMP 1992, we also consider ideological parties who have preferences over platforms.

⁶Initially, both parties begin with a random platform.

⁷The landscape metaphor is common in models of adaptive search. Applications to political science include Axelrod and Bennett (1993).

platforms evolve, a state of affairs familiar to observers of American party politics. Second, even though an incumbent party can always be defeated, they often remain in office. In this computational and mathematical models lead to opposite conclusions.⁸ In our computational experiments, incumbents remain in office because their adaptive challengers are unable to locate winning platforms. Third, even though parties tend to converge, the rates of convergence differ systematically.

2.1.2 Preferences, Landscapes, and Outcomes

Our computational spatial voting model (KMP 1992) suggests a relationship between voters' preferences, electoral landscapes, and outcomes. The intuition behind electoral landscapes is straightforward: parties in search of more votes try to find points of higher elevation on the landscape. Landscapes may be rugged with many local optima, or they may be smooth with one large hill. On rugged landscapes, if parties have only limited and imperfect information, and can only move incrementally, they may have difficulty finding winning platforms because local optima may lead them away from winning platforms. Furthermore, because information about a landscape is imperfect, the slope of the landscape becomes paramount. To see this, note that if a party wants to move to a higher elevation, a steep slope in one direction is easy to recognize and even imperfect information will tend to lead the party in that direction. In contrast, if a party faces a rugged, gradual slope, small polling errors can lead to big mistakes. With gradual slopes, parties may not be able to recognize which direction will lead to enough votes to win.

To learn how voters' preferences influence electoral landscapes and platform changes, in KMP 1994a we alter the distribution of preferences in empirically relevant ways, and then compare both the shape of landscapes and the resulting party behavior across distributions. Which characteristics of voters' preferences might make an electoral landscape complicated? Our model highlights one plausible set of characteristics: strengths across issues.

We introduce three types of correlations between ideal points and strengths in the model: *centrist*—voters place more weight on issues on which they have moderate views, *extremist*—voters place more weight on issues on which they have extreme views, and *uniform*—voters place equal weight on every issue. To simplify exposition, let ideal positions, x_{ji} , belong to $[-1, 1]$. The second

⁸Mathematical models often rely on an exogenous incumbency advantage to find that incumbents win re-election (Calvert 1985; Baron 1994).

Table 1: The Impact of Various Preference Types

Type of Preference	Strength	Slope of Landscape	Separation of Platforms
Centrist	$s_{ji} = (1 - x_{ji})$	steep	least
Extremist	$s_{ji} = x_{ji} $	gradual	most
Uniform	$s_{ji} = \frac{1}{2}$	mid-range	mid-range

column in Table 1 indicates how strengths are distributed for the various preference types.

The slope of a landscape is determined by how many new voters are attracted by a small position change. Suppose voters are centrist about a social insurance program. Only those who have moderate or centrist views care a lot about the issue, while those with extreme views do not make voting decisions based on this issue. Moves toward the center by a party on social insurance will likely win a party a lot of votes without losing many votes. This is because voters with ideal points near the center care a lot about the issue, while voters away from the center do not. The large number of votes to be won by moving to the center will be reflected in a large, steep hill in the center of the electoral landscape. Now say voters are extremist about abortion policies. Only those who have extreme views care a lot about the issue. Small moves toward the center on abortion will not win many votes, because voters with ideal points near the center do not care much about abortion policy. No single steep hill forms in the center of the landscape, but instead, there may be several local peaks. Each local peak corresponds to a stable policy position, and the hill supporting a local peak corresponds to portions of the platform space in which the party has incentive to move away from the center.⁹ One obvious intuition about adaptive search on landscapes

⁹Recall that voters in our model have quadratic utility functions, which amplifies the effect of change with distance. Small moves by nearby parties matter less among voters than small moves by far away parties. If voters are centrist, and parties are relatively far from the center of the issue space, small moves toward the center of the space by parties will win parties lots of centrist votes. If voters are extremist, and parties are near the

Table 2: Computational Results on Preference Types

preference type	platform separation
uniform	20.23 (0.54)
centrist	12.24 (0.38)
extremist	30.11 (0.62)

is that steep slopes should be relatively easy to climb. In the context of our model, a party confronting a steep slope should recognize improving platforms. These platforms will tend to lie near the center of the issue space, especially with centrist voters. Therefore, with centrist voters, we should expect the parties to be “closer” together ideologically than with extremist voters. The fourth column of Table 1 lists the predictions of party platform separation. The more centrist voters predominate on an issue, the more likely parties will have similar positions on the issue.

