SFI TRANSMISSION

COMPLEXITY SCIENCE FOR COVID-19

STRATEGIC INSIGHT: **The countervailing pressures of economic pain and disease containment are keeping the COVID-19 pandemic at a noisy equilibrium.**

FROM: **Jon Machta, University of Massachusetts Amherst; Santa Fe Institute**

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There's a magic number in epidemic modeling. When it exceeds 1, new infections grow exponentially, spreading like wildfire. When it falls short of 1, the virus succumbs to exponential death.

That basic reproductive number, R_0, is simply the average number of new infections each existing case will spawn. If every coronavirus victim infects, on average, two new people, R_0 (pronounced "r-naught") is 2 and the virus doubles its way around the globe. If R_0 is near zero, whether due to the host's immune system or lack of contact with others, the virus dies out quickly.

R_0 equaling 1 is a critical equilibrium. Here the caseload stays constant on average but is subject to large fluctuations.[1] In this essay, we argue that R_0 for COVID-19 in the U.S. is likely to hover near 1 or oscillate around 1 for a lengthy period. This is an example of a phenomenon known as "self-organizing criticality," which is observed in many complex systems. Another example of self-organized criticality is seen in earthquakes. Here the inexorable motion of tectonic plates is balanced by friction until some part of the fault ruptures and moves suddenly. The result is a critical equilibrium with frequent small earthquakes and rare large earthquakes.

The natural value of R_0 for COVID-19 in a naïve population appears to be above 2, leading to rapid spread in the absence of mitigation. Mitigation efforts have brought R_0 below 1 in many countries but at great economic cost. In some countries such as Taiwan and New Zealand, very effective mitigation has brought case numbers to a low enough value that relatively inexpensive measures now suffice.

However, in the U.S. with its large and heterogeneous population and weak central leadership on this issue, suppressing R_0 below 1 is currently extremely costly and likely to remain so for the foreseeable future. In this regime, where mitigation is extremely expensive but having an exponentially exploding number of cases is also unacceptable, R_0 will remain near 1. If R_0 exceeds 1, then exponential growth quickly leads to unacceptable rates of infection, hospitalization, and death, but as R_0 is suppressed by the crude tools of social distancing and business closures, the economic pain creates great pressure to open up the economy, pushing R_0 back above 1. The combination of these two forces generically and robustly keeps R_0 close to the critical value of 1. Many current models of the epidemic include both the biology of infection and policy responses in more or less detail[2] (for example, see [2]) but it is useful to understand that these models will display the general feature of self-organized criticality so long as the two strong and opposing forces are incorporated.

A simple, two-variable class of dynamical models demonstrates the idea of self-organization to $R_0 = 1$. These models cannot be used to make quantitative predictions, but they reveal important qualitative features. The two variables are C(t), the number of new infections at time t, and $R_0(t)$, the (time-dependent) reproductive number. Roughly speaking, time is measured in weeks. The equation for the number of new infections, $C(t+1) = R_0(t)C(t)$, is simply the definition of R_0. The dynamics of the reproductive number is the new ingredient, and it takes the form $R_0(t+1) = R_0(t) + F[C(t), R_0(t)]$. There is wide latitude in constructing the function $F[C,R_0]$. It should be negative when the number of new infections is above an acceptable threshold, pushing the system toward mitigation. $F[C,R_0]$ should be positive when R_0 is below 1, leading to large economic costs that push the system away from mitigation. Depending on the details of the function F and the initial conditions, there are two generic behaviors. Either $R_0(t)$ will evolve to the critical value of 1 and C(t) will settle to a steady state value given by solution to $F[C, 1]=0$. The result from a simulation of a specific choice of F and initial conditions is plotted Fig. 1.