The connection between landscape slope and adaptive party behavior works just as predicted. Table 2 shows data from the sixth election of computational experiments using hill-climbing parties. The results shown are means and standard errors from 500 trials in a computational experiment using 2501 voters and ten issues. In the algorithms, parties polled 251 randomly selected voters, and parties tested forty platform alterations. We performed difference of means tests on the computer-generated data and find strong support for our conjecture about party behavior: extremist preferences lead to the most platform separation followed by uniform preferences and then centrist preferences. To supplement these findings, we also measured the slope of the actual landscape and find that it varies in the expected way. According to several measures of slope, centrist landscapes have significantly greater slope than uniform landscapes, which in turn have significantly greater slopes than extremist landscapes.

2.1.3 A Mathematical Analysis of Slope

In KMP 1994b, we found a mathematical relationship between the distribution of voters’ strengths and the slope of electoral landscapes. For reasons

center, moves by parties toward the center will win few votes, while moves by parties away from the center will win lots of extremist votes.

of tractability, KMP 1994b considers a simplified version of our model in which platform changes occur on a single issue of a multidimensional issue space. By restricting our analysis to a single dimension, we can isolate the effect of variations in preferences on landscapes. We make three assumptions about voters' ideal points. First, voter ideal points are uniformly distributed on issue 1 on $[-1,1]$. Second, voter strengths were as in Table 1. Third, for voters at each ideal point in $[-1,1]$ on issue 1, the utility difference between the challenger and the incumbent on the other $n - 1$ issues is uniformly distributed on $[-b, b]$ where $b \geq 1$. These assumptions enable us to calculate the vote total that the challenger party receives as a function of its position on issue 1, y , the incumbent's position, z , and the divergence of opinion on other issues, parameterized by b . The challenger's vote total equals the measure of the agents whose votes he receives. We can then compute the change in vote total as a function of the candidates position and we can state the following three claims.

Claim 2.1.1 *For any (y, z, b) with $|y| > 0$ the slope of a landscape formed by centrist preferences is strictly steeper than the slope of a landscape formed by extremist preferences.*

Claim 2.1.2 *For any (y, z, b) with $|y| > 0$ the slope of a landscape formed by centrist preferences is strictly steeper than the slope of a landscape formed by uniform preferences.*

Claim 2.1.3 *For any (y, z, b) with $|y| > 0$ the slope of a landscape formed by uniform preferences is strictly steeper than the slope of a landscape formed by extremist preferences.*

As these claims show, the results from the mathematical analysis agree with the findings from the computational analysis.

2.1.4 An Empirical Investigation

In KMP 1995a, we find empirical support for the computationally generated hypotheses about the relationship between voter preferences and party separation suggested by KMP 1994a and KMP 1994b. We focus on presidential elections in the United States. Recall that the models predict that the more extremist voters (in our sense of the term extremist) predominate on an issue,

the more parties will diverge or separate on that issue. The more centrist voters predominate, the more the parties will have similar positions.

Measuring the policy positions of parties at a given time is notoriously challenging. Scholars have suggested various methods of data collection and estimation, each having strengths and shortcomings depending on the purposes of research (Aldrich and McKelvey 1977; Enelow and Hinich 1984; Granberg and Brown 1992; Enelow, Mandel, and Ramesh 1988; Page and Jones 1979; Brady and Sniderman 1985; Laver and Schofield 1990). Since we are concerned with how citizens influence party behavior and how parties appeal to citizens, (that is, since perceptions of ideological distance matter every bit as much as “true” ideological distance between and among parties and voters), to investigate our model we rely on polling data measuring citizens’ perceptions of party issue positions.

In the American National Election Surveys (ANES), which are extensive, mass surveys of Americans prior to major elections, respondents are asked to place themselves, candidates, and parties on scales (usually seven-point scales) referring to ideologies and controversial political issues. In results reported in KMP 1995a we use the mean of respondents’ perceptions of a party’s position on an issue to estimate the true position. The use of seven-point scales and the use of means of respondents’ perceptions raise troubling methodological issues, and we address these issues in detail in KMP 1995a. To summarize that discussion, we agree with Brady and Sniderman (1985) that voters have reasonably accurate and stable *aggregate* perceptions of party positions on issues. Moreover, we use several different sample groups to estimate party positions, and our findings are robust over these different measures.¹⁰ Here we summarize only the results using the difference between the mean evaluations of each party among all respondents.