Alternatively, for other choices of F and initial conditions, $R_0(t)$ oscillates around 1 and C(t) oscillates as shown in Fig. 2. Figure 3 shows the behavior or $R_0(t)$ for the two cases of Fig. 1 and 2. Finally, one can add noise to either equation in various ways. When weak noise is added to the system, these qualitative behaviors persist but become quite noisy, as shown for specific simulations in Fig. 4 and 5.

In conclusion, one can expect R_0 to stay near 1 and the number of new cases to stay relatively high for an extended period, but with lots of unpredictable and perhaps oscillatory behavior along the way. The only way out is with an effective treatment or to dramatically lower the cost of an R_0 below 1 through natural herd immunity, a vaccine, or effective testing and contact tracing.

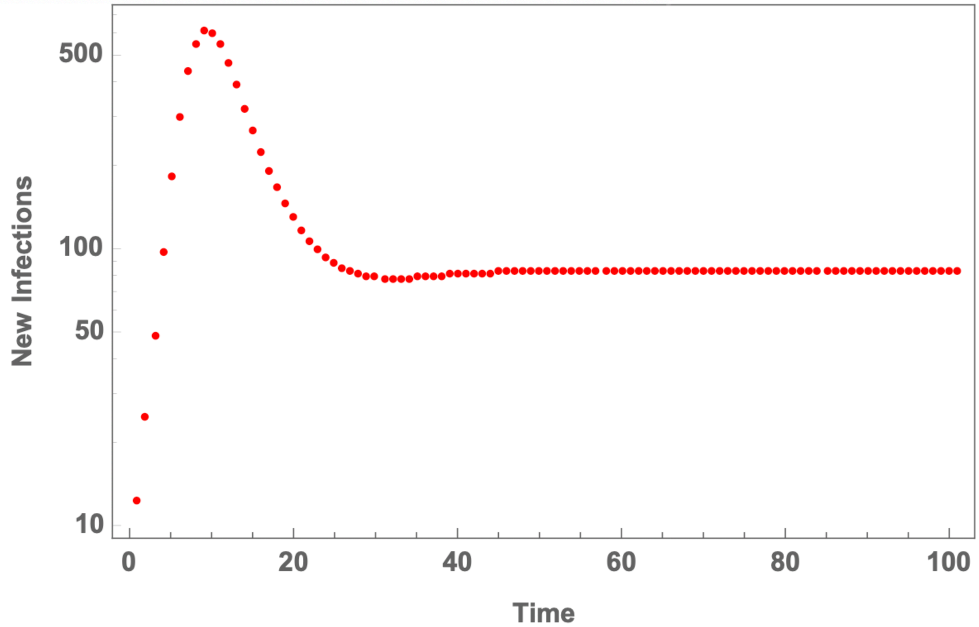

Figure 1: *New infections vs time (steady)*

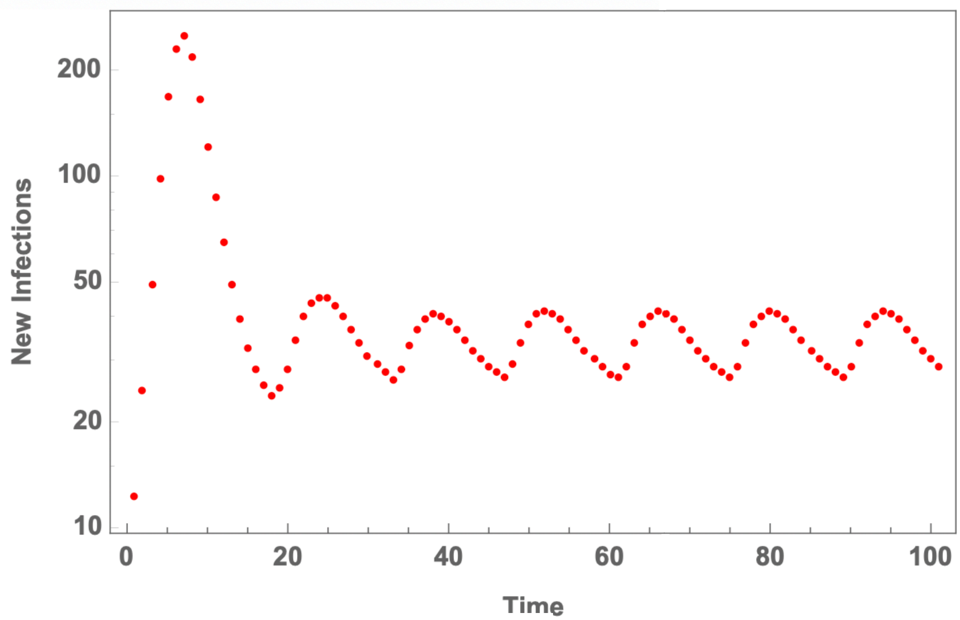

Figure 2 *New infections vs time (oscillatory)*

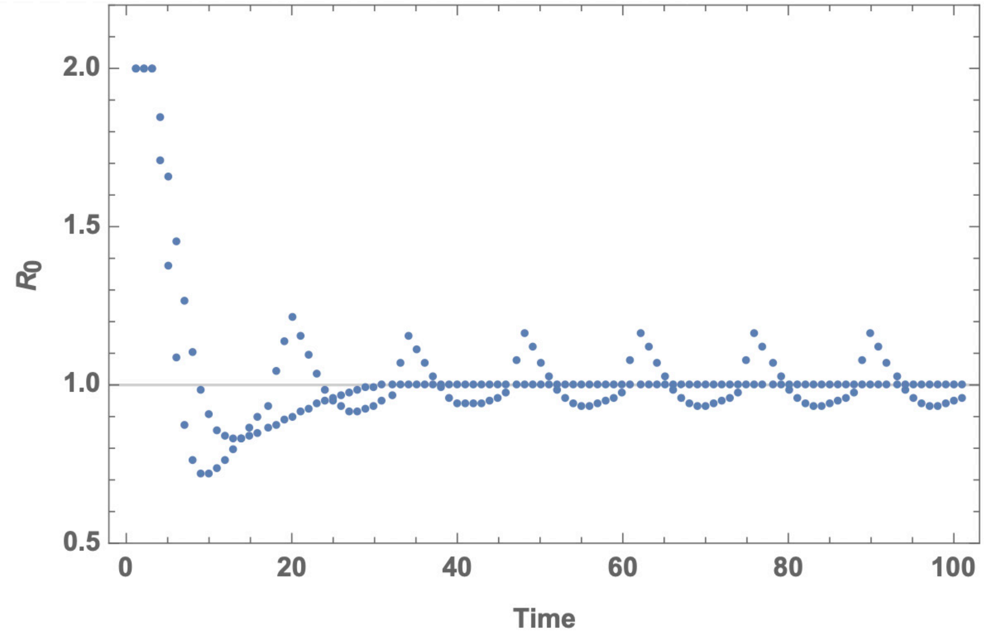

Figure 3 *Reproductive number vs time (both steady and oscillatory regimes shown)*