The ANES also contains data on respondents’ views on the most important problems facing the country. Open-ended responses were coded according to nine categories, such as foreign policy, the economy, social welfare, and racial issues. We use these data to measure strengths and how strengths relate to policy preferences.

Our measure of extremism uses three components.¹¹ The first component is *perim*, the percent of respondents who considered that issue the most

¹⁰The literature on scaling party positions is quite large. See in particular Aldrich and McKelvey (1977).

¹¹We ran all tests with several measures of extremism. All results are congruent with those summarized here.

important problem facing the country. Second, z^{sub} is the average of the absolute values of the z-scores of responses on the seven-point scale for the subpopulation considering the issue important. Note that the z-scores used in z^{sub} are calculated according to the entire set of respondents on the seven point scales and the most important problem question. As z-scores give a measure of the standardized deviation of respondents from the mean, the average of the absolute values of the z-scores will offer a single measure of the aggregate outlier status of respondents on an issue. Third, z^{tot} is the average of the absolute values of the z-scores of responses on the seven-point scale for the entire population of respondents. Putting these three components together we have

$$extremism_{ie} = \frac{perim_{ie} \cdot z_{ie}^{sub}}{z_{ie}^{tot}},$$

where $perim_{ie}$ is the percentage of voters who felt issue i was the most important issue in election e . The measure, $extremism_{ie}$, captures how the distribution of the subpopulation which cares about issue i in election e differs from the distribution of the entire population of respondents, weighted by the percentage of the population that cares about the issue. The measure will be high when extremist voters weigh heavily on the issue and low if centrist voters weigh heavily on the issue. For example, if the subpopulation is 25% more extreme than the total population, and 25% of the population thought the issue was most important, then $extremism = (1.25) \cdot (.25) = .313$.

Our model predicts that parties' divergence on a given issue will depend on the distribution of voters' strengths on that issue. An assumption of linearity between *separation* and *extremism* is a good approximation of the results of our model, and we reproduce a scatter plot which reveals a positive linear relationship between the two measures. Each point on the scatter plot is an issue, so the correlation ($r = .51, p = .001$) between the two measures offers initial support for our hypothesis. The more extremist the population on an issue, the more the parties will separate on the issue. Using the candidate measure (the difference between the means of party candidate evaluations), the correlation is still fairly strong ($r = .31, p = .05$).

Place Figure 1 Here

The linear relationship between party separation and extremism is detailed further in ordinary least squares (OLS) coefficients. The first column

of Table 3 shows the coefficients for a simple bivariate regression of *separation* on *extremism*. The coefficient is significant by standard measures and in the expected direction. Moreover, to make sure only one component of the extremism measure is not causing the correlation with *separation*, we regress *separation* on the two parts of the extremism measure. The second column of Table 3 shows OLS coefficients for *perim* and the ratio of z^{sub} and z^{tot} . These data indicate that both components are important in predicting values of *separation*. Furthermore, the two component variables are not significantly correlated ($r = -.28$, $p = .11$). Candidate separation is not presented, though for all results presented here and in KMP 1995a, using the measure of candidate separation leads to similar coefficients and errors.

These OLS coefficients may be biased, however, because the error terms among the issues are certainly correlated. The cases are issues which overlap in time (there are, for example, seven issues from the 1976 survey) and issues measured over time (the question on aid to minorities was asked for all six election surveys). In fact, our data on issues resemble panel data, which have both cross-sectional and time-series properties. Suitable procedures for analyzing such types of data (especially in cases involving missing data) are discussed in Baltagi (1995) and Kmenta (1986, Ch. 12).

Two methods for estimating panel data coefficients, essentially controlling for time and cross-section effects, are commonly discussed. A random effects model allows the intercept to vary by each subset of data, while a fixed effects model, otherwise known as a least squares dummy variable model (LSDV), sets an intercept for each subset of the data.

In Table 3 the third and fourth columns show coefficients for these two methods. As is commonly noted in the econometrics literature (Hsiao 1986, Ch. 3), coefficients between a random effects and fixed effects model vary, sometimes dramatically. However, the coefficients of *extremism* for both models are positive and significant by standard measures. Even separating the two characteristics of the extremism measure works well with the data (shown in KMP 1995a), indicating robustness of the results.

It is important to note that our main purpose in the empirical work in KMP 1995a is to demonstrate that party separation and extremism covary, controlling for the effects of time. A more fully-specified model, incorporating other pressures on parties to change—for example, pressures from activists and donors—would add further explanation for why parties take certain policy positions. We leave such questions for future research.

There is always the possibility of the results being highly sensitive to

Table 3:

Independent Variable	Party Separation (OLS)	Party Robustness (OLS)	Party Separation LSDV (OLS)	Party Separation Random Effects (REML)
<i>extremism</i>	1.97 (.46)		1.27 (.49)	.86 (.45)
z^{sub}/z^{tot}		1.30 (.70)		
<i>perim</i>		2.12 (.46)		
Cov. Par. Estimates				
<i>issue</i>				.09 (.06)
<i>year</i>				.05 (.04)
R^2	.26	.29	.20	
Adj. R^2	.24	.24	.20	
SE	.41	.41	.35	
n	33	33	33	33
Stan. Dev. Estimate				.24
REML log likelihood				-13.13
Akaike's Information Criterion				-16.13
		18		

For each entry, the first row is the OLS or maximum likelihood coefficient, and the second row is the robust standard error (White 1980).

particular measures or to the use of survey data. In KMP 1995a, we show that our results based on voters' preferences are robust when accounting for different measures and the possible contamination of results due to biased perceptions by survey respondents.

2.1.5 Discussion

Our research offers encouraging evidence in support of computational models. The original computational model (KMP 1992) generated findings in accord with empirical evidence: challengers unable to locate winning platforms, and moderate, though distinct, platforms in two-party systems. It also generated a puzzle: why do parties appear to differ more on some issues rather than other issues? This led to a more detailed computational investigation in KMP 1994a, which spawned both theoretical (KMP 1994b) and empirical investigations (KMP 1995a). We also find that more formal mathematical and empirical analyses inspired by the initial computational results, support the insights from the computational models.

2.2 A Computational Tiebout Model

In KMP 1996, we extend our computational spatial voting model to a Tiebout setting. In our Tiebout model, the polity is broken up into multiple jurisdictions within which political competition or referenda are used to determine policy. Once policies have been chosen, citizens relocate to jurisdictions with the most favorable policies for them. Ideally, as citizens sort themselves among jurisdictions according to their preferences, total utility increases. Tiebout's (1956) original formulation was an attempt to disprove Samuelson's (1954) conjecture that public goods could not be allocated efficiently. The core "Tiebout hypothesis" has since been extended to include additional propositions. Prominent among them is that Tiebout competition, as a result of enforcing efficiency, renders local politics unimportant: a political institution able to attract and retain citizens cannot waste resources, that is, it must be efficient (Hoyt, 1990). This argument does not preclude the possibility that political institutions may differ in their ability to sort citizens according to preferences, which is the focus of our model.

Our computational model allows a direct comparison of political institutions as sorting devices. Guiding our analysis is the idea that the performance of a political or economic institution depends upon its ability to structure

micro-level incentives to be in agreement with macro-level goals (Schelling, 1978). In many important situations, micro-level incentives are consistent with multiple equilibria (Axelrod and Bennet, 1993; DeVany, 1994), and one role of institutions may be to steer agents towards the best configuration, or at least bias outcomes towards better configurations.

Through computational experiments, we have found that some political institutions lead to a natural annealing process which improves sorting. Annealing is a concept unfamiliar to most social scientists, so we explain it in depth below. We find that political instability, such as voting cycles, can improve outcomes in a multi-jurisdiction environment when the degree of instability correlates positively with the heterogeneity of voters' preferences. Or, alternatively stated, institutions whose stability negatively correlates with the goodness of sort outperform more stable institutions. These findings occur under a variety of conditions and parameters in the model.

In KMP 1996 paper, we consider three institutions, two-party competition, democratic referenda, and proportional representation. Here we describe only the first two; they are sufficient to provide the primary intuition. In the formal model, we assume a set of N_a agents, each of whom resides in one of N_j possible jurisdictions. Within any jurisdiction, the local government takes positions on a set of N_i local public issues. For simplicity all such positions are binary. Examples of such issues include the presence (or absence) of a public good (say a community swimming pool) or policy (like smoking in public buildings). Let $p_{ji} \in \{Y, N\}$ give the position of jurisdiction j on issue i , and let a *platform*, $P_j \in \{Y, N\}^{N_i}$, give the vector of decisions p_{ji} across all N_i issues in jurisdiction j . Finally, define a *configuration* as a mapping of agents to jurisdictions.