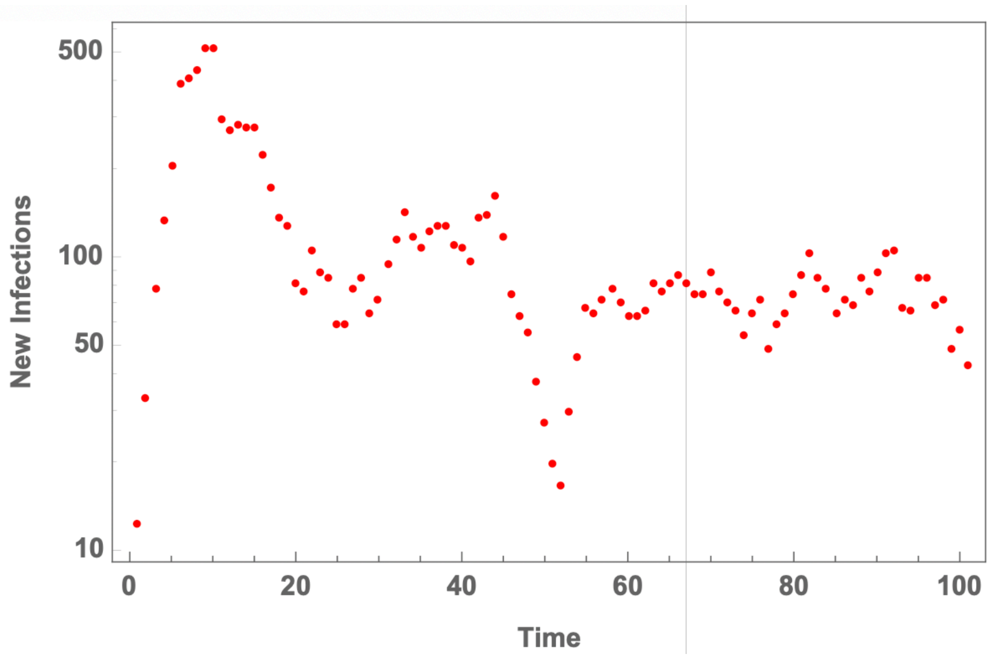

Figure 4 *New infections vs time (same as Fig. 1 but with noise)*

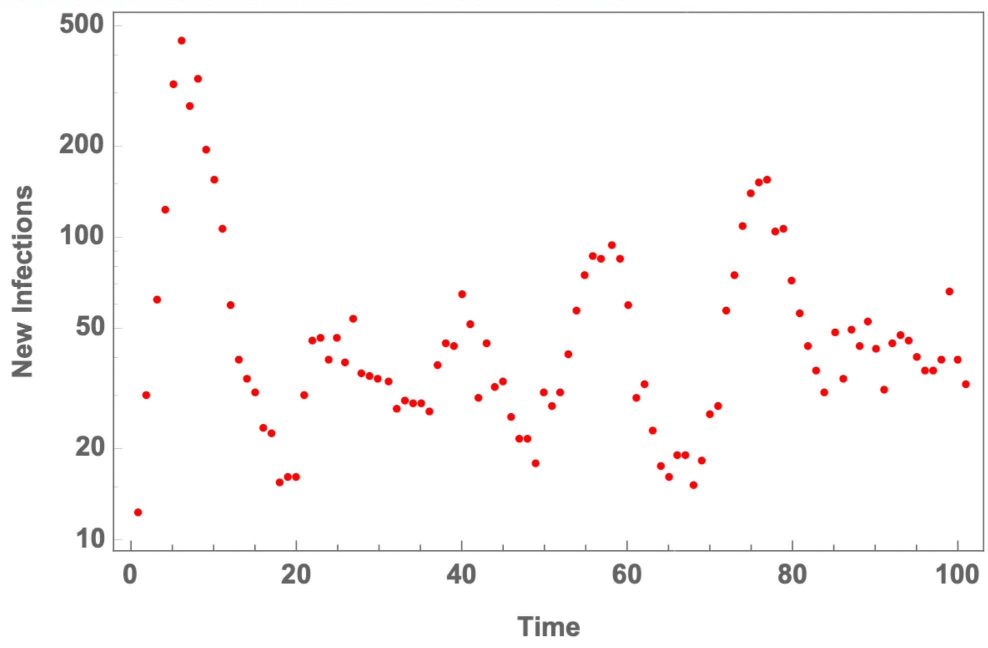

Figure 5 *New infections vs time (same as Fig. 2 but with noise)*

ENDNOTES

1 Moore, TransmissionT-024 (2020); Redner, Transmissions T-028 (2020)

2 Kissler et. al., "Projecting the transmission dynamics of SARS-CoV-2 through the postpandemic period", *Science*, published online April 24, 2020.

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