Agents have linearly separable preferences on the issues and their per unit value for each issue lies in the interval $[-\frac{400}{N_i}, \frac{400}{N_i}]$ distributed uniformly. Let ν_{ai} give agent a 's per unit utility for issue i . Thus, agent a 's utility from P_j is given by

$$u_a(P_j) = \sum_{i=1}^{N_i} \nu_{ai} \cdot \delta(p_{ji}),$$

where $\delta(Y) = 1$ and $\delta(N) = 0$. A straightforward calculation verifies that the expected value to an agent of an arbitrary platform equals zero, and the expected value of her optimal platform equals one hundred.

We model democratic referenda as majority rule on each issue.¹² The as-

¹²In the case of a tie, we assume that the policy on the issue is N.

sumption that there are no external effects between projects implies that sincere voting is a dominant strategy for all agents. The outcome of democratic referenda is the *median platform*, p_j^m , for the jurisdiction, where $p_{ji}^m = Y$ if the number of agents in j with $\nu_{ai} > 0$ exceeds the number of agents in j with $\nu_{ai} < 0$; otherwise $p_{ji}^m = N$.

The platform P_j maximizes utility at jurisdiction j given a configuration if and only if on every issue the mean agent value and the median agent value have identical signs. Generally speaking, democratic referenda locate a policy of high aggregate utility given a configuration. A perceived advantage of democratic referenda is its stability: the policy prediction is unique and an individual agent migrating into or out of a jurisdiction rarely changes the median platform. We show, however, that this stability stifles sorting in multiple jurisdiction environments.

We model two-party competition using the previously discussed adaptive party model. Adaptive parties advocate policy platforms, and each agent votes for the party proposing the platform which yields her higher utility. Two-party competition is not as stable as democratic referenda, which, as we have just discussed, produces a unique outcome equal to the median voter's preference on each issue. Even with the linearly separable preferences considered here, policy predictions cannot be guaranteed to be unique without severe restrictions on preferences (Plott, 1967). In fact, the top-cycle set, a commonly used solution concept that assumes any platform that could be victorious over any other possible platform via some sequence of pairwise elections is a potential solution, can encompass the entire space.

2.2.1 Annealing and Instability

The central insight in our Tiebout paper is that the instability inherent in two party competition may be beneficial in a Tiebout model, provided that instability and utility have a negative correlation. Our argument hinges on the relationship between the level of political instability within jurisdictions and the degree of homogeneity of preferences at each jurisdiction. At this point we should clarify that at present we do not have formal proofs for many of the ideas we put forth. What follows is an informal explanation of our results and findings from computational experiments.

We begin with an example which shows how the Tiebout equilibria with respect to two-party competition may be preferred to the Tiebout equilibria with respect to democratic referenda in a multiple jurisdiction model. Prior

to describing this example, we must clarify what we mean when we say that a configuration is a *Tiebout Equilibrium with respect to an institution*. Given an institution, we first need a rule for the set of policies in each jurisdiction which can result from each configuration. For democratic referenda, this rule consists of the median policy, p_j^m . For two-party competition, we assume it consists of all platforms such that each platform belongs to the top-cycle set for its jurisdiction. Finally, a configuration is a Tiebout Equilibrium with respect to an institution if for any set of policies in the jurisdictions, no agent wants to relocate.

Example: Improved Sorting: There are two jurisdictions: α and β , eight agents: a, b, c, d, e, f, g , and h , and three issues: 1, 2, and 3. Define preferences as follows:

Preferences			
Agent	issue 1	issue 2	issue 3
a	+1	+1	+1
b	+1	-1	+0.5
c	+1	+0.5	-1
d	+1	-1	-1
e	+1	-1	-1
f	-1	-0.5	-1
g	-1	+0.5	-1
h	-1	+0.5	-1

Assume the following configuration of agents to jurisdictions: α contains agents a, b , and c , and β contains agents d, e, f, g and h . If democratic referenda is the political institution, then the policy platform in α is YYY and in β the policy platform is NNN. It is easy to show that no agent wants to relocate. Therefore, the configuration of agents and the platforms form a Tiebout Equilibrium with respect to democratic referenda. A simple calculation shows that the aggregate utility equals 4.0. We now show that two-party competition does not support this configuration of agents to jurisdictions as a Tiebout equilibrium. In two-party competition the platform YYY in α can be defeated by YNN.¹³

¹³YNN can be defeated by either YYN or YNY. Thus, these three platforms along

Note that democratic referenda are preferred to two-party competition on utilitarian grounds. The sum of the utilities to agents a , b , and c from the platform YYY equals 4.0, while the sum of their utilities of the other platforms in the top-cycle set YNN , YNY , and YYN equals 3.0, 3.5, and 3.5 respectively. If we assume for the moment that the set of policies resulting from two-party competition is randomly chosen from this set, then the lack of a unique outcome from two-party competition yields lower aggregate utility.

With multiple jurisdictions, the lack of a unique outcome from two-party competition may induce sorting and increase aggregate utility. Including jurisdiction β in the analysis, we find that two-party competition does not support this configuration of agents as an equilibrium. More importantly, this configuration lies in the basin of attraction of an equilibrium configuration which has a higher aggregate utility.¹⁴ First, notice that if the agents in α elect a candidate advocating a platform from the set $\{YYY, YYN, YNY\}$, they will not create a strict incentive for any agents from β to move to α . If, however, a candidate wins election in jurisdiction α advocating the platform YNN , then agents d and e strictly prefer jurisdiction α to jurisdiction β , and they will relocate. Jurisdiction α would then contain agents a , b , c , d , and e and YNN would be the unique member of the top-cycle set (as well as the median platform). Jurisdiction β would now contain agents f , g , and h and have the median platform NYN as the unique member of the top-cycle set. It is easy to show that this configuration of agents is a Tiebout Equilibrium with respect to two-party competition. It is also a Tiebout Equilibrium with respect to democratic referenda. More importantly, the aggregate utility of this configuration equals 5.5 which exceeds the aggregate utility of 4.0 from the previous configuration.

To provide deeper understanding of why this example is representative of a general phenomenon and not an anomaly, we can rely on insights from research applying *simulated annealing* to nonlinear optimization problems. Simulated annealing is a sequential search algorithm applied to a real valued function, f . A simulated annealing algorithm begins by assuming a neighborhood structure over the function's domain. Each point, x , in f 's domain belongs to a neighborhood, $N(x)$, which contains at least one point different from x . Simulated annealing works much like hill climbing: at each step a

with YYY form the top-cycle set.

¹⁴By basin of attraction we mean the set of connected configurations that lead to higher aggregate utility than the status quo.

new point in the neighborhood of the status quo point is tested and becomes the status quo point if it has a higher value under f . Where simulated annealing differs from hill climbing is that it also accepts points which have lower values with some probability. This probability depends on the difference in the function values, $\Delta(x, \hat{x}) = f(x) - f(\hat{x})$, and a *temperature*, $T(t)$, which is a decreasing function of the time spent searching, t . Formally, if x is the status quo point and \hat{x} is the lower-valued neighboring point, the probability of acceptance is written:

$$p(x, \hat{x}, t) = e^{\frac{-\Delta(x, \hat{x})}{T(t)}}$$

Notice that if the difference in function values, $\Delta(x, \hat{x})$, is large relative to the temperature, $T(t)$, then the probability of acceptance is low. The temperature can be interpreted as the degree of leniency. A high temperature allows for almost any new point to be accepted. A low temperature allows for a new point to be accepted only if the difference in function values is small. The temperature decreases to zero according to an *annealing schedule*. When the temperature nears zero, search converges to a local optimum with respect to the neighborhood structure.

A substantial body of theory exists to explain the performance of simulated annealing (*Algorithmica* (1991) Vol. 6). Essentially, simulated annealing locates good local optima because mistakes bias search away from local optima with low values and onto local optima with high values. This is more likely to occur when optima with low (high) values have small (large) basins of attraction. In such spaces, simulated annealing performs “as if” it can recognize whether a local optimum’s value was relatively high or low, and escapes lower valued local optima. We propose that two-party competition performs similarly in a Tiebout model: *two-party competition makes more (fewer) mistakes in relatively low (high) valued configurations enabling it to act ‘as if’ it recognizes the value of a local optima and to escape it (remain in it) if it has a low (high) value.*

Two characteristics of Tiebout competition generate this fortuitous bias in error-making. First, aggregate utility for a configuration of agents is positively correlated to the homogeneity of preferences at each jurisdiction. Second, more homogeneous preferences result in more stable outcomes from two-party competition.¹⁵ Combining these two effects, if agents in a configuration are not very homogeneous at each jurisdiction, their aggregate utility

¹⁵An aspect of this second characteristic has been addressed formally by McKelvey

will be low and two-party competition may tend to roam about the space of platforms rendering the configuration unstable. If the agents in a configuration are homogeneous at each jurisdiction, their aggregate utility will be high and two-party competition will select from a small set of platforms. This second configuration is more likely to be stable.

2.2.2 Computational Findings

The computational model begins with a procedure in which agents' preferences are created and in which agents are assigned randomly to jurisdictions, with each agent being assigned to each jurisdiction with equal probability. We next begin a series of relocation decisions by agents and platform decisions by the political institutions until agents have had ten opportunities to relocate. For some political institutions, an equilibrium will have been attained at this point.

Following standard Tiebout models we assume that an agent moves to the jurisdiction providing her the highest utility. As mentioned previously, mathematical theory does not provide a unique platform prediction for the two-party model, except in special cases. Therefore, we rely on the adaptive party model described earlier in this paper. The findings below are from hill-climbing parties which take forty polls each of the entire population of their district. The total number of citizens equals one thousand.

We first consider a single jurisdiction model. The findings are not surprising. Democratic referenda outperforms two-party competition. Democratic referenda produces an outcome of the median on each issue, which produces nearly maximal utility given our assumptions on the distribution of preferences. Democratic referenda had an aggregate utility of 2.49, while two-party competition had an aggregate utility of 1.84.¹⁶ The differences are statistically significant at a 99% level.

With multiple jurisdictions, agents can sort according to their preferences. Tiebout's theory predicts that aggregate utility will increase with the

(1986) who showed that the size of the uncovered set decreases with the level of symmetry of preferences.

¹⁶The numerical findings we present here are from fifty trials with one-thousand agents and eleven issues. Utilities have been normalized so that the expected maximal utility to an agent equals one-hundred and the expected utility to an agent of a randomly selected platform equals zero. These findings appear to be robust to large variations in parameter values and search heuristics.

Table 4: Multiple Jurisdictions: Utility

Institution	3 Loc's	(s.d.)	7 Loc's	(s.d.)	11 Loc's	(s.d.)
Democratic Ref.	34.40	(2.31)	48.51	(2.04)	55.64	(2.05)
Two-party Comp.	34.74	(1.83)	49.35	(1.74)	56.64	(1.96)

Table 5: Number of Agent Relocations

Institution	3 Loc's	(s.d.)	7 Loc's	(s.d.)	11 Loc's	(s.d.)
Democratic Ref.	864.16	(121.90)	863.2	(73.55)	887.3	(37.22)
Two-party Comp.	915.68	(103.71)	1162.5	(61.03)	1293.7	(50.43)

number of jurisdictions. We present findings from models with three, seven, and eleven jurisdictions, and for both institutions considered, aggregate utility increases with the number of jurisdictions. The increases from the one jurisdiction model to the three jurisdiction model are most dramatic. More importantly, the *performance of the political institutions reverses*: two-party competition now outperforms democratic referenda. These differences in aggregate utility (standard errors of the distributions in parentheses) are also statistically significant at a 99% level.

Earlier we described how, with multiple jurisdictions, two-party competition might induce more sorting of agents. Configurations of agents which had relatively low aggregate utility would be less likely to be stable with respect to two-party competition. A small policy change might lead to a small migration, which in turn might lead to yet another policy change and still more migration, until eventually a stable configuration has been located. One approach to capturing the extent of sorting is to keep track of the total number of agent relocations for each political institution. Two-party competition results in significantly more relocations than democratic referenda with three, seven, and eleven jurisdictions.

Our argument as to why two-party competition outperforms democratic referenda boils down to a conjecture that instability is positively correlated with the degree of heterogeneity of voters' preferences. To test whether this is true, we create populations of agents of varying degrees of heterogeneity. We then measure the number of changes in platforms over time and verify whether the number of platform changes increases with the heterogeneity of

preferences. We manufacture populations of agents with specified levels of heterogeneity as follows: each agent’s ideal point was made a convex combination of a *base preference* and an *individual preference*. Both an agent’s individual preference and the common base preference were drawn from the same distribution as agents’ preferences in the basic model. We can write an agent’s preferences as $(1 - \theta) \cdot \text{base} + \theta \cdot \text{individual}$. We refer to θ as the *degree of heterogeneity*. We examine a one jurisdiction model with two hundred and fifty agents. The restriction to one jurisdiction guarantees that none of the changes in platforms are attributable to the shifting population. We vary θ between 0.5 and 1.0.¹⁷ We define the variable *flips* to be the number of policies which change in each election. For democratic referenda, the number of flips equals zero for all θ . This occurs because with a fixed population, democratic referenda results in a unique outcome, and in this scenario no agents move into or out of the jurisdiction. For two-party competition, a different picture emerges: at $\theta = 0.5$, the number of flips is approximately one-quarter (0.24), at $\theta = 0.8$, the number of flips increases to over one-half (0.53), at $\theta = 0.95$, the number of flips exceeds two and a one-half (2.65), and finally, at $\theta = 1$, the number of flips is nearly three and a half (3.36).¹⁸ As expected, the average number of flips increases with the degree of heterogeneity. These findings strongly support our assumption that as preferences in a jurisdiction become more homogeneous the propensity for platforms to wander decreases.

2.2.3 Discussion

In constructing a computational Tiebout model, we find that institutional instability can generate better sorting provided that the instability is correlated negatively with the homogeneity of preferences. This result, though counter-intuitive to social scientists, may not be surprising to those who study nonlinear systems. The institutions we model mimic a class of annealing procedures which have been found to be effective in finding good solutions to highly nonlinear problems. Thus, the computational model reveals a new insight (an insight, for which other computational results provide an explanation). Tiebout models are just one example of a much broader

¹⁷Values of θ lower than this create agents whose preferences are too homogeneous: almost all have the same preferred platform.

¹⁸The findings presented are from fifty series of ten elections. The differences reported are significant by standard statistical measures.

class of models in which agents sort among alternative configurations. These include models of coalition formation and economic organization. Under decentralized sorting mechanisms, these systems can get trapped in suboptimal configurations. If, however, there are means by which these poor configurations can be annealed in an appropriate manner, then the global system can escape these configurations and achieve superior outcomes. Whether other institutions also naturally anneal, or whether institutions with built-in annealing features can be constructed, is a potential research area within computational social science.

3 The Future of Computational Models in Political Economy

Adaptive computational models in political economy will likely become more prevalent because they offer several advantages to researchers. The models provide a flexible, yet precise, means of capturing a variety of phenomena. In particular, they allow the creation of dynamic models of adaptive agents. The ability to analyze adaptive behavior opens up important territories for research, permitting the creation of new classes of models that cannot be analyzed with previous tools. Moreover, such models also have the potential to elucidate and enhance existing theoretical tools. For example, computational techniques can be used to define the limits of rational choice approaches by demonstrating where static equilibrium models and dynamic adaptation models diverge. Irrespective of these advantages, the use of computational models is likely to increase if for no other reason than the changing relative cost advantages of computational versus mathematical modeling.

In this paper we have summarized our use of computational methods in the context of two basic models in political economy. Our decision to focus on well-known problems is not coincidental. Only by returning to the basic models that define our disciplines and challenging the fundamental assumptions of these models will computational approaches lead to real breakthroughs. By studying what we think we already know, we reveal the strengths and weaknesses of both computational and mathematical approaches. Computational approaches can change the way social scientists understand social phenomena by forcing them to reconsider core assumptions.

While the advantages of computational models make a compelling case

for their implementation, practitioners must address important methodological issues. The scientific validity of such models can be evaluated only if methodological norms are established. Computational modeling is no different than any other scientific enterprise; there are instances of brittle results being driven by both ignorance and political agendas. With time, the ability of the scientific community to evaluate these models will be enhanced.

In our own work, we have handled potential shortcomings with a computational approach in a variety of ways. As discussed, we have incorporated multiple adaptive algorithms to establish the robustness of our results. We have analyzed the effect of issues such as timing on updating rules (Page 1996). The fact that such changes can have an impact on theoretical predictions is an important lesson for both computational and traditional modeling efforts. (In particular, traditional modeling often trivializes issues of timing and procedure by, for example, assuming instantaneous information updating.) We have also concentrated our modeling efforts on the creation of simple computational worlds that provide a framework within which the adaptive agents can “fill-in” the details. For example, in Miller (1988, 1996), rather than defining a set of “black-box” strategies for the agents, a genetic algorithm was used to allow agents to adaptively select their strategies from an enormous set of potential strategies. Finally, we have examined the importance of heterogeneity in strategy encodings and in the heuristics used to locate strategies (Hong and Page 1995). Our efforts and those of others are far from comprehensive. At present little consensus exists as to what makes a robust computational model different than just “a bunch of examples.” In the future, the ability to confront and solve methodological problems will be as important to the acceptance of computational models as the ability to generate substantive theoretical breakthroughs.

We are neither advocating nor predicting a dramatic change in the social sciences, where computational techniques replace mathematical techniques. Instead, we foresee the future of computational modeling as a steady development of new methods that extend and occasionally overturn existing theoretical insights. Perhaps the greatest contribution of computational methods will be an indirect one. Such models allow theorists to interact with a convenient “artificial world.” Such interaction extends a theorist’s intuition and allows the computer to serve as a collaborator, in both the inspiration and refinement of theoretical ideas. By introducing new principles and conceptual tools, computational modeling will lead social scientists in valuable new directions.

